MODERN DECISION MAKING

A GUIDE TO MODELING WITH DECISION SUPPORT SYSTEMS

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Samuel E. Bodily
The Colgate Darden Graduate School
of Business Administration
University of Virginia

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To Jolene,
Adam, Diana, and Jill

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PREFACE

For some time now, the needs of the generalist manager for guidance in creating quantitative models have not been met. Some books prepare people with strong mathematical skills to work as specialists. Other books give generalists an *appreciation* of models, but not the skills to *create* models. What has been missing is a book that can help generalists learn to do their *own* modeling in support of their *own* management responsibilities.

The need for such materials has now become particularly acute and the opportunity especially ripe because of recent developments in computer decision support software and the prevalence of computing power in the manager's office and home. If you need a computer model, you no longer need to hire a consulting firm or bring in company staff to write a model in Fortran. You can create the model in your office and run it in minutes in a high-level modeling language. In this book the examples are done with IFPS (Interactive Financial Planning System of Execucom Systems Corporation, Austin, Texas), but a long list of other financial planning systems available for mainframe computers have similar features. Most of the modeling tasks in this book can be carried out on a variety of software (see Chapter 1) on a microcomputer.

Because English-like expressions are used in the IFPS models, they may be understood quickly. You can change the model as you wish, and the new results will appear almost instantaneously. Even complex problems that formerly may have taken months of labor to write in earlier languages can now be written and run literally overnight or over a weekend.

General modeling languages readily available are now flexible and powerful, and nonspecialists can quickly learn to use them. They tie into large data bases in the firm or available commercially. And they are available for personal computers in high-level languages that are compatible with the same language on a mainframe computer.

This book is intended for two audiences: (1) practicing managers seeking help in developing their own modeling skills on the job, and (2) MBA students and managers in short courses (and advanced engineering, undergraduate business, and public administration students) who will use the book as part of a course. State-of-the-art modeling languages are employed to aid the manager in building a model. But the thrust of the book is not to hammer out solutions to intricate problems, as in the various management science, operations research, or computer science texts oriented toward those who specialize in such fields. Mathematical sophistication is not assumed, yet the book develops skills that enable the reader to use quantitative tools and computation.

The book aims to develop managers' creativity by providing methods and skills that enable them to express their own perception of problems. Powerful tools, such as the influence diagram, enhance the individual's ability to design and communicate models. But modeling is a craft which can only be learned through first-hand experience in carrying out modeling tasks.

Cases provide the context for acquiring these creative skills. Those users of the book in category 1 who will not have the benefit of classroom discussion of the cases, will find study of the book more edifying if they can carry out their own modeling tasks as they read through

the chapters and examine the case material. Exercises at the end of chapters (with selected solutions in the back) help the reader to develop some of the skills needed to carry out the larger tasks of the cases. Most of these exercises relate to problems arising from general experience and do not require a special background.

The cases are best suited for students with some experience, or exposure at least, to business problems. They are organized by functional topic into sections such as marketing, finance, planning, etc. after all of the chapters of the book. Practicing managers will easily find the case material that pertains to certain problem areas they may face. The cases were kept separate from chapters to enhance the value of the classroom use of the cases. Often a major task in developing models for decisions is to decide what the problem is and to structure an approach to it. Placing the cases with specific chapters would take away the opportunity to diagnose the problem and the stage of development of the model. In addition, most of the cases deal with material in several of the chapters. There are specific assignment questions related to particular cases at the end of chapters. But this should not suggest that a case referred to at the end of a chapter deals only with the portion of the modeling process dealt with in that chapter. The intent was to provide broadly conceived cases that would help the user of the book develop a holistic approach to problem solving and avoid the pigeonholing of problems which is prevalent in many introductory courses and texts.

It helps if the reader is familiar with introductory business concepts (e.g., the income statement and net present value) in working through the cases. Generally the cases are based on situations in existing corporations. Consistent with the book's philosophy of learning modeling by doing it, the cases each involve a task to be done in the design, creation, and use of a computer model.

In keeping with the purposes of the generalist, a wide range of problem structures is used in the book. Thus, both certainty and uncertainty models are included, with risk analysis integrated into the approaches used when certainty is assumed. Both single-objective and multiple-objective decision situations are treated. In all situations, managers have the flexibility to mix their subjective judgments and preferences with objective data in the model.

The greatest benefit of this book is that it can give generalists, some of whom consider themselves inadequate in quantitative or computer skills, the confidence that they can:

1 Develop and run models in time for their own decision making on the job using high-level languages on mainframe and microcomputers.

2 Communicate their model analysis to critical superiors who may have little appreciation for quantitative modeling.

3 Apply a flexible, free-form framework to problems with multiple objectives and uncertain components arising in a variety of functional areas, industries, or firms.

4 Have what may be the most critical managerial skill: the ability to handle messy, unstructured situations using their own preferences and perceptions of the problem.

ACKNOWLEDGMENTS

I am indebted to the Darden Graduate Business School of the University of Virginia for providing time for me to write this book. The Case Research Program of the Darden School generously provided funds for time, travel, and research assistants for the research and writing of the cases in the book. An award of a Sesquicentennial Associateship from the Institute for Advanced Studies at the University of Virginia enabled me to be on leave for a semester to finish the book.

The book has benefited greatly from the able aid of several research assistants at the Darden School. Michael B. McEnearney worked with me on cases and examples for a year. Michael O'Donnell, William Long, and Jeffry L. McNair each spent a summer developing cases. Not only did they co-develop with me much of the case material, but their thoughts served to focus the ideas of the book and their enthusiasm sustained my excitement with the work.

Many companies and individuals willingly cooperated with the field research needed for the cases. Luckily, there are many managers who recognize the pedagogical benefits of field-based experiences for students of modeling and generously contribute their time and experience, as only they can.

The materials in the book have evolved through use in courses at the University of Virginia, the University of Washington, and in management educational programs of the IBM Corporation. I thank the participants in those courses and faculty members of those universities for their comments and encouragement.

Sherwood C. Frey, Jr., of the University of Virginia deserves special thanks for comments on the content of the book and on the design of courses using the material. Richard P. Ten Dyke of IBM jointly wrote the "DiscoMall, Inc." case, and the following authors are responsible for the respective cases: Elwood S. Buffa, UCLA and James S. Dyer, University of Texas, Austin, for "California Oil Company" case, and Charles A. Holloway, Stanford Business School, for "Westview Environmental Planning (A)." Larry Brown of IBM corporation made a detailed review of an early draft of the book and provided several pages of his own thoughts on modeling, which, with his permission, have been woven into the book. The field work which served as a basis for two of the cases was carried out by Darden School students Charles Barnard ("Stevens and Company") and Robert Martin ("John Denison") as part of a supervised business study done to fulfill educational requirements.

Many people had a chance to correct errors in this book. I have had at least a dozen lunches with students to talk about typographical or substantive criticisms in various drafts. Beverly Seng of the University of Virginia provided editing comments on early drafts. A number of reviewers have examined it. Copy editors have had their chance with it. Spelling proofreaders on my personal computer were used on parts of it. To all of them I am grateful, even to Wang, my Compaq, and my IBM PC for word processing. Yet, I suspect there are mistakes in it still (I am sorry to say) that have gotten by all of us, which your sharp eye may catch. I would be glad to know about the mistakes (I would even be happy to know what you like about the book). Who knows—I may buy you lunch to talk about it.

Samuel E. Bodily

MODERN DECISION MAKING

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GETTING STARTED

CHAPTER 1

INTRODUCTION TO MODELING

This book is about creating and using decision models with the aid of a computer decision support system (DSS). For our purposes a *decision model* is any quantitative or logical abstraction of reality that is created and used to help somebody make a decision. It consists of quantities and their relationships. For example, if you were considering the purchase of a piece of real estate, you would project revenues and costs over the next 10 years and a residual value of the property after this time and then put this information together to help you assess what the property is worth. You may wish to express the relationship of revenues and costs to inflation and then investigate a variety of inflation scenarios. A decision model would contain all of your forecasts and all of the relationships among the variables. It would provide an estimate of the value of the property for any inflation scenario you may envision.

A DSS is the conduit for creating, revising, checking, and using the model. In its crudest form, the DSS may consist of a spreadsheet planning system, such as VisiCalc (or one of its many cousins), or an equation solver, such as TK!Solver. Most of the things you might like to do in using this book could be done reasonably well with even the simplest of such software.

The professional, however, will quickly develop an appetite for a higher level of decision-

making support. Think of the complete DSS as a high-level language that allows for natural, English-like expression of the model; that is able to access corporate and vendor data bases; that has easy-to-use graphics for displaying the results; and that contains powerful computational features for activities such as "what-if," sensitivity analysis, goal seeking, extrapolation, risk analysis, and optimization. In addition, think of the DSS as a system that supports the manager in treating ill-structured, messy problems and extends and enhances the manager's own understanding and judgment rather than providing a unique solution.

The examples in this book use the Interactive Financial Planning System (IFPS)[1], which has all of the features just mentioned in an easy-to-use package that runs on a mainframe computer. A companion package for the personal computer, IFPS/Personal, is fully compatible with IFPS on the mainframe and will run most of the example models in this book. IFPS/Personal does not support risk analysis and optimization and it does not solve problems with the same rich variety of features available on IFPS.

Many other mainframe financial planning systems have features similar to IFPS, and the user of this book could use these systems to carry out the modeling tasks and develop models for the examples. The list of usable systems at the time of printing of this book includes the following:

System	Vendor
CUFFS	CUFFS Planning and Models, Ltd. New York, N.Y.
EIS	Boeing Computer Services Co. Seattle, Wash.
Empire	Applied Data Research Princeton, N.J.
Express	Management Decision Systems Waltham, Mass.
FCS-EPS	Evaluation & Planning Systems, Inc. Houston, Tex.
Foresight	United Information Services Overland Park, Kan.
GSA/GSM	Prediction Services, Inc. Manasquan, N.J.
IMPACT	MDCR, Inc. East Brunswick, N.J.
Model	Lloyd Bush & Associates New York, N.Y.
MSA/FMS	Management Science America Atlanta, Ga.
Simplan	Simplan Systems Chapel Hill, N.C.
Stratagem	Integrated Planning, Inc. Boston, Mass.
System W	Comshare Ann Arbor, Mich.
XSIM	Interactive Data Corp. Waltham, Mass.

There are many software possibilities on the microcomputer as well, too many to provide a complete list. In addition to the many cousins of VisiCalc and IFPS/Personal, the list of microcomputer software available at the time this book went to press would include:

[1]IFPS and IFPS/Personal are registered trademarks of Execucom Systems Corporation of Austin, Texas.

Software	Vendor
1-2-3	Lotus Development Corporation Cambridge, Mass.
Encore	Ferox Microsystems, Inc. McLean, Va.
MBA	Context Management Systems Torrance, Calif.
Multiplan	Microsoft Bellevue, Wash.
System W	Comshare Ann Arbor, Mich.
TK!Solver	Software Arts Wellesley, Mass.
VisiCalc	VisiCorp San Jose, Calif.

This is a book on modeling, not computer language, and it is intended for users of these other systems as well as IFPS users. In most cases the non-IFPS users will be able to understand the IFPS models easily and translate them quickly into the language of their own systems.

1.1 WHY MODEL?

You build a model to help you make a decision or to help someone else's decision. The help comes in two ways. First, the decision maker can respond to much more complexity than one person can easily grasp and resolve. Second, the model, through computer support, can keep track of many details and perform rapidly all of the computations. This allows the modeler to devote attention to judgments made about the individual details and composite results produced by the model.

1.1.1 Necessity

Models are built from necessity. They are done reluctantly when simpler approaches will not suffice. They are not a goal in themselves, even though they can be fascinating, almost seductive in pulling you from the decision at hand.

No one wants a model. People making decisions want the help that models can efficiently give. The model is not part of a goal—the decision is the goal. The model must be limited to a small effort relative to the importance of the decision. Low-stake decisions will be modeled only if they are repetitive or generalizable enough to be levered into a high-stake problem.

Learning to model requires adapting one's language in order to communicate the model and its results effectively. The medium with which to communicate models both to computers and to others is now provided by modern high-level modeling languages. This book adds other design and communication tools to aid the process of translating a messy problem into a model. While it is necessary for many people to become accustomed to new language and certain conventions of communication to create models, few people need to specialize in and study the language.

In this book computer language and mathematics are treated as English would be in a freshman English course; we all need to use English, but understanding language is not a goal in itself, as it might be for an English major. The modeler who is adept in a particular modeling language is like the writer of diplomatic communiqués. The translator can take pride in accurately rendering the subtlety of the subject. Yet neither the language nor the model is the end in itself; they are means to the end of better decisions.

1.1.2 Better Decisions

The model has helped you if you get a *better decision.* The decision can take into account more of the relevant facts and how those facts apply to the decision. The decision can deal with many relationships among things that influence the outcomes of interest. The decision can include the interaction of influences over a much longer period of time so that the decision does not just respond to the most obvious short-term considerations. The decision is better because the model has allowed a sensitivity analysis: The outcome has been studied as different assumptions are methodically varied. The impact of uncertain factors on the surety of results can be understood. The decision maker can understand which assumptions most affect the outcome.

1.1.3 Insight

A model gives you *insight* into your subject. You can explore the balances and tradeoffs among the factors that enter the decision. You learn the structure of the subject—the relationships among influences. It is useful to break a problem into pieces and put it back together in a model just to understand its anatomy. Diagnosis of other problems for which you do not build a model will be better because of this understanding.

1.1.4 Aid to Presentations

A model can be an *aid to presentations*. The model makes explicit the beliefs about interactions of aspects of the subject. Your presentation to the decision maker(s), or your depiction of your own decision, uses that structure. It shows your understanding of the problem as compactly as possible. You can concentrate on the important aspects, as shown by the results of modeling, rather than the obvious aspects.

1.1.5 Intuition

Complex systems behave nonintuitively. The model gives you insight into these nonintuitive behaviors that come from time lags between action and response, from interactions, and from the damping of one influence by another. The model provides *intuition about the whole*, starting with intuition about the parts.

The model tells you which gaps in your knowledge matter. Necessarily, you always work with incomplete understanding and data. Most gaps do not affect the decision much, but your model analysis tells you which pieces of information are important. This sets your agenda for research. If time and money allow, you know which areas of study will most improve the quality of the decision.

1.2 A MODELING DISCIPLINE

The model must be reliable. It must accurately reflect the assumptions of its builder. A disciplined development of the model is essential. The rest of this chapter describes such a discipline, one that works well. Subsequent chapters develop the elements of that discipline piece by piece.

1.2.1 Simplicity

The model must be kept simple, both to help make it reliable and to limit the investment in it. The model should be extendable. New influences may need to be taken into account and new questions may be asked about the subject matter of the model. Thus the process of model building recognizes two facts. First, the model will grow and shrink in future uses; thus the

modeler must use tools that allow flexible editing. Second, a useful discipline for modeling cannot be described in terms of a flowchart. It is impossible to include all of the branching points in which it may be necessary to go back and redo a previous stage of the process or skip ahead to a later stage and work backward. Two important watchwords are *simplify* first-cut efforts to the bare essentials and *reconsider* any structure already applied to the problem in going on with the process.

1.2.2 Communication

A modeler must communicate with a computer that runs the model. This is the least important form of communication. More importantly, model builders communicate with themselves, through the model, about the structure of the subject that is modeled. The model must be clearly written so it can be quickly understood. This clarity also helps make the model reliable.

The model also provides communication to other people. It represents the embodiment of the beliefs of those who developed its structure; its clarity is essential.

1.2.3 The Manager's Roles in Modeling

A full appreciation of the specific purpose of the discipline requires an understanding of the relationships between the manager and the model. Throughout their careers, managers will work with decision models in three roles. The manager serves as model builder and *analyst*, as initiator and *user* of the model, and as the model's beneficiary or *client*. These roles are illustrated in the Stevens & Company case in this book. In that case a real estate firm that brokers large estates and agricultural properties is interested in modeling a large timber and farm property for prospective investors. The company engages a student of a master of business administration (MBA) program to carry out the modeling work. In this situation, the MBA student is the analyst, the broker is the user, and the prospective buyer of the property is the client.

While settings of problems may vary, these roles are generally identifiable in virtually all problems, although one person may play more than a single role. The Unicron cases illustrate the roles in a corporation, for example. A financial staff member is the analyst, the vice president of finance is the user, and the members of the new products review committee (senior executives) are the clients.

Typically, MBA graduates play the role of analyst more often in the early years following graduation. As they move up in the organization, they take on a greater portion of the user role (and have analysts reporting to them). As they become responsible for more and more decisions, they increasingly are clients with respect to the models.

Each role requires separate skills and an appreciation of the tasks faced by those in the other roles. As a user, the manager will learn to define objectives and performance measures for decision making, to structure the variables that will be used in the model, and to evaluate and use the support offered by the model. As an analyst, the manager will learn how to write the model; how to carry out the assessment of numbers needed in the model; how to conduct an analysis of any combination of certain, risky, and time-dependent variables; and how to communicate to supervisors the work done on the model. In both the user and the analyst roles the manager will need to know how to present the model design and results to clients. As a client, the manager will learn to *evaluate* and understand the work of others and learn appropriate ways to *intervene* in the modeling effort.

The skills necessary to be effective in each of these three roles are developed in this book. The discipline of modeling is described assuming the reader is interested in learning all three roles. Indeed, in many of the cases in the book, proper understanding of the situation and the appropriate tasks to carry out requires that the problem be examined from the perspective of each role.

1.3 GETTING STARTED

Managers often have the greatest difficulty knowing how to start a model. Their problems may appear to be huge and untidy, and they may have little confidence and experience in creating a model. Thus a major focus of this book is model structuring.

How do you start modeling? First, understand the decision that is to be made. This sounds simple, but very often a problem is so complex that no one has clearly stated what they are trying to decide. This phenomenon was illustrated vividly to me in working with executives on a new-product-development problem. Although this was a big project on which many people had already done a lot of planning, I could not get them to tell me what the decision to be made was. It might have been a go or no-go decision on the product, or it may really have been a product redesign question since the commitment to the product in some form appeared firm. Although a lot of time and work had already been spent on the problem, no one was sure exactly what the decision was to be made and what were the available alternatives. In that case, a little time spent defining the questions at the outset could have saved an enormous amount of time later.

Finding the objectives to be achieved in making the decision is not always obvious or implicit in the decision. Make sure what is being maximized before creating a model to decide how to do it.

Chapter 2 presents ways to define decision variables, to establish the objectives of decision making, and to identify attributes associated with the objectives. A key contribution of this material is the assurance that it can give the modeler that the objectives of the decision maker can be translated into attributes that measure the degree of achievement of the objectives.

Having identified the variables of a model, the next step is to decide how these variables relate to one another. Chapter 3 presents tools for structuring a model design from the variables. The influence diagram displays all variables, and arrows running from one variable to another indicate the direction of influence. Variables that are assumed to be known with certainty are distinguished from those that are uncertain.

The diagram also uses three distinctive types of influence. The simplest influence is one where the level of one variable determines the level of another variable with *certainty*. A *random* influence implies that a variable only partially affects the level of another and an unpredictable effect also partially determines the level of the influenced variable. The final type of influence is a *preference* influence, wherein the desirability of an attribute is influenced by another variable or attribute.

The step of creating the influence diagram is the most important one in establishing the framework of the model. Often you find a "major" influence is not linked to an attribute at all. When this occurs, demote it to a minor consideration to be handled outside the model. Sometimes holes in the influence diagram indicate that intermediate variables are missing. Not only does the finished influence diagram make you ready to get down to the nitty-gritty of writing the model, it is the most convenient form for presenting the model to others.

1.4 CREATING AND USING A MODEL IN A HIGH-LEVEL LANGUAGE

The next step is to decide on the form of relationship for each influence. Keep relationships simple; there is no prize for introducing the most unusual mathematics. Later steps of the modeling discipline will revisit these choices and you will have an orderly way to improve the ones that matter.

This does not mean that within the bounds of simplicity and conciseness the model should not be as accurate a representation of reality as possible. The modeler needs to become familiar with the wide range of possible relationships among variables that may be expressed in order to create realistic models. Chapter 4 describes a menu of mathematical functions to use in expressing a relationship between a dependent variable and one or more influencing variables.

The examples use the features of IFPS, but the modeling capabilities are available on most high-level planning or modeling languages. Most relationships can be produced with a little effort even on microcomputer spreadsheet planning software.

Random variables are modeled with a menu of available probability distributions. When random variables are used, the models are solved using the technique of Monte Carlo simulation (described in Chapter 5) in order to see the uncertainty in the attributes.

Chapter 5 describes how to complete the model to obtain preliminary results and then how to improve and validate the model. After checking the pieces of the model, you test the accuracy of the model in its composite form. A first-cut run of the model is the place to begin.

What-if exercising helps in understanding and verifying the model and provides answers to decision questions. Sensitivity analysis continues this process, revealing the significance of intermediate variables in determining attribute levels. Use this understanding to decide if and how to extend the model.

At this stage, you can test specific goals using the model. For example, you can check whether it is possible to attain some target level of performance on a specific attribute. This begins to move the problem from the realm of what if to the realm of "what's best." A what's-best analysis requires evaluating the relative desirability of alternative levels of outcome attributes. This raises questions of how to treat multiple objectives and uncertain attributes.

1.5 INCORPORATING DECISION PREFERENCE INTO MODELS

Chapter 6 describes the major ways to make choices accounting for multiple objectives. Starting with dominance and other approaches that do not require tradeoffs among attributes, the chapter goes on to describe procedures for weighting attributes. These procedures are easily incorporated directly into a model.

Chapter 7 provides the modeler with methods for incorporating attitude toward risk into the model. An important aspect of the approach of this chapter is the discussion of how to treat risk when one is modeling corporate risk preferences, where there is no single person to express attitude towards risk.

The modeler may want a more realistic model of preferences than was provided in Chapter 6. In particular, it may be necessary to treat interactions among attributes and nonlinear values for attributes. For example, an attribute such as square feet of space in a model to help in choosing a home may be such that its value is nonlinear. That is, the value of an additional 100 square feet of space may decline as the size of home increases. Preferences for this attribute may interact with another variable or attribute. For example, the location of the house may greatly affect the desirability of various home sizes. These issues are treated in Chapter 8. Besides discussing the implications of the various preference models, the chapter gives guidance on assessing preferences.

The final two chapters deal with specific application problems. Chapter 9 shows how to use models for group decision making. The principal application of this would be to the committee decision problem, but more general collective choice problems and bargaining problems are considered. Voting rules, scoring rules, procedures for achieving consensus, and group preference functions are presented and incorporated into decision models.

Chapter 10 treats issues that arise when modeling risky projects that stretch over significant amounts of time. Several approaches are described. The discussion then centers on when and why each possible approach should be used.

The discipline developed in this book will prepare you to take an unstructured problem, decide on the important variables and attributes, structure influences among these variables, apply your own or a group's preferences to them, exercise the model, and make a decision. What is then needed is familiarity with a wide range of applications and some experience developing models in a decision context. The cases making up the rest of the book provide this opportunity to internalize the discipline.

1.6 USE OF CASES AND EXERCISES

Cases are organized by functional or application area as shown in the table of contents. Nonetheless, the cases are intended to develop specific modeling skills. Assignment questions to develop those skills are given for particular cases at the end of each chapter. Some cases are integrative; no assignment questions are given for them. A principal component of learning to create a model is developing judgment about what is the major modeling task. Thus the diagnosis of what to do with the integrative cases is a key part of the learning that comes from cases.

Clearly, the learning that will come from the kinds of cases in this book will be enhanced by classroom discussion. Where this is not available, discuss the problems with colleagues, or use the cases to identify similar problem areas in your own area of responsibility. Work through these personalized cases as you read the chapters.

With each chapter are exercises. Spend some time with these as you read the chapters. Some of the exercises have sample solutions at the end of the book. Recognize that few of these

EXHIBIT 1.1 Influence diagram of the modeling discipline and the book's organization.

exercises have a single right answer, and these solutions should serve only as guides. Complete the work you intend to do on the exercise before flipping to the back of the book. Clearly, the value of the material for the nonclassroom user is tied to the degree of self-discipline you can muster to work through the exercises and use the cases along with the reading of the chapters.

1.7 SUMMARY

What book on modeling would be complete without a *model* of its conceptual design? The best way to express the essence of a model in a nutshell is through an influence diagram. Thus it is appropriate to end this chapter with an influence diagram of the book and its associated modeling discipline (see Exhibit 1.1). Undoubtedly, this exhibit will shed more light on the schema for the book after the material on influence diagrams in Chapter 3 has been read. It would be wise to look at the exhibit briefly now, then refer to it after reading Chapter 3 and later.

The rectangles in Exhibit 1.1 represent choices made by the modeler outside the modeling process. The modeler must decide what are the boundaries of the problem and who will play the roles of user, analyst, and client with respect to the model. Circles refer to variables in the modeling process, or in other words, the component tasks faced by the modeler in reaching the ultimate goal of making a decision. Note that these tasks center on the model of the problem itself. Of course the ultimate measure of the value of modeling is the *decision*, represented by an oval.

KEY TERMS

decision model
better decision
insight
aid to presentation
intuition about the whole
simplify—reconsider
roles in modeling: analyst, user, client
modeling discipline

EXERCISES

1.1 Why are you interested in this book? What are your typical modeling tasks? What role(s) do you play in these modeling tasks—analyst, user, or client? Select two or three of these tasks to think about as you go through this book.

1.2 Based on your prior experience with quantitative modeling, assess your own philosophy of modeling. Make a few notes on the process of going from problem to model to decision. How does it compare with the ideas in this chapter?

CHAPTER **2**

VARIABLES AND OBJECTIVES

This chapter describes the initial steps to take in creating a decision model, which was defined to be any quantitative or logical abstraction of reality that is created and used to help somebody make a decision. Computer decision support will normally, but not necessarily, be employed in writing and running the model. Virtually all decision models contain one or more of the following: system variables that describe the environment and the effects of the decisions, structural linkages or interdependencies among these system variables, and preferences regarding ultimate outcomes of decisions. Before even starting to build the model, you must answer these questions: What system variables belong in the model? and What objectives of the decision maker may be used to measure the desirability of alternative outcomes? We will confine the discussion in this chapter to these two questions. Later chapters address the issues of how to model the relationships among variables.

2.1 VARIABLES

A quantitative model developed for decision making or planning is nothing more than a mathematical description of the relationships among some variables. There are many ways to categorize variables, but most useful here is to divide them into decision, intermediate, and outcome variables. *Decision* variables are those controlled by the decision maker; they vary in accord with the alternative selected by the decision maker. *Intermediate* variables are any variables

necessary to link decisions to outcomes. *Outcome* variables are those used by the decision maker to measure performance and in this book will be called *attributes*.

It may be obvious in a particular situation which variables are involved in a problem, yet the identification of the variables as decision, intermediate, and outcome variables is the telling feature of the nature of the decision model. Consider, for example, the following three variables: a shepherd's whistle, the location of a sheep dog, and the location of a flock of sheep. It would be natural for the shepherd's whistle to be the decision variable, the location of the dog to be the intermediate variable, and the location of the sheep the outcome variable. Alternatively, if the dog's location were the decision variable, the location of the sheep the intermediate variable, and the shepherd's whistle the outcome variable, we would have quite a different model of the problem. This would conform to the dog being master and deciding how to make the shepherd whistle. It is left to the reader to devise how to identify decision, intermediate, and outcome variables so as to portray choices made by the sheep concerning how to control the dog through the shepherd. Clearly the crucial first step in modeling is to identify the three types of variables.

Any variable is one of three types: binary, discrete, or continuous. A *binary* variable takes on a value of 0 or 1. It is useful in a go or no-go or on or off situation (e.g., whether product B is to be included in the product line or not). *Discrete* variables may take on any of a finite number of values. For example, the answer to the question Which manufacturing machine? might have five possible values (the integers 1 through 5) to signify five potential machines. (The number may imply some ranking of alternatives or might instead simply label alternatives, in which case the letters A through E would serve as well as the numbers 1 through 5.) Discrete variables may take on any of a finite set of values that need not be integer or in sequence. For example, the choice of a family passenger vehicle may be made from the set of discrete possibilities 4, 5, 6, 9 reflecting the number of passengers carried by the vehicle. A *continuous* variable, on the other hand, has an infinite number of possible values, all lying within a specified range. An example of a continuous variable is a decision variable representing facility size where any number of square feet between a minimum and a maximum size may be selected.

Variables may also have an additional classification as random or exogenous. A *random* variable models uncertainty and will therefore be described using probability. An *exogenous* variable is one that is external to the firm or entity being modeled and cannot be influenced by the decision variables. Exogenous variables are of two types: those that affect intermediate variables and those that affect decision variables. Exogenous variables that affect intermediate variables often arise from "the economy" or from competitive behavior, but they may arise in numerous ways. An example of an exogenous variable affecting a decision variable is a government policy that restricts the choices that a firm can make. Exogenous variables may also be random.

Consider a general planning situation involving questions of capital investment and design of a product line. The variables in this situation may be categorized as follows:

Decision variables	**Intermediate variables**	**Outcome variables**
Size of plant?	Investment capital	Operating profit
Make products A and B or just A?	Working capital	Cash flow
Which manufacturing machines?	Production expenses(R)	Net present value
	Unit sales(R)	Internal rate of return
	Demand for product(R,E)	Market share
	State of the economy(E)	Retention of scarce skilled workers
	Competitive action(E)	

R = random variable
E = exogenous variable

In this example the decision variable Size of Plant? is continuous; Make Products A and B or Just A? is binary; and Which Manufacturing Machines? is discrete. The intermediate and outcome variables are all continuous variables. Note that a variable may be both random and exogenous (e.g., Demand for Product).

It is the modeler's judgment that determines when a variable is designated random or exogenous. Virtually all intermediate variables could, in one sense, be considered random, since they cannot be pinned down with precision. On the other hand, for many purposes it is not necessary to consider any randomness in the variables. One of the modeler's tasks is to decide for which variables the uncertainty is "significant" relative to the purpose for the model. Similarly, modelers must judge which variables are within the boundaries of their control and thus affected by decision variables, and which are not and thus considered exogenous. Experience and the discussion of later chapters will help develop the judgment necessary for these modeling decisions.

Sometimes it is hard to begin a model. The first step is to state clearly what decisions are to be made. That step clarifies some of the messiness of the situation. Then think about how those decisions are related. Are some contingent on others? Are they separated in time?

For those familiar with decision diagrams, it may help to draw a decision tree. Exhibit 2.1 shows the correspondence between the elements of a decision model and those of a general decision tree. In the decision tree, there may be many decision variables which are denoted by x's, intermediate variables by y's, and outcome variables by z's. Use trees not only to begin the modeling but also to begin a presentation or explanation of what the model tells you.

Decision trees can help to collect the variables to be included in the model, particularly for those with prior experience using them. But for many problems a chronological sequence of alternatives and unfolding events does not apply. Some problems are static, involving cause-and-effect relationships at a single point in time. Consider, for instance, a problem concerning alternate ways to avoid air pollution. The decision variables might be the capital investment and energy needs of various kinds of pollution control equipment and who will pay for them. The intermediate variables may be the level of production at the site, external air conditions, and demographics at the site. Finally, the outcome variables may be health effects, the impact on various tax rates, and political consequences.

A common weakness in decision models (and the associated decisions) is inadequate specification of outcome variables. Often the variables chosen as outcome variables and used to measure performance are more accurately decision or intermediate variables.

Consider an advertising example. Some companies measure their advertising effort purely by expenditures. Expenditure level, however, is an allocation decision, not a measure of performance. Using the level of expenditures as an outcome variable is not very helpful for decision making. Going one step further, other companies measure advertising performance

EXHIBIT 2.1 General decision tree and related decision model.

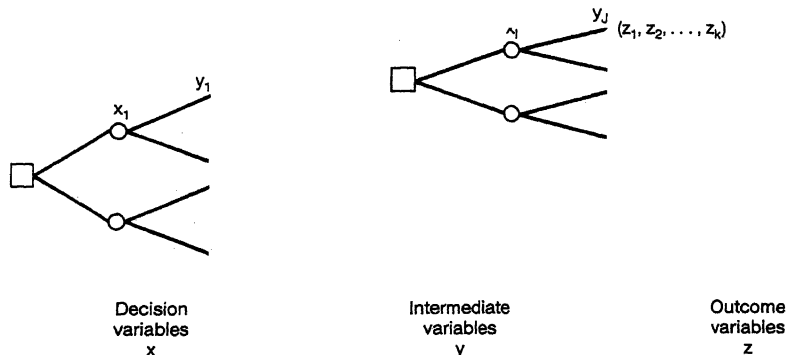



EXHIBIT 2-2

EXAMPLES OF THE CORRECT USE OF DECISION AND INTERMEDIATE VARIABLES ERRONEOUSLY USED AS OUTCOME VARIABLES

Setting	Decision variables	Intermediate variables	Outcome variables
Advertising	Expenditures	Coverage of ads: No. of people viewing ads No. of exposures per person	Customers influenced Sales
Emergency Medical Service	No. of ambulances Training expenditures	No. of ambulance runs No. of splints per year	Morbidity Mortality

by *reach*, that is, by the number of people who view the company's ads, and/or by the *frequency* of viewing. This measure is an improvement over expenditures, but the objective of advertising is not simply to reach people but rather to educate them, to improve a product's image, and ultimately to *influence purchase decisions and sales*. Ideally, a decision model would link the decision (in this case how much to spend on ads) to the true outcome variables, those based on the ultimate objectives of the decision maker.

Consider an example in the area of emergency medical service. Often a community measures this service by the number of ambulances or the amount of training of the personnel, which are, again, decision variables. An improvement would be to use measures of system activity, such as the number of ambulance runs or the number of splints applied per year. However, the figures for these measures might be impressively high without achieving the real objectives—saving lives (reducing mortality) and reducing the impact of disease and injuries (morbidity). The splints must be on the right people and the ambulance runs must do some good. Thus, mortality and morbidity are the true outcome variables. In Exhibit 2.2 the use in this setting of more appropriate outcome variables and decision or intermediate variables is illustrated.

There are reasons why decision or intermediate variables are often used in place of outcome variables. They are easier to obtain, can be more accurately measured in many situations, and may be less controversial than outcome variables. For example, there would be less argument about how many ambulance runs were made than about how many lives were saved. In addition, reliable estimates for more legitimate outcome variables are very difficult to find. Yet there seems little point in developing a decision model using the wrong performance measures. Proxy measures for the correct outcome variables would be better than inappropriate variables, no matter how easily measured. How to select proxy measures will be discussed later in this chapter.

The outcome variables for a decision situation are not generally obvious, nor do they appear automatically after some casual consideration. They come from realistic objectives, which are, in turn, the result of explicit consideration of what needs to be accomplished. Careful thought about objectives will enhance the modeling effort and reduce the need for later revisions.

2.2 STRUCTURING OBJECTIVES AND ATTRIBUTES

Adopting some reasonable definitions will help the discussion; let us use those of Keeney and Raiffa (1976). An *objective* indicates the direction to move in order to do better; hence words like "minimize," "maximize," and "improve" appear in the statement of the objective. An example of an objective would be To maximize productivity. An objective differs therefore from a goal or milestone (e.g., To land on the moon by 1970), which is either met or not met. Objectives may never be completely achieved; it is their degree of achievement that is important. Generally not all objectives can be simultaneously achieved (e.g., it may not be possible to maximize productivity while minimizing investment). Therefore, tradeoffs among the objectives must be made.

There may be *subobjectives* related to each objective (e.g., To improve the productivity of labor or To improve the productivity of investment capital are subobjectives of Maximize productivity). Associated with each objective (or subobjective) is a measurable attribute that will indicate the degree to which the objective is met. For example, the attribute Dollar Value of Production per Employee might be associated with the subobjective To improve the productivity of labor. The word "attribute" is synonymous with the often-used terms "measure of performance," and "measure of effectiveness." An indication of the units of measurement and a description of how the attribute is to be measured are generally a part of the definition of an attribute in each situation.

The process of developing objectives and attributes is a creative one which cannot be formalized into rules and procedures. We can only suggest activities to aid the process and provide some guidelines as to when the process is complete.

Usually one starts with a single, general, often ill-defined objective, such as Maximize the good of the corporation. More concrete subobjectives can be specified by answering precisely what is meant by the good of the corporation. In most corporations it is the job of the board of directors to answer this question. In other situations, a source of appropriate objectives is a panel of experts, individuals with both experience and special skills relating to the area of concern. Objectives may also be found in the literature on the problem or in the practice of similar institutions.

Observing how individuals make decisions in practice will expose objectives, even if the objectives have never been stated. How do people talk about the problem? What information is required by decision makers before making a choice? Thus in a strategic marketing decision, for example, one may discover whether the objective is to capture a market or to be profitable by noting whether the discussions focus on market share or cash flow.

2.2.1 Subjective Index

A number of ways are available to treat an objective for which there is no apparent measurable attribute. One approach is to develop a *subjective index*. For example, a physician's objective To minimize pain eludes measurement. Yet a pain index can be developed in which a patient or an expert subjectively scores possible outcomes on a scale defined in terms of some common ailments. Any tradeoffs between pain levels and other attributes (such as Effectiveness of Treatment) would be assessed over this pain index as well. In the medical field this approach has resulted in commonly referenced indexes such as the Veterans Administration disability index or the New York Heart Association index of cardiac dysfunction.

2.2.2 The Proxy Attribute

Another approach is the use of *proxy* attributes—those attributes that reflect the achievement of an objective but are not direct measures. For instance, a proxy attribute for the objective Minimize pain might be Days in Bed. Assessing tradeoffs with proxy attributes is more difficult than with the actual attribute. The mind must deduce the relationship between the proxy attribute and the true objective (e.g., what pain level corresponds to a specific number of days in bed). Using a subjective pain index simplifies the assessment of the decision maker's preferences. On the other hand, it would generally be more difficult to model the relationship between the medical decision and the subjective index than the relationship between the decision and the proxy attribute.

An objective and a proxy attribute might be related in any of the following ways:

1 Indirect The proxy attribute measures a closely related objective. For instance, the attribute Earnings does not measure directly the objective Increase dividends to investor but rather the related objective Increase profits.

2 Precursor The proxy attribute measures a predecessor of the true objective. For example, the attribute Recognition of Product does not measure the objective Maximize product sales, but rather a precursor objective.

3 Combined The attribute is related to a combined set of objectives, perhaps in an indirect way. For instance, Share of Market is an attribute that may serve as a proxy for a number of objectives related to achieving economies of scale in both production and marketing costs.

Proxy attributes are generally designed to be measured objectively or physically. Alternatives are scored on the proxy attribute. Then the decision maker must convert the proxy score (either mentally or on paper) into a value score according to his or her innate preferences.

2.2.3 Direct Preference Assessment

A different (and final) approach to defining attributes is to combine the scoring of attributes and subjective preference assessment into a single process of *direct preference assessment*. In this approach, alternatives are scored subjectively (say on a scale 0 to 100) for each objective. For example, in making admissions decisions for a professional school one important objective is to admit the most intelligent students. In a direct preference approach, the decision maker may simply give the applicant a subjective rating on intelligence (along with other attributes such as experience and professional commitment). This is contrasted with a proxy attribute approach where the applicant is scored on measurable attributes (e.g., admission test scores or undergraduate grade-point average), and any tradeoffs or preference judgments are made in terms of these scores. The direct preference assessment procedure is a quicker one-step approach, but it presupposes an ability on the part of the decision maker to make consistent, unaided, subjective judgments.

2.3 HOW TO KNOW WHEN THE SET OF OBJECTIVES AND ATTRIBUTES IS RIGHT

Having developed a list of objectives and associated attributes, one faces the inevitable question of when to stop. There are several tests to apply to a set of objectives to determine if that set is the one to use. One should ask if the set is

1 Complete Are all important factors included?

2 Minimal Are there redundant attributes? A test of these first two considerations is the "test of importance" credited to Ellis (1979), which asks Could the best course of action be different if this objective were missing (or added) from (to) the list of objectives? If there is no conceivable situation where a choice would be altered by including an attribute, then it may be left off the list.

3 Comprehensive Is each attribute unambiguous? If the level of the attribute is known, is it clear to what degree the objective has been achieved? For instance, if the objective is To improve the rate of return to the investor, the attribute Return on Sales would not be considered comprehensive. This is because a product line with a lower return on sales might actually provide a better return to the investor than another product line with a higher return on sales. For example, the first product might require a lower working capital investment.

4 Measurable Can the effects of decision variables on the attribute be measured and can the preferences of the decision maker for levels of the attribute be assessed? For example, if the objective is To improve company influence and leadership in society, an attribute might be Degree of Public Acceptance of the Company's Goals, but this attribute is very difficult to measure objectively. In a case such as this it will usually be best to identify one or more proxy attributes which may be only indirectly related to the objective but which can be measured. For example, the proxy attribute might be Percentage of Population Who Have a Favorable Perception of the Company. All measurements will be subject to many sources of error. The most accurate measurements should be relied on most.

A number of passes at the problem will often be required before a set of objectives and attributes is obtained that satisfies the above requirements. Generally, after the first time through the list will be much longer and less specific than it will be after the refinement. Once it is settled what the attributes are, they will be used as the outcome variables in building a model or in directly comparing alternatives.

2.4 ILLUSTRATIVE EXAMPLE: OBJECTIVES AND ATTRIBUTES FOR THE CHOICE OF A COMPUTER SYSTEM

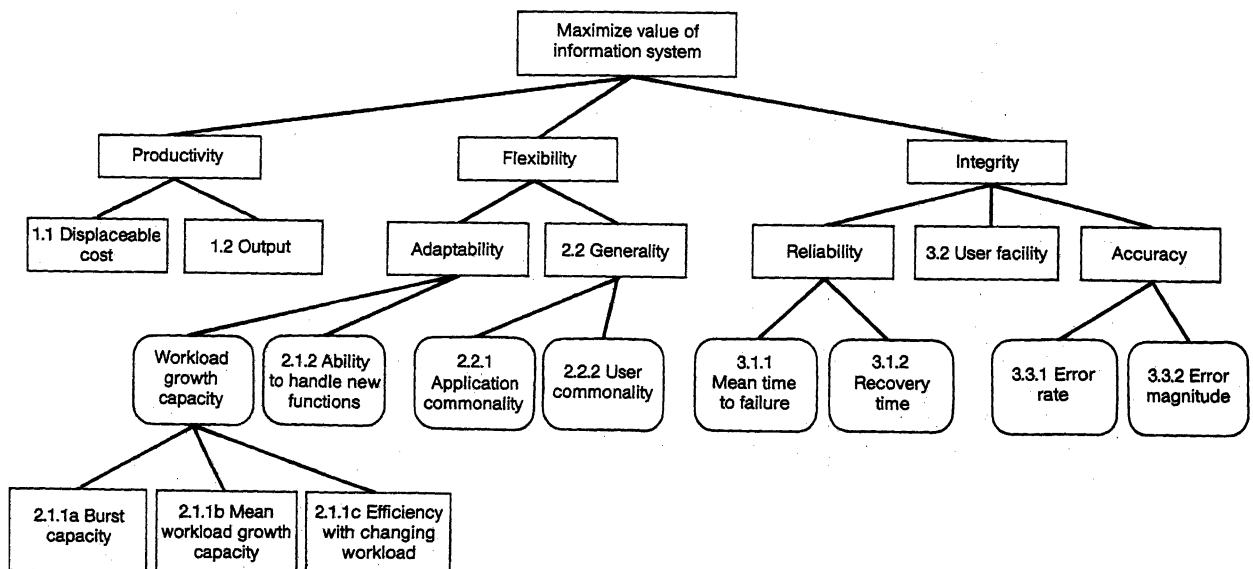
The following example illustrates the development of a set of objectives and attributes. A company was facing some decisions about the kind of computer information system it should have. Among the decisions to be made were selecting a computer and associated software systems assuming a given spending limit and planning for the operating and service personnel to run the information system. An overall objective for these decisions was To maximize the value of the information system to the company. This general statement was, of course, not definitive enough to be useful. Clarifying the objective involved dividing it into subobjectives that were more detailed and specific. This process was conducted by the company managers who were to make the decisions with the aid of an outside consultant.

To generate more concrete objectives, the consultant asked the questions: What constitutes a valuable information system? and How does one determine a high-quality information system? Discussion of these questions led to the following statement of major objectives:

1 Maximize productivity is defined as the ability to get the job done.
2 Maximize flexibility is defined as response to unknowns of the future.
3 Maximize integrity is defined as soundness of the system.

These objectives were still broad and needed more clarification in order to be operational. Probing for more definition, the managers came up with another tier of subobjectives, as shown in the third row of Exhibit 2.3 (e.g., Maximize displaceable cost). Later these objectives were further divided into the lower-level subobjectives of the hierarchy (e.g, Workload growth capacity).

The managers felt that stating and organizing objectives into this hierarchical framework

EXHIBIT 2.3 Objective hierarchy for evaluating an information system.

was useful for *qualitative* insight even at this preliminary stage, whether or not it was later used to *quantify* performance. They also felt that specifying subobjectives in more detail than necessary would help them to quantify higher-level objectives with less effort and greater validity.

Having developed the objective hierarchy of Exhibit 2.3, the managers faced additional questions: (1) Should the objectives be formalized into a more detailed hierarchy? and (2) At what level in the hierarchy should the attributes be quantified for decision making? The first question was answered using the four tests mentioned in the previous section. The test of importance and the test for the minimal set resulted in a decision not to specify the objectives in any greater detail. The second question was answered with the adoption of the objectives in Exhibit 2.3. (Some of the managers individually generated more specific subobjectives to aid in their own understanding and in the estimation of attributes for the numbered objectives.) The final objectives are not all at the same level in the hierarchy. For example, Maximize displaceable cost is two levels higher in the hierarchy than Maximize burst capacity. The managers felt that the level of necessary detail varied from objective to objective.

Once the objectives had been chosen, the next task was to specify the attributes associated with each one. Recall that a complete description of an attribute includes an indication of what units are used and how the attribute is measured. Exhibit 2.4 presents the final results of

EXHIBIT 2.4 ATTRIBUTES AND THEIR MEASUREMENT FOR THE INFORMATION SYSTEM OBJECTIVE HIERARCHY

Objective	Attribute	How measured?
1.1 Maximize displaceable cost	$ saved with new system	Expert estimate
1.2 Maximize output	Number of queries answered	Expert estimate, tracked monthly
2.1.1a Maximize burst capacity	Capital/98% workload fractile	System specifications
2.1.1b Maximize mean workload growth capacity	Years of growth of mean workload capacity	Expert estimate
2.1.1c Maximize efficiency with changing workload	Fixed/total cost	Expert estimate
2.1.2 Maximize ability to handle new functions	% of new functions (weighted by importance) that can be handled	Expert estimate
2.2.1 Maximize applications commonality	% of current applications supported	Count
2.2.2 Maximize user commonality	% of current users supported	Count
3.1.1 Maximize mean time to failure	0–1 year	Tests, data from other installations
3.1.2 Minimize recovery time	0–7 days	Tests, data from other installations
3.2 Maximize user facility	0–100 rating of user sample	Subjective from users
3.3.1 Minimize failure rate	% of inquiries resulting in no information or error	Tests, data from other installations
3.3.2 Minimize failure magnitude	0–100 rating of seriousness of errors or failures	Subjective from experts

discussions on the attributes. The amount of subjectivity in the measurement of the attributes varies substantially, from a simple count or a test result to a totally subjective score. One of the objectives, Maximize user facility, was to be scored using direct preference measurement. Some of the attributes are proxy attributes. For instance, Numbers of Queries Answered is an indirect proxy measure of Maximize output (a subobjective of productivity): it does not account for the importance of the queries. It does, however, have the desirable properties of being familiar to managers and of being easily and routinely measured.

Generating objectives and attributes in this example required a lot of thought from the managers involved. They considered the time well spent, even if the analysis were not to be carried further, because the result of their efforts was a set of attributes for which they could easily score alternatives and express preferences. This favorable outcome was not the case with several interim sets of objectives and attributes from which this final set evolved. The attributes were now available to be used as outcome variables in evaluation models for new information systems.

2.4 SUMMARY

A decision or a planning model describes the interactions among a set of variables. Before creating the model, it is necessary to decide what variables will be used, including all decision variables, intermediate variables, and outcome variables.

A frequent problem in constructing decision models (or in making decisions for that matter) is in confusing outcome variables with decision or intermediate variables. This may mean that a completed model is useless or even detrimental. Consider what may happen, for example, if we measure the quality of our national defense strictly by the amount of money poured into it (which is really a decision variable) rather than by its ability to defend us.

A weakness of models is often due to ill-defined attributes. A careful structuring of objectives and associated attributes prior to building a model or making a decision can overcome these potential pitfalls.

The process of developing objectives and attributes will vary with the decision being made or with the decision maker. A guideline is to begin with a general, overall objective and work, in hierarchical fashion, to more specific and clearly defined objectives.

There are several tests which will determine if the set of objectives defines the problem sufficiently. The set of objectives should be complete, minimal, comprehensive, and measurable.

A key element in successfully setting objectives is to match with each objective a measurable attribute. If problems arise with quantifying objectives, three approaches offer some help. The first possibility is to use a proxy attribute, which indirectly measures performance on the associated objective and, therefore, may be easier to estimate. A second approach uses a subjective index, developed by the decision maker or an expert to displace an attribute that is difficult to measure. A final approach is to combine preference judgments and the scoring of alternatives together through direct preference assessment of a subjective attribute.

KEY TERMS

decision variable
intermediate variable
attribute: outcome variable
variable type: binary, discrete, continuous
random variable
exogenous variable
objective
attribute: measure of performance
subjective index
proxy attribute
direct preference assessment
attribute hierarchy
tests of objectives: complete, minimal, comprehensive, measurable

EXERCISES

2.1 Define (one or more) decision variables for each of the following problems and decide whether each variable should be treated as binary, discrete, or continuous:

a A bond underwriter has to decide which of a set of proposed bond issues that the firm will underwrite. It is possible that the firm could accept all of the bonds or none of them.

b A manager has received applications for a newly created position that reports to him. From the list, he will pick the person to be hired.

c The executive of an electronics firm will decide whether to release a new microcomputer and what its price will be.

2.2 Consider a decision problem of importance to you now, or in the past, or possibly in the future. For example, it may be to choose a new employer; to choose for purchase a personal computer, automobile, or house; to choose a spouse, etc. For this problem:

a Define a set of objectives you would have in making the decision.

b Identify for each objective some measure of performance or attribute. Use proxy attributes, subjective indexes, and direct preference attributes as appropriate.

c Now apply the tests of a set of attributes given in this chapter to your set of attributes. Organize the attributes into a hierarchy, if appropriate, and decide what level in the hierarchy you would choose to use for your problem.

2.3 Evaluate the following three aggregate objectives relative to their appropriateness for guiding decisions for organizations generally, either private firms, or public institutions:

a Survival of the organization

b Growth (in quality or size of the organization)

c Maximization of the well-being of the organization (or profit of owners of the firm).

2.4 A hospital administrator is developing a simple model to help establish hospital policy. Identify which are decision variables, intermediate variables, and attributes (or outcome variables). Which of the variables might be treated as exogenous? as random? The variables to be included are Morbidity Rate of Population Served by Hospital, Number of Beds, Patient Occupancy Rate, Number of Patients Rehabilitated, and Length of Hospital Stay.

2.5 The Financial Accounting Standards Board (FASB) in a recent study (FASB, 1976) identified the following generally accepted attributes of accounting information: Relevance, Reliability (composed of two subcriteria: Objectivity and Bias), Measurability, and Comparability. Develop your own working definitions of these attributes and decide how they may be quantified in practice.

CASE ASSIGNMENTS

2.1 Westview Environmental Planning (A), page 251.

Using the information in the case (and where necessary your own interpretation of the problem facing Mr. Mills), develop a hierarchy of objectives which you feel adequately represents Mr. Mills's concerns. Using this hierarchy, specify a set of attributes which should be used in evaluating the alternatives. Specify the way in which you would measure each attribute—what are the units of the attribute? Do not attempt to assign number to alternatives, but do think about how you would obtain the numbers for the various attributes.

2.2 William Taylor and Associates (A), page 277.

a Assume the role of Brian Taylor and develop a hierarchy of objectives for choosing an organizational form for William Taylor and Associates. Suggest an attribute for each objective and describe how you would estimate or measure them. Try to develop a hierarchy with only a few objectives at the highest level. Select the level of detail in the hierarchy where you would choose to work through the analysis needed for this problem.

b What objectives are appropriate, in general, for selecting the organizational form for a firm?

c What are appropriate overall objectives for a firm? Should you focus exclusively on maximizing cash flow to shareholders? Stock price? Is it legitimate to include objectives relating to survival of the company and independence from outside interference? How important is the selection of objectives and attributes in corporate modeling?

2.3 Green Valley Foods, page 257.

- **a** Identify decision variables for this planning problem. Are these variables binary, discrete, or continuous?
- **b** What objective(s) would you have for this problem? What attribute would you use for each objective? In what units would you measure each attribute?

2.4 Whirlpool Research and Engineering Division (A), page 207.

What objectives and attributes would you use in treating this problem?

Chapter **3**

THE INFLUENCE DIAGRAM: A TOOL FOR STRUCTURING RELATIONSHIPS AMONG VARIABLES

This chapter is intended to help a manager bridge the gap between a problem and the mathematical model the manager would like to design as a decision aid. In many cases the mathematical model, often expressed as a series of equations, serves poorly as a tool for communicating or structuring a model. In this chapter a graphic device called an *influence diagram* will both display the problem and frame the concept of the model. This diagram may be easily developed from a decision tree, but it is more general and less cumbersome to use for many problems than a decision tree. It is especially valuable to the user of DSS because it is the natural predecessor of the English-language line model used in many new modeling systems.

Like the decision tree from which it evolved, the influence diagram is a powerful aid to presentations. Displaying it (perhaps how it evolved from a simple form to its final form) clearly shows the chief beliefs embodied in the model. Such a presentation can assure a user of the model or a client of the model builder that a messy situation has been understood and brought under some control.

3.1 THE INFLUENCE DIAGRAM

Ease of use and clarity of result are important to any model. Managers with little or no prior experience with computers often must build models that describe their experience and understanding. The tools used for expressing the manager's beliefs should fit the manager's cognitive style, and that often means the tools bear little resemblance to the language needed for carrying out the computer analysis. An ideal tool would fit the manager's thought processes and yet lead directly to the formal language needed to express a model to a computer. The influence diagram has these characteristics.

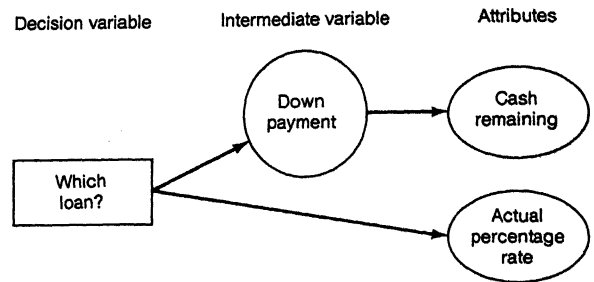



EXHIBIT 3.1 A simple influence diagram for a borrowing decision.

The influence diagram, as it will be used here, is a display of all of the decisions, intermediate variables, and outcome attributes that pertain to a problem, along with the influence relationships among them. By influence we mean a dependency of a variable on the level of another variable. This is a more general use of the influence concept than has been used by others in modeling work.[1]

We will use the convention that a rectangle in an influence diagram represents a decision variable, that a circle is an intermediate variable, and that an oval (or ellipse) signifies an attribute. Exhibit 3.1 shows an influence diagram for a simple borrowing decision. The decision variable indexes which loan is taken. This choice influences the down payment needed, which in turn influences the buyer's cash position. The decision also influences the actual percentage rate on the loan. This example has two outcome attributes: the applicant's cash remaining and the actual percentage rate on the loan.

3.2 VARIABLES AND INFLUENCE ARROWS

A variable in the influence diagram is one of three types described in Chapter 2: binary, discrete, or continuous. The variable may also be exogenous or random. Related variables in an influence diagram are connected by an arrow that indicates the direction of influence. By definition, exogenous variables have no decision variables as direct or indirect predecessors (preceding in influence relationship, not necessarily in time) in the influence diagram. Any variable influenced by a random variable is also a random variable. Any random variable will be noted with a tilde (~) above the variable.

The influence arrow means that the value of the influencing variable is set first and is used in determining the level of any variable that it influences. Three types of influence arrows are used in an influence diagram. A single straight arrow indicates a *certain influence*. An arrow with a squiggle in it stands for an *uncertain* influence. And a double straight arrow signifies a *preference* dependency.

Exhibit 3.2 illustrates these types of influence arrows. For example, the level of fixed assets influences the amount of depreciation as a certain influence. If fixed assets increase, then without question, depreciation will increase. This is indicated by the single straight arrow.

In example B of Exhibit 3.2, Price, a decision variable, will affect Sales, in this case an outcome attribute. But the influence is uncertain. That is, if Price is changed upward, we would expect Sales to be lower, but we are unsure by how much.[2] This is represented by an arrow with a sharp bend in it.

[1]The influence diagram as a formal tool has developed in at least two forms. The earliest use was probably that of J. Forrester (1968), where the influence diagram represents a causal loop in a feedback system. More recently at the Decision Analysis Group at SRI International (see D. Owen (1979) and Howard and Matheson (1980)), the diagram has served to map the variables in a decision problem. The diagrams in this book, which incorporate outcome attributes and distinct types of influence for uncertainty and preference, are more general than the SRI diagrams. These diagrams are also applicable to many uses in addition to modeling feedback loops in system dynamics.

[2]The uncertainty is due, perhaps, to market and competitive factors that we could explicitly account for with separate variables in an elaboration of this model. But we have chosen to lump these other variables together in the uncertain influence, at least at this stage of modeling.

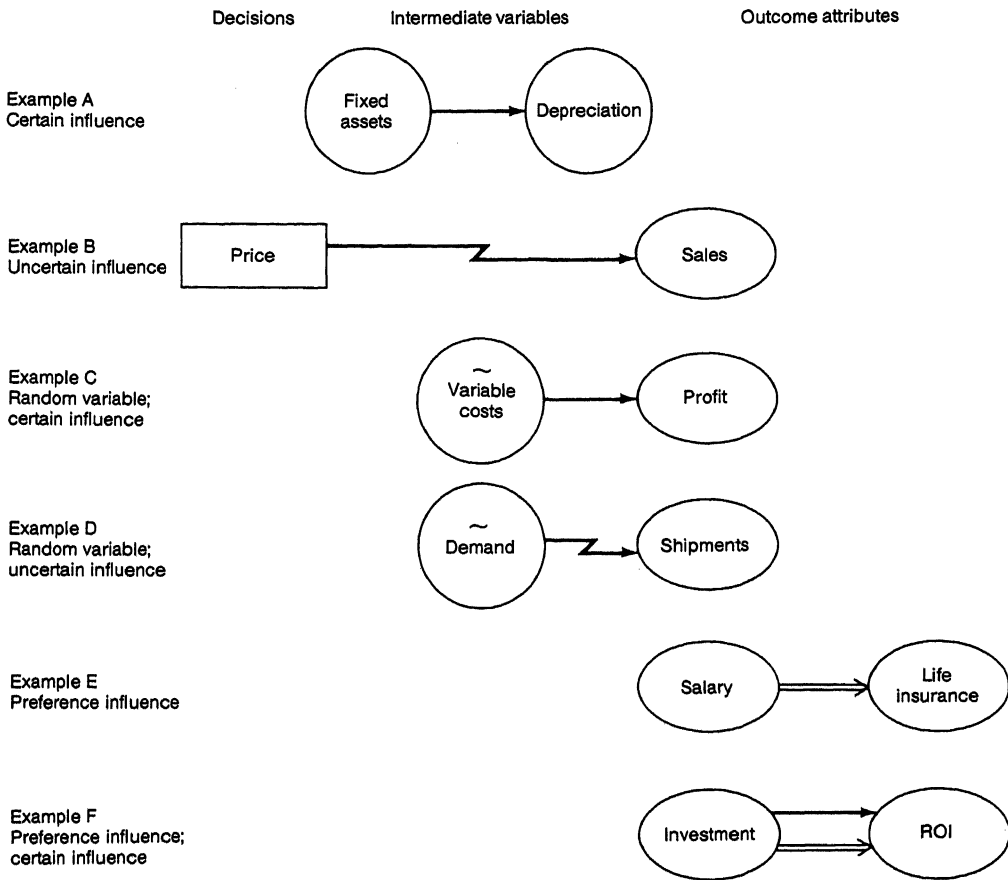



EXHIBIT 3.2 Symbols used in an influence diagram.

A straight arrow indicating a certainty influence may apply even if the influencing variable is a random variable. For example, Variable Costs may be treated as random in a model and, hence, will make Profit uncertain (see example C). However, a straight line of influence between them indicates that, if the level of Variable Cost (now random) were revealed, Profit would likewise be known. This treatment distinguishes a situation in which all uncertainty in a variable is due to uncertainty in a predecessor variable (no squiggle in the arrow) from one in which a variable known with certainty has an uncertain influence on another (a squiggly arrow).

There are important situations where randomness in a variable is due both to randomness in some influencing variable and an uncertain influence. For example, Demand for a product may be random and influence Shipments (example D). But even if Demand were known with certainty, there would still be additional uncertainty in the number of Shipments. (Maybe the number of items produced is variable, for example.) This would suggest that the variables are related through an uncertain influence (see example D).

Note that in Exhibit 3.2, the variables Sales, Profit, and Shipments will be uncertain because an influencing variable is random or an influence is random, but we do not mark these variables with a tilde. Tilde's are used only on variables where randomness originates. When building or using the model, it must be recognized that if there is any randomness anywhere along a path of influence leading up to a variable, that variable will be random. One random variable, if it influences every other variable in a model, can make them all random.

So far the influence relationships that have been described may relate decision variables to intermediate variables or to outcome attributes. They may also relate intermediate variables to outcome attributes, or they may simply relate one intermediate variable to another.

The third type of arrow is used primarily between outcome attributes. A double-line arrow indicates that the decision maker's preference for an attribute is influenced by the level of the predecessor attribute. For example, in a personal financial model, preference for the amount of Life Insurance to carry may be influenced by Salary level (example E). If Salary increases, for example, then higher levels of Life Insurance would be more desirable.

The preference influence should not be confused with the other influences that have been discussed. A preference influence reflects an influence on the *desirability* of the influenced variable, not its *level*.

The preference influence may exist alongside the other type of influence. For example, consider a model for evaluating capital investment where the return on investment (ROI) may be affected by the amount of dollar investment and both ROI and Investment are attributes. It may also be the case that the desirability of various levels of ROI is affected by the amount of Investment. This situation would be drawn as in example F, where there is a single line of influence and also a double line of preference influence running from Investment to ROI.

The types of variables may appear in virtually any precedence order in the influence diagram. In fact, outcome attributes or intermediate variables resulting from one set of decision variables may themselves influence other decisions. For example, Exhibit 3.3 shows an influence diagram having two decisions. First an undergraduate student decides on the Level of Effort to put into his or her studies. A resulting outcome attribute is Grades. But Grades may in turn influence the decision made by a graduate admissions committee whether to admit the student into their program. This decision in turn influences the Career Path that the student pursues.

Two-way influence relationships between variables are also allowed. However, if the variables are treated as random variables, two-way influences become so complex that the solution goes beyond the purposes of this book and will not be covered in later chapters. Two-way relationships between random variables do appear in econometric models and are treated in texts on econometric model building.

Exhibit 3.4*a* illustrates a two-way dependency between the variables Interest Expense and Debt Level. This relationship would be expressed in a line model by two simultaneous equations. For example, the equations might be as follows:

$$\text{Interest Expense} = .08\ (\text{Debt Level})$$
$$\text{Debt Level} = \$10{,}000 + \text{Interest Expense}$$

Having the arrows go in both directions is a way of introducing a simple loop of influence. One might also have a loop involving many variables without using two-way influences. In Exhibit 3.4*b*, for example, both Income and Expenses affect the deficit, which in turn affects Borrowings, which in turn affects Expenses through Interest expense. One reason for drawing an influence diagram prior to writing a mathematical model is to detect loops such as this. Anytime such a loop exists, simultaneous equations are needed in the model and special instructions may be required to enable the model to run.

It is often useful in an influence diagram to have separate variables representing the same item measured in different time periods. For example, Exhibit 3.5*a* is an influence diagram showing the relationships between Assets and Depreciation in different time periods. Year n Assets determine Year n Depreciation, which in turn affects Year n + 1 Assets. This case does not involve a loop since Year n Assets and Year n + 1 Assets are really separate variables.

EXHIBIT 3.3 Student's influence diagram showing an attribute influencing a decision.

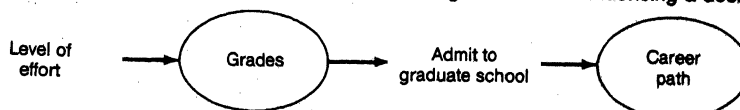
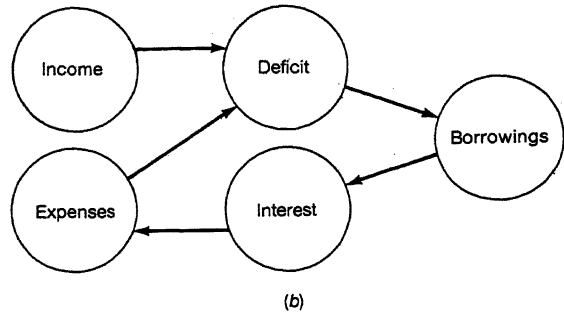
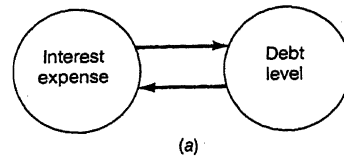



EXHIBIT 3.4 Influence diagrams showing (*a*) a two-way influence and (*b*) a loop.

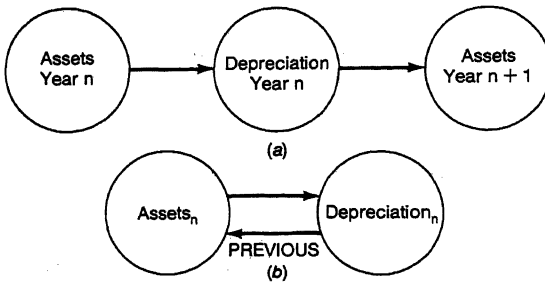



EXHIBIT 3.5 Influence diagrams with variables indexed by time.

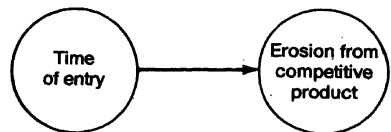



EXHIBIT 3.6 Time as a variable.

Rather than create separate variables for each time period, which may result in a large, cluttered influence diagram, the modeler may instead place subscripts on variables to show that they vary through time. If a variable—Assets, say—is influenced by the previous value of another variable—Depreciation, say—this may be indicated by writing PREVIOUS on the arrow. Thus Exhibit 3.5*b* is another way of writing the general relationship in Exhibit 3.5*a*. A variable influenced by the value of another variable m periods prior would have an arrow coming into it from the other variable with PREVIOUS m beside it.

Sometimes it is easiest to introduce the effects of time by using a separate variable for time. Suppose, for example, that in a new product introduction model, one of the variables is the erosion in sales of the new product that would result from the future entry of a competitive product. The magnitude of that erosion may depend on when the competition introduces their product. This relationship is shown in Exhibit 3.6.

3.3 BUILDING THE STRUCTURE OF A MODEL

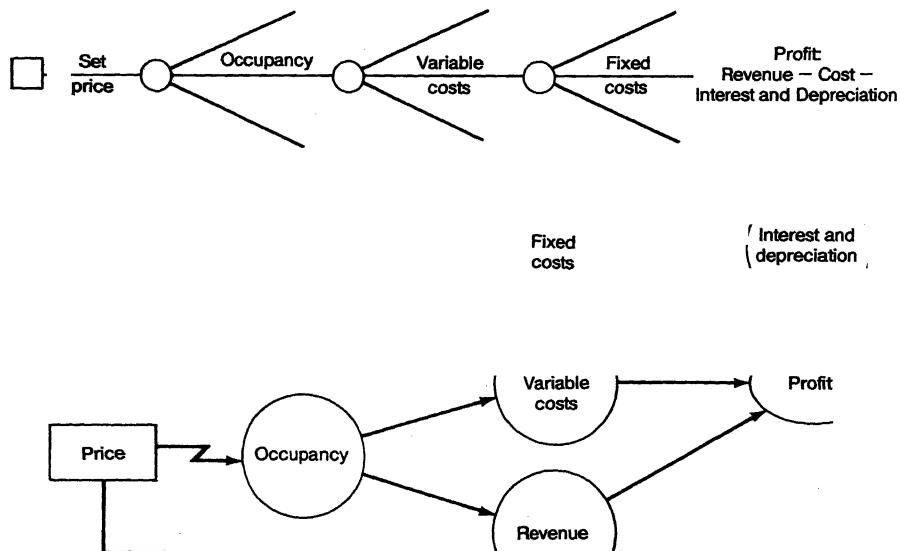
In designing large models, you will quickly discover that any attempt to write down the model in one step is not advisable. You will find it easier to begin with an outline of parts of the model, then to flesh out the details. To create an influence diagram, it is often wisest to start with the two ends of the problem, that is, with the most elemental decision and with the ultimate outcome attribute, then work to fill in the linkage between them.

For example, in real estate management, the elemental decision may be the rental price to set on apartment units. The ultimate attribute is the Profit for the rental units. Obviously, what is needed is the complete set of relationships necessary to get from the price decision to the Profit attribute. Starting from the decision variable, you could build the decision diagram by recognizing that Price will affect Occupancy, which affects Revenue, and so on. Or starting with the Profit attribute, you could work backward by recognizing that Profit is influenced by cash flow and the amount of money invested. The influence diagram is completed by filling in the variables and appropriate arrows that connect revenue and cash flow.

Often the influence diagram is elicited from the decision maker in an interview. Again, start by establishing what the decision variables and the outcome attributes of the problem are using the ideas in Chapter 2. Having done that, a question to ask in order to draw out the rest of the intermediate variables and their relationships would be What variable would you most like to know the value of in order to determine the value of your outcome attributes? If the attribute is Profit, the decision maker might say Revenue, then ask what variable the decision maker would like to know to determine Revenue; he or she will say Price, and so on.

Coyle (1977) suggests six possible justifications for an influence between variables: conservation considerations (mass balance), direction observation, instructions (or policy statements) of a system, accepted theory, hypothesis or assumption, and statistical evidence. It is useful to think of the justification that may be appropriate, not only to avoid modeling erroneous influences but also to be able to understand what the influence is.

Some people find that the easiest first step in modeling is to structure the problem using a decision tree. A decision tree can always be converted into an influence diagram simply by including the variables at the nodes of the decision tree in the diagram and by adding arrows flowing between them in the same direction as the tree opens up. Exhibit 3.7 illustrates the

EXHIBIT 3.7 A real estate management decision tree and associated influence diagram.

influence diagram that corresponds to a decision tree for the real estate management problem mentioned previously. Note that the influence diagram has certain flexibilities not available in the decision tree. In the decision tree you must either put Fixed Costs in front of or behind Variable Costs (thus suggesting that it may influence or be influenced by Variable Costs), but the influence diagram can show that no influence relationship exists between them. If the decision tree of Exhibit 3.7 were drawn in complete detail, it would show Fixed Costs branches for each outcome of Variable Costs, even though the Fixed Costs outcomes would not change with the outcome for Variable Costs. Thus, the influence diagram can show relationships more concisely and accurately than a decision tree.

An influence diagram can also indicate a wider range of intricate relationships than a decision tree. Exhibit 3.7 shows Revenue being influenced directly by Price and also being *indirectly* influenced by Price through Occupancy. This subtlety could not be depicted by a decision tree. In an influence diagram, a variable may be influenced by any number of predecessors that may have any structure of relationships among each other.

Even though an influence diagram can be created for any decision tree, not every influence diagram can be converted into a decision tree. Loops in the influence diagram, such as in Exhibit 3.4, cannot be accommodated in a decision tree. Thus the influence diagram is a tool of greater generality for structuring models than a decision tree.

Another feature of an influence diagram is that it is expandable. A variable can be broken down to more fine-grained variables, and additional influencing or influenced variables can be added with ease for more detail or completeness. In the real estate management problem, for instance, the aggregate variable Interest and Depreciation could be divided into Interest and another variable Depreciation with the determinants of these variables defined separately.

A major reason the influence diagram is a boon to modeling is that it lets you get a model working accurately with a simple structure. You can expand a working model with confidence, whereas a model that starts out complex can be very slow and demoralizing to get running and to check out. The specific needs of the model and the judgment of the modeler will, of course, dictate the level of detail in the influence diagram. It is wise to develop the diagram in stages, with detail added only as needed.

3.3.1 Caveats in Building an Influence Diagram

The influence diagram appears to be a simple enough tool, but first-time modelers often unconsciously fall into traps in using them. The following items should help prevent both misuse and overuse:

1 The influence diagram is not

a A flow chart; it does not indicate things such as "units shipped from point A to point B."

b A precedence chart, such as might be used in PERT planning. An arrow does not mean "must be followed by."

c A representation of hierarchical structure, as in an organizational chart. An arrow does not mean "is an element of."

2 Think of variables as levels of some quantity and arrows as effects of variables on each other. Make sure it is clear what higher or lower levels mean. Use a variable such as Cost or Revenue rather than Economic Performance.

3 Divide variables into two or more variables only when more detailed expression is needed for clarity. Thus inflation is fine unless the distinction between Consumer Price Index and Producer Price Index is needed.

4 If (and only if) an influence arrow needs more explanation, insert another variable. For example, if the influence of Price on Sales is not clear, expand it to Price influences Demand, which in turn influences Sales.

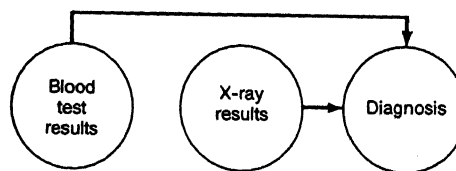
3.4 THE INFORMATION STRUCTURE OF AN INFLUENCE DIAGRAM

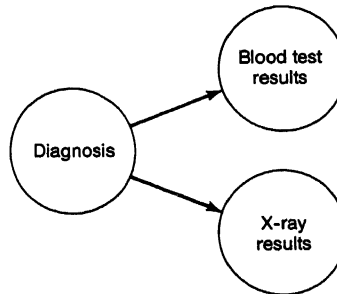
The influence diagram tells a great deal about the information that is assumed to be available for a decision and the structure of that information in a model.

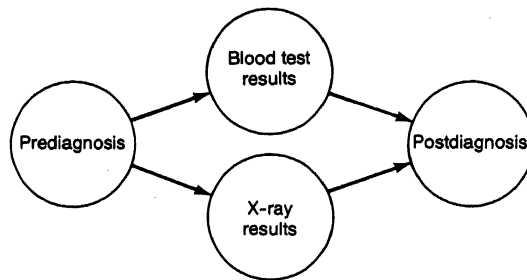
Arrows leading *into* a decision node represent informational influences. These identify variables that will be known at the time a choice is made. For example, suppose in a medical decision problem, a decision must be made on the type of Surgical Procedure to perform on the patient. An informational influence of the Diagnosis on the surgery decision would be represented by

In a situation such as this, the choice of Surgical Procedure may be clearcut, given Diagnosis. Contrast this with the situation where there is no such informational influence, for example, where Surgical Procedure will reveal Diagnosis. This situation may be represented by an influence running the other direction:

These two situations exhibit the extremes of information. In the first case it is assumed that there is perfect information on the Diagnosis when the decision is made, and in the second case it is assumed there is no information available. In Chapter 5, we will contrast these two situations in order to determine the *Value of Information*, that is, what information is worth in terms of improving decisions.

Another aspect of information depicted by an influence diagram is the structure of *conditioning influences*. All arrows leading into an intermediate variable modify that variable. For example, suppose there are results from two separate tests that influence a medical diagnosis as follows:

Clearly, Diagnosis will then be affected by these results. This represents quite a different information structure than if there were no test results available, or, as the opposite informational

extreme, if the diagnosis were known before test results and therefore affected test results, as below:

Here Diagnosis is unconditional, that is, it is not influenced by any variable. But the test results are conditional on Diagnosis.

Sometimes, the true situation can only be reflected by designating multiple versions of the same variable to reflect various states of information. For example, the following diagram may more accurately reflect the development of information and its effects on Diagnosis.

When the influence is uncertain (a squiggly arrow), then the information used in setting the influenced variable is, of course, probabilistic. This information is less reliable than information effects indicated by a straight arrow. More will be said about the value of probabilistic information in Chapter 5.

The influence arrow, whether straight or not, may or may not indicate a cause and effect relationship. It does reflect *association* between two variables, but not necessarily that the influencing variable *causes* changes in the influenced variable. For example, the number of chickens in the United States is associated with the number of eggs produced annually. You may choose to use in an influence diagram either an influence from the number of eggs to the number of chickens or an influence from the number of chickens to the number of eggs. To use either influence it need not be settled whether chickens cause eggs or vice versa in order to incorporate the association or influence in a model.

3.5 THE COMPLETED DIAGRAM

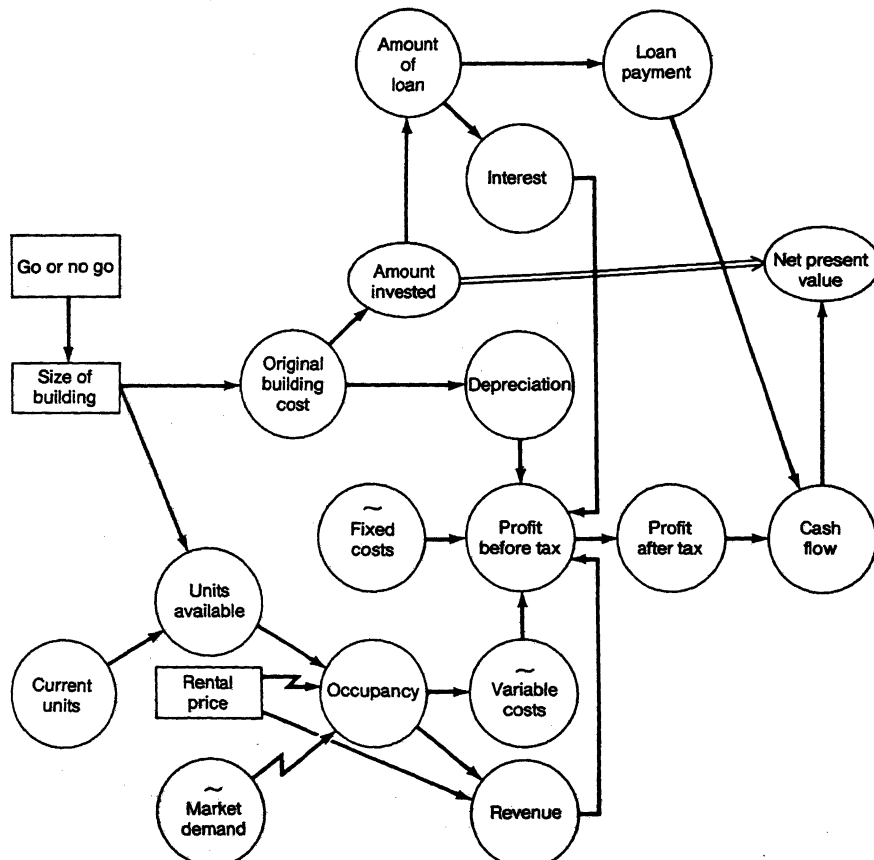
A completed influence diagram expresses the structure of a formal model. It provides participants in the modeling process a means of communication. It also serves as the framework for expressing more specifically the exact nature of the influence relationships. A variety of specific mathematical forms of relationship can be given for each arrow in an influence diagram. Some possibilities are discussed in the next chapter.

Exhibit 3.8 shows a completed influence diagram of a real estate investment model that was to be used by a real estate firm in evaluating new apartment-building projects. The company faced three decisions in any new project: (1) whether to take it on, (2) how big a building to build (or whether to expand a purchased building), and (3) what to charge for rent.

The diagram has only two uncertain influences: the influence on Occupancy of Rental Price and of uncertain Market Demand. Two other variables, Fixed Costs and Variable Costs, were treated as random.

The diagram contains two outcome attributes, Amount Invested and Net Present Value. A preference influence runs from Amount Invested to Net Present Value to reflect the decision maker's feeling that a larger investment should generate greater Cash Flow. Interestingly, there is also an influence from Amount Invested to the level of Net Present Value. This influence is indirect, because Amount Invested affects the initial Cash Flow. Again, such separate influences present no ambiguity, since they are totally different concepts of influence.

The company felt that enough similarity existed among different projects to allow one general model to evaluate all projects being considered. This general model is described adequately by the influence diagram. Two additional steps are necessary to generate the model from the diagram, however. First, the mathematical forms of relationship (functions) that apply to each arrow in the diagram must be identified. The functions for most of the arrows would be the same for all projects considered (for example, the same form of depreciation rule would be applied to each project). The second step is to supply specific numbers to these functions. These numbers would, of course, vary by project. Chapter 4 discusses the choice of mathematical function and the estimation of numbers. Elements of this real estate problem are continued as illustrative examples in that chapter.

EXHIBIT 3.8 Completed influence diagram for real estate investment decision.

KEY TERMS

influence	certain influence
uncertain influence	influence loop
preference influence	structure of information

EXERCISES

3.1 Try to depict your understanding of how a variety of economic variables affect each other. Develop an influence diagram that relates the following macroeconomic variables to each other:

Savings
Risk-Free Interest Rate
Deficit Spending by the Federal Government (Tax Revenue less Expenditures)
Inflation
Money Supply
Gross National Product

3.2 Structure an influence diagram for an inventory system. Include at least the following variables: Orders, Backlog Orders, Production Capacity, Production Level, Shipments, and Inventory.

3.3 Draw an influence diagram that would be used to build a model for studying the use of work time of a stockbroker. Include at least the following variables: Phone Calls received per Day, Phone Calls made per Day, Length of Phone Calls (you may wish to distinguish between those received and those made), Orders Taken per Day, Reading Time per Day, Time Spent Generating New Business. On each influence arrow, place a positive or negative sign indicating whether a higher level of the influencing variable tends to make the influenced variable move up or down.

3.4 Modeling Small Group Dynamics

Suppose you wish to study the characteristics and dynamics of a small group working on a required activity imposed from outside the group. In particular, you want to know how different levels of required activity will affect the group. It has been decided that a simple model would include four variables which change over time:

I, the intensity of interaction between group members
F, the level of friendship which exists between group members
A, the amount of activity within the group
E, the amount of activity imposed on the group by the external environment

These variables can be treated as averages over all group members or as some overall measure for the entire group. I, A, and E could be expressed in terms of hours. F would be measured on some subjective scale. E may be considered an exogenous decision variable (one use of the model would be to give recommendations on how high to set E) and the other variables, I, F, and A, may be both outcome variables (or attributes) as well as intermediate variables.

The model might be used to simulate I that takes place within an MBA study group. E would relate to the group work required by the program. A would include reading cases, library research, and studying course materials or preparing presentations for class. I takes place when the actions of one group member set off the actions of another. A is necessary for I, but not conversely—an individual member of the group can work alone. Situations which would include I are discussing cases with other members of the group, passing information from one to another, doing group projects, or simply conversing idly. Some I is necessary in order to perform the required task, but some of it is a by-product of F formed by being a member of the group. The same can be said about A; it interacts with F, and some level of A would take place even without externally imposed work.

Draw an influence diagram involving the four variables.

3.5 Draw an influence diagram for the elements of a predator-prey ecosystem. Include the following variables: Food for the Predator, Food for the Prey, Number of Predators, Number of Prey Kills by Predator, Number of Kills by Humans (of prey and predator separately).

CASE ASSIGNMENTS

3.1 *Green Valley Foods*, page 257.

Recalling your efforts in identifying variables and objectives from Case Assignment 2.3, now structure the variables and attributes into an influence diagram.

3.2 *DiscoMall, Inc.*, page 233.
Begin to structure an influence diagram for a model to carry out the financial analysis. This is an involved case and the modeling may at first seem overwhelming. Therefore, force yourself to stick with just the major variables in this first-cut structuring effort. It may help you to keep the following questions in mind when thinking about a model for this problem. What is DiscoMall selling? To whom? How do they make money? What will determine their success or failure?

CREATING AND USING A MODEL IN A HIGH-LEVEL LANGUAGE

Chapter **4**

MODELING DEPENDENCIES

The process of developing a quantitative model may be described in five steps:

1 Identifying the objectives of the decision maker and the associated performance measures or attributes, the decision variables, and all other variables to be used in the model
2 Structuring the influence relationships among the attributes and variables of the model
3 Specifying the form of relationship for each influence
4 Estimating the numbers that are to be used in the model
5 Using the model to analyze the problem, including a sensitivity analysis and a risk analysis

This chapter discusses step 3 of this process. It assumes the first two steps have been carried out as described in Chapters 2 and 3. That is, the problem structure has been delineated in the form of an influence diagram which includes all decisions, intermediate variables, and attributes. For each influence arrow in that diagram, a specific type of dependency is needed. Only when these dependencies are established and numbers are estimated for variables will there be a working model. Step 5 will be the subject of the next chapter.

Step 4 is covered only very lightly in this chapter and in the rest of the book. Estimation is a crucial step that must be done well. It is not treated in depth here, however, since good presentations of a variety of methodologies are available in other sources. Most introductory texts in statistics [e.g., Hamburg (1977)] or quantitative methods for business [e.g., Vatter et al. (1978)] include a discussion of regression analysis and the standard forms of data analysis. Another useful source is the *Handbook of Forecasting* (Makridakis and Wheelwright, eds., 1982).

While providing only a few helpful but informal pieces of advice on estimation, this book provides through the cases what others may not. A very important part of skill development is grappling with realistic and challenging problem situations. The stumbling block of managers

is often not technical ignorance of regression and other statistical methods of estimation. References or technically trained staff can help with methodological problems. What managers need to develop is judgment about modeling. This requires first an understanding of the meaning of various forms of models, which is the topic of this chapter. Secondly, the manager needs experience, which is obtained from trying out ideas on a few exercises and cases.

As each form of relationship the manager is likely to encounter or need is presented we give examples of situations where these relationships would exist and ideas on how to test for misapplications. These aids, along with the cases, serve to build the manager's experience and judgment.

4.1 SPECIFYING THE FORM OF A RELATIONSHIP

At the outset, we should clarify the distinctions among steps 2, 3, and 4 mentioned at the beginning of this chapter. Each of these steps defines a specific aspect of the relationship between any pair of related variables. These aspects may be concisely summarized as step 2, influence; step 3, function; and step 4, numbers.

Suppose, for example, that two variables in an influence diagram are Profit Before Tax and Profit After Tax. Of course, the influence arrow (step 2) would run from Profit Before Tax to Profit After Tax. With respect to this influence, Profit Before Tax would be called the *independent variable* and Profit After Tax the *dependent variable*.

The function (step 3) could be specified by writing

Profit After Tax = (1 − tax rate) * Profit Before Tax

where * is a symbol for multiplication. (Addition, subtraction, and division are given by the usual symbols +, −, and /, respectively.)

The numbers (step 4) can be included by quantifying every item in the expression except the variables. Thus the relationship is completely expressed by a statement such as the following:

Profit After Tax = .54 * (Profit Before Tax)

Separating the task into the two steps of choosing the function and setting the numbers is not pedantry. Both function and numbers must be right for the model to be of value. They can be tested separately. If (1 − tax rate) is wrong and should be, say, .52, the model will give erroneous results. Similarly, if the function is wrong, the model will give erroneous results, even though the numbers may be right for many possible values of the independent variable. For example, taxes are due only when Profit Before Tax is positive. The function should therefore be expressed as

Profit After Tax = IF Profit Before Tax .LE. 0 THEN Profit Before Tax
ELSE (1 − tax rate) * Profit Before Tax

This statement uses the IF . . . THEN logic of the IFPS to determine whether Profit Before Tax is less than or equal to 0 (denoted .LE. 0)[1]. If so, then no taxes are taken out in setting Profit After Tax, otherwise taxes are deducted according to the tax rate. Clearly, for some values of independent variables, errors can be more serious from misstating the function than from miscalibrating the numbers. Of course we would like to have both aspects of a model correct.

The example just given demonstrates that writing down a function conveniently requires conventions of language, such as in the IF . . . THEN logic. Often a function may be communicated most quickly by a graph. A plot of Profit After Tax against Profit Before Tax is

[1]Other conventions for numerical comparison using IF . . . THEN logic in IFPS are .LT. (less than), .LE. (less than or equal), .EQ. (equal), .GE. (greater than or equal), .GT. (greater than), .NE. (not equal).

given in Exhibit 4.1. The general relationship of a function can be shown without supplying the number scale to the axes. The tax rate number could be filled in later in step 4.

There are various devices now available for transferring the graph to a computer. In fact, if you have the right hardware, the computer will accept whatever you can draw. Among the devices available for delivering graphical data to the computer are light pens and graphics digitizers. With a light pen, the graph is drawn on a cathode-ray tube (TV) screen. Graphics digitizers may be flat electronic tablets on which you draw the graph using an electronic pen, or they may consist of an electronic "mouse" that you move along a flat surface to sketch out the graph. The mouse rides on a roller that transfers your movements along the surface into X and Y values of points on the graph you are drawing.

Many times you are without the hardware to give graphical information directly to a computer, or your software may be inadequate to make any modeling use of relationships expressed graphically, or you simply want to communicate a relationship compactly in writing. You then need language to express functions. This should not be a language that is foreign to you, but one that is natural and English-like. There are now available many high-level modeling languages that allow you to express relationships among variables simply. They are designed to calculate and present the results of assigning specific numbers to independent variables quickly and easily.

IFPS has been chosen to present the examples in this book due to its modeling flexibility and natural, easy-to-use features. It is not unlike other financial planning languages on mainframe computers now found in most large corporations, but it is among the most widely used. Because IFPS uses English-like lines to write models, the examples in this book require little translation to be used with other mainframe modeling languages or those for microcomputers.

In the rest of this section, we describe a long list of possible functions that may be used in building a model. An example of each will be given both graphically and as lines in the IFPS language. Where the conventions of IFPS are used, function names will appear in uppercase letters (as in IF . . . THEN). For the remainder of this section, we assume certainty in the relationships; none of the examples include random variables. In Section 4.2, random variables and uncertain relationships are introduced.

4.1.1 A Menu of Functions

Linear Perhaps the most common general function is the straight-line, or linear, function,

$$Y = a + b * X$$

where a and b are constants and X and Y are variables. Suppose that two variables in a model are Total Cost and Production Volume. Further, you have decided that the latter influences the former. In many instances this is a good candidate for a linear relationship of the following form:

Profit after tax

Slope = 1 − tax rate

Profit before tax

EXHIBIT 4.1 Graph of a relationship.

Total Cost = a + b * Production Volume

The assumption inherent in this expression, as in any linear expression, is the following:

> A one-unit increase in the independent variable (Production Volume) will result in the same increase in the dependent variable (Total Cost) for any starting level of the independent variable.

In this example, a would be interpreted as the fixed costs of production, and b the variable costs per unit of production. Naturally, both a and b are positive numbers for this example. In the general linear model, however, a and b may be positive or negative numbers. Exhibit 4.2 shows linear relationships for a variety of values of a and b. Note that a is the intercept of the line with the Y axis and b represents the slope of the line.

Quadratic Sometimes the increase in the dependent variable for a unit increase in the independent variable changes for different levels of the independent variable. You may wish to have a relationship where the dependent variable changes at an increasing or a decreasing rate; and it may be appropriate for the increasing rate or decreasing rate to be linear. This would make the relationship quadratic. For example, suppose in a model Market Share is influenced by Advertising Share (company advertising divided by industry advertising). It has been determined that as Advertising Share goes up, Market Share rises. Suppose that advertising builds on itself in that a one-point increase in Advertising Share brings an *increase* in Market Share that is directly proportional to the level of Advertising Share. Then the following quadratic form for the relationship would be appropriate with b (a constant) positive:

Market Share = a + b * Advertising Share * Advertising Share

A graph of this expression for specific values of positive a and b is shown in Exhibit 4.3.

It is convenient to use some shorthand here rather than multiplying the independent variable twice. The expression X^2 will mean the same as X * X. More generally X^n will denote X * X * X * . . . * X n times. The IFPS convention for this expression is Y = XPOWERY(X,n).

If Market Share were to decrease at an increasing rate, the quadratic form would have a negative b. Such a case is shown in Exhibit 4.4 along with curves for other values of a and b. Each quadratic form satisfies the general property:

> The increase or decrease in the dependent variable increases at a rate that is linear in the dependent variable.

Power Curves The linear and quadratic relationships begin a natural progression of possible relationships. The next step is the cubic relationship $Y = X^3$, then comes the quartic relationship $Y = X^4$, and so on. An example of a natural cubic relationship would be

Warehouse Space Needed = a + b * Diameter^3

where Warehouse Space Needed is measured in cubic feet and Diameter in feet measures the size of spherical items to be stored in boxes in the warehouse. In this case we would expect b to be 1.0 and a to be the amount of space for maneuvering in the warehouse and for storage of equipment used in the maintenance of the building.

Exhibit 4.5 shows the family of curves $Y = X^c$ for c = 3, 4, 5. As c gets larger, Y increases at an ever-increasing rate. A more general family of curves could be expressed through

$Y = a + b X^c$ = a + b * XPOWERY(X,c)

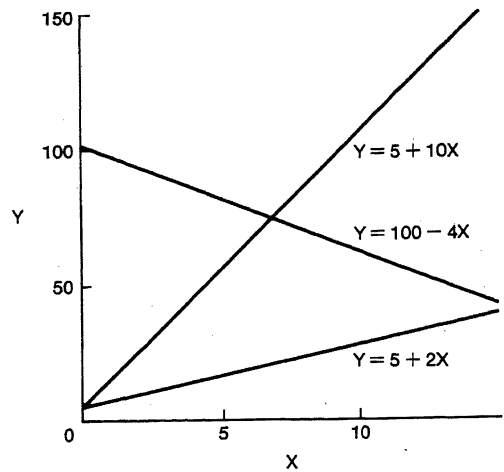



EXHIBIT 4.2 Linear relationships for a variety of values.

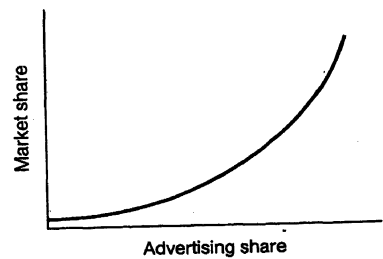



EXHIBIT 4.3 An increasing quadratic relationship.

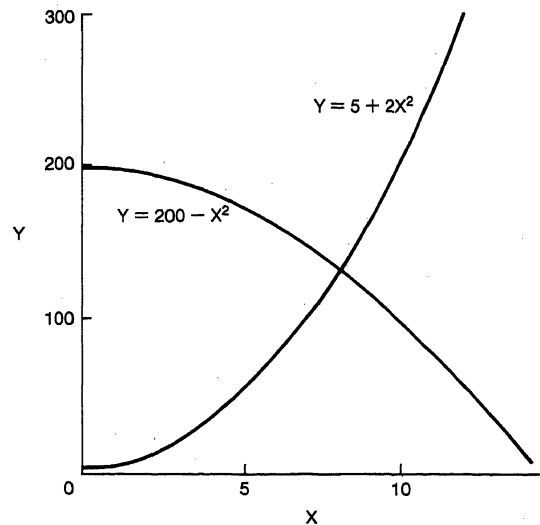



EXHIBIT 4.4 Some possible quadratic forms.

where a would be the intercept of the line on the Y axis. The constant b scales the values of Y and consequently the rate of increase. A higher b implies a higher rate of increase of Y.

Fractional Power Curves There is no reason that the exponent of X cannot be a number between 0 and 1. For example, Y may be related to the square root of X, that is $Y = X^{.5}$ = XPOWERY(X,.5). This may be appropriate in some instances where there are economies of scale to relate Total Cost to Production Volume. The expression would be

$$\text{Total Cost} = a + b * \text{Production Volume}^{.5}$$

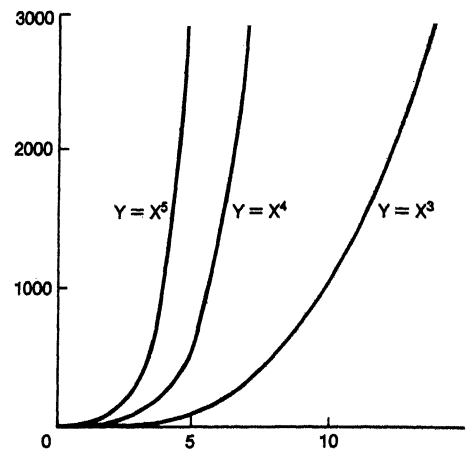



EXHIBIT 4.5 Examples of power curves.

where a again represents fixed costs. Clearly, nonfixed costs double when Production Volume quadruples. Exhibit 4.6 shows a family of curves with the general shape $Y = X^{1/c} =$ XPOWERY(X,1/c) for c = 2, 3, 4.

Exponential A very useful relationship, particularly for modeling growth, is the exponential curve. Population, for example, would normally grow exponentially in Time. Thus the relationship may be written as follows:

$$\text{Population} = e^{c*\text{Time}} = \text{NATEXP}(c * \text{Time})$$

where e = 2.71828 is a natural mathematical constant (like pi). The constant c expresses the growth rate, which may be a positive or negative number. Exhibit 4.7 shows examples for both positive and negative c, and therefore increasing or decreasing Y. The exponential exhibits the following property:

The increase or decrease in Y as a percentage of Y per unit of change in X is constant.

EXHIBIT 4.6 Fractional power curves.

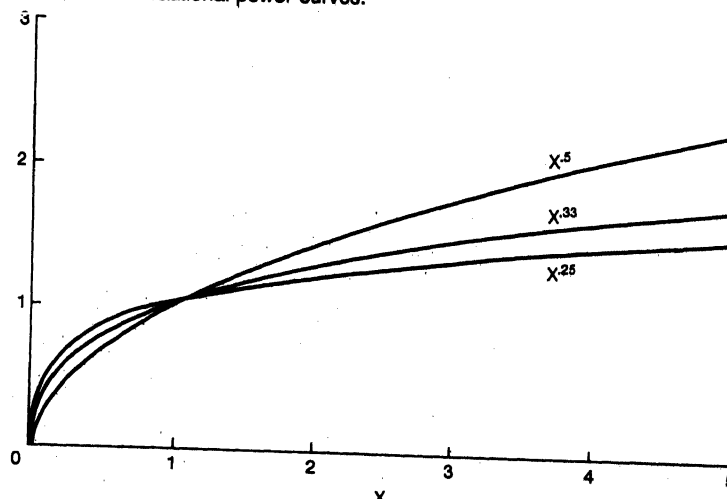
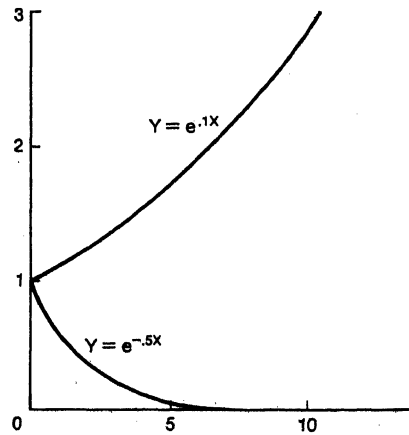



EXHIBIT 4.7 Exponential relationships.

Thus the population at year n + 1 is c percent more than the population at time n for any n in the exponential relationship. Many economic growth variables, price levels, and social indicators fit the exponential model through time.

Logarithmic Sometimes it is useful to use the inverse of the exponential model, the logarithmic function. The number Y, such that when used as the power of e gives an amount equal to X, is known as the *logarithm* of X. For example, suppose we are modeling a competitive marketing situation and we wish to evaluate the Time to Entry of a competitor's product. We believe the competitor will enter only when the market is several times larger than now, the exact multiplier being a variable called Critical Size. We project that the market will grow exponentially at a rate of 20 percent. Then

$$\text{Time to Entry} = 5 * \ln(\text{Critical Size}) = 5 * \text{NATLOG}(\text{Critical Size})$$

where ln is the mathematical symbol for the natural logarithm. To verify that this is the inverse of the exponential form of market growth, write an expression that is equivalent to the one above

$$\ln(\text{Critical Size}) = .20\,(\text{Time to Entry})$$

This is equivalent to

$$\text{Critical Size} = e^{.20\,(\text{Time to Entry})}$$

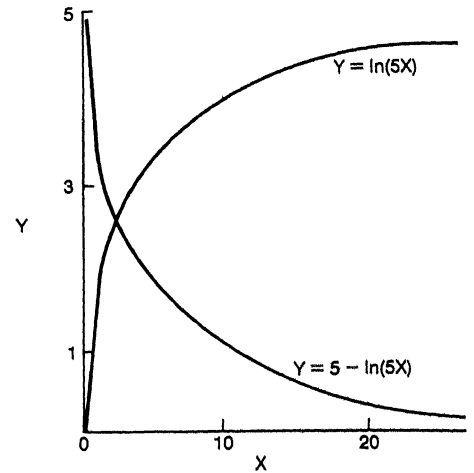
which is the familiar form of exponential growth.

Exhibit 4.8 shows forms of

$$Y = a + b * \ln(X)$$

for b taking both positive and negative values. Note that ln (X) goes to 0 at X = 1 and to negative infinity at X = 0. The curve exhibits increasing Y, but at a decreasing rate, as in the fractional power curves. For large X, the slope goes to zero; the curve approaches horizontal.

Logistic The S-shaped logistic function is a natural model of a variety of situations, particularly life-cycle growth. For example, the initial growth of sales of a new consumer

EXHIBIT 4.8 Logarithmic curves.

product may have an S shape: start-up growth is slow, then picks up to a fast growth stage, then tails off as the market becomes saturated. The general expression

$$Y = \frac{}{1 + be^{-cX}}$$

has parameters setting how high the curve goes (a), how wide it is (c), and how much of an S there is to it (b).

Exhibit 4.9 shows different versions of the S-shaped logistic function. It is the modeler's prerogative to pick the curve that looks right or best fits a theoretical model or data. Conferring with people experienced in the business at hand can often define a point or two along the curve and implicitly some of the parameters of the curve.

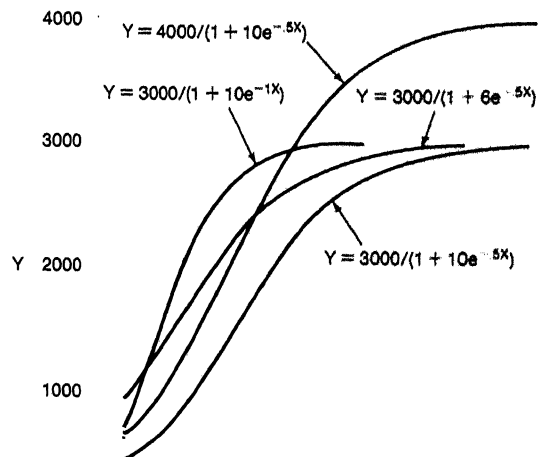



EXHIBIT 4.9 The logistic family of s-shaped curves.

The S shape might just as easily be used to model the "death" stage of a product. For example the following IFPS statement gives a model for the entire life cycle of a product:

Sales = IF Time .LE. 10 THEN 10000/(1 + 20 * NATEXP(-.9 * Time))
ELSE 10000 - (10000/(1 + 10 * NATEXP(-.8 * (Time - 10)))

The IF . . . THEN expressions in IFPS allow the division of the range of Time into two parts. When Time is less than or equal to 10, the logistic curve that is expressed following THEN is used. Otherwise, the logistic curve following ELSE is employed. Exhibit 4.10 shows a graph of Sales versus Time, where the expression just given results in a logistic-function growth stage (prior to Time = 10) followed by a similar mirror-image logistic-function death stage. The death stage has a different shape from the growth stage since the parameters in the logistic function are different. For example, b = 10 in the death stage and b = 20 in the growth stage, making the death stage more compressed with less of a bend to it.

We will stop now pulling out new functions from our mathematical bag of tricks. Enough shapes have been given for most common needs, particularly when you consider the range of combinations that can be had by adding shapes together.

In general it is best to start with simple shapes and with assumptions that you understand. Do not waste time and effort refining the shape of the curve until preliminary work with the model shows that the precise shape of the curve matters. It rarely will! It often helps to graph expressions for likely values of parameters in order to be sure first that the general shape of the curve is appropriate and then to find the right values of the constants. Again, while we do not describe how to do it here, there are many references and expert statistical help available to most managers to carry out an estimation of the right constants. But the manager often must decide on intuitive grounds if the general shape of the curve and its assumptions are appropriate.

What do you do when the shape you feel you need is not one of the ones that has been presented? Do not panic! Many modeling systems make it possible for the computer to take almost any shape curve you can draw, at least in an approximate way. The system may take your curve and turn it into a piecewise-linear curve, which looks as though someone used a straightedge to connect some points on the curve (Exhibit 4.11).

The following statement uses the IFPS function INTERPOLATION ON to give the curve shown in Exhibit 4.11:

Sales Volume = INTERPOLATION ON (Sales Price,1.1,11000,1.2,10900,'
1.4,10500,1.5,9500,1.6,8000,1.7,7000,1.9,6000,2.1,5500,2.4,5000)

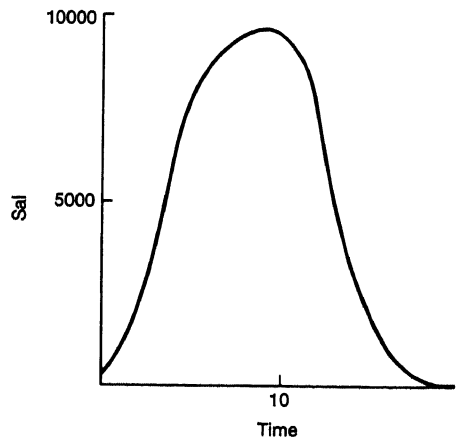
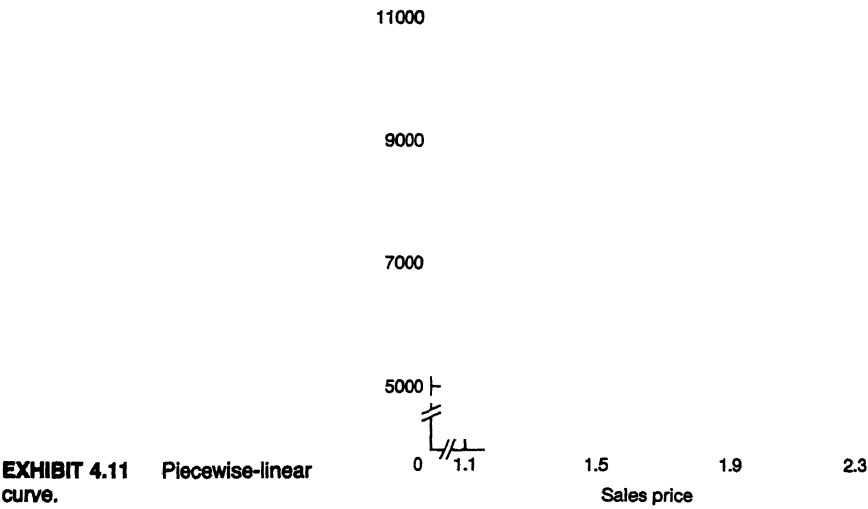



EXHIBIT 4.10 Product life-cycle curve using logistic functions back-to-back.

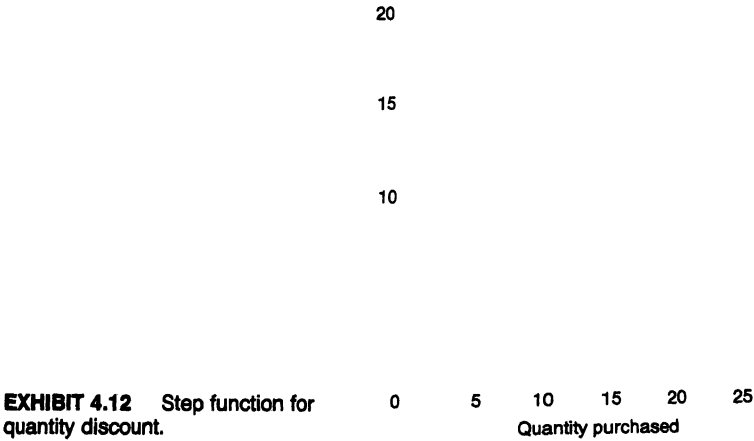
EXHIBIT 4.11 Piecewise-linear curve.

The numbers defining the function are simply pairs of X and Y values for points on the curve. If you want the approximation to the true curve to be more accurate, you simply use more points in the INTERPOLATION ON function. How many? You must make a judgment on how close the piecewise-linear curve needs to approximate the true curve.

Sometimes you want a function in which the vertical (or dependent) variable is discrete. If the independent variable is also discrete, then you simply build a table showing the values of Y that correspond to the values of X. If the influencing variable is continuous, however, you may need a step function to turn the dependent variable into discrete values. For example, suppose a discount is given for quantity purchases of microcomputers. The step function in Exhibit 4.12 relates the Discount Percentage to Quantity Purchased. In IFPS the expression for this curve is

Discount Percentage = STEP(Quantity Purchased,0,0,2,5,10,10,25,20)

The numbers in this expression are the string of XY pairs for the beginning of each new stair step.

A step function could be used to create a binary dependent variable by using just one step in the stairs. Alternatively, the same effect could be had using IF . . . THEN logic. For example,

EXHIBIT 4.12 Step function for quantity discount.

suppose there is buried in some model a variable which tracks whether sales of a product are satisfactory over time. The target for sales of the product is 120. Then the relationship of a binary variable Sales Condition dependent on Sales could be written:

Sales Condition = IF Sales .GT. 120 THEN 1 ELSE 0

The binary variable is set to 1 when Sales exceeds 120 and to 0 otherwise.

There are some features of most modeling languages that are not of themselves very useful to express Y as a function of X, but when used in conjunction with features already described can be quite helpful. These are ABS, ROUND, ROUNDUP, and TRUNCATE in IFPS. ABS finds the absolute value of numbers; that is, it removes the minus sign from negative numbers. ROUND finds the closest integer value. ROUNDUP increases numbers with fractional parts to their next higher integer value. TRUNCATE removes all fractional parts; it could have been named "round down."

4.1.2 Modeling the Influence of More Than One Independent Variable

In many models, more than one independent variable may influence a dependent variable. Perhaps the simplest function of more than one variable is the MAXIMUM (or MINIMUM) function. For example, Unit Sales may depend on two variables, Production Level and Demand. Clearly, one cannot sell an amount that exceeds demand, nor any more than you have produced. So

Unit Sales = MINIMUM(Production Level,Demand)

This minimum may be used when there are more than two independent variables. In IFPS, as many as nine arguments are allowed in the MINIMUM or MAXIMUM function.

Of course more than one of the menu of functions in the previous section may be used together to define a new shape. The expression

$$\text{Sales} = a + b * \text{Time} + c * \text{Time}^2$$

has both a linear and a quadratic component to it. The shape that results will depend on the relative magnitudes of b and c. Exhibit 4.13 shows plots for several different values of the constants b and c.

You can get a feel for the shape that results from adding two functions by envisioning the two functions stacked on top of each other (e.g., the case b = c in Exhibit 4.13 looks as though the quadratic form is placed on top of the linear form on a point-by-point basis). When the weightings placed on the two functions are unequal, it is trickier to see how they combine; but the higher the relative weighting of b (or c), the more pronounced is the linear (or quadratic) effect. Some experimentation (or prior experience) helps in seeing the shape that results from adding two or more functions together.

In this example, there really is only one independent variable, Time, even though we used both linear and quadratic functions in this independent variable. When there are two or more independent variables, different shapes may be used for expressing the relationship of the dependent variable to each independent variable. For example, family expenditures may increase linearly with the age of children at home and logarithmically with the number of children at home.

In order to determine what combination of shapes to use, it is appropriate first to envision the relationship of the dependent variable to each independent variable and then think about what happens when the separate terms are added together. Again, graphing the relationships will help, although if you want to see the relationship to two independent variables, you will

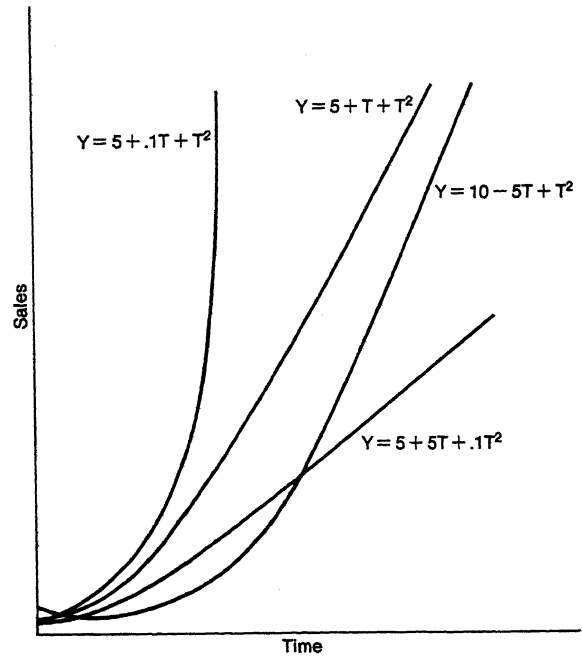



EXHIBIT 4.13 Sum of linear and quadratic functions.

need to produce a three-dimensional graph. If you do not understand a prospective model, get expert help or use a simpler model.

Clearly, there are a great many possible combinations of functions and shapes, and there are other functions that we have not mentioned. There is great flexibility to express the kinds of relationships that are most appropriate. Consider one more way to increase the modeling options.

We have mentioned and given examples of adding together two or more shapes. Another possibility is to multiply them to get a product form. An example of the product form is a model that is often used to estimate sales for a product:

Sales = Market Size * Market Share

where Market Size refers to total sales in the market segment to which the product belongs. This is a linear product model. That is, if one of the independent variables is held constant, the dependent variable varies linearly with the other one.

A product model need not be linear in each independent variable. For example, consider the following model of the number of houses available as a function of Time and Household Size of the U.S. population.

Housing Stock = 200 * NATEXP(.01 * Time) * (1/Household Size)

The logic of this model is that Housing Stock is related to the population and the number of houses per person. The first term, 200 * NATEXP(.01 * Time), is a surrogate for population (which in this example rises exponentially at about 1 percent per year). The second term, 1/ Household Size, is a surrogate for the number of houses per person. If the number of people living together goes down, of course, more houses would be needed per person. Housing Stock is then the product of the two terms.

More exotic ways of combining two variables to get another are no doubt possible, but because the ways of combining variables are so diverse and are used so rarely (compared to the summation and product forms above), categorizing them all here is not worthwhile. As a

modeler, you are limited only by your imagination. However, when you are being very creative, be careful to test fully that the model makes sense to you. The exotic relationship must flow from the actual structure of the activities represented in the model.

4.2 PROBABILISTIC DEPENDENCE

One of the major problems of managers who want to create or understand models is that, while they may be perfectly comfortable with models that assume certainty, as soon as uncertainty is introduced, they suddenly see the model enshrouded in mystery. In this section, however, uncertainty in some of the model's variables is treated as a natural development of the model that assumes certainty.

An influence diagram may be used to express the types of uncertainty that may be introduced in a model. Exhibit 4.14 shows four ways that a random variable may arise in a model. (All of these examples come from Exhibit 3.8.) In Exhibit 4.14*a*, a variable is uncertain in and of itself, not because of influence from any other variable. In Exhibit 4.14*b*, a variable that is assumed to be certain has an uncertain influence on another variable. Exhibit 4.14*c* has no randomness in the influence relationship, but because the influencing variable is random, the influenced variable is also random. Finally, in Exhibit 4.14*d*, a random variable influences another with an influence that is itself random.

Taking the first case, consider how one can model randomness in a variable that is not influenced by any other variable. This may be done by defining a ***probability distribution*** for the random variable. First, the interval over which the random variable ranges is identified. Then the relative likelihood of each possible value within that range is established. One way to do this is simply to draw a curve of relative likelihood or frequency over the interval.

Sometimes, instead of drawing the relative likelihood curve, it is simpler to think of percentiles of the curve. The 50th percentile (also known as the median) is the point below which the variable is expected to fall 50 percent of the time, or in other words, it is equally likely for the variable to be above or below the median (ignoring the tiny probability that it falls right on it). In general the *k* percentile (also known as the .k fractile) is such that k percent of the time the variable will fall below or at that level.

A graph of the percentile value versus the random variable is called the ***cumulative distribution function***. It is so named because the percentile value at a point X is essentially the cumulative probability that the random variable is X or less. In Exhibit 4.15, a cumulative distribution curve is shown in the middle of the top row. A relative likelihood curve that corresponds to this cumulative distribution curve looks like the left picture of the top row of that exhibit.

Exhibit 4.15 gives a menu of forms that the frequency distribution may take in the IFPS language. The list is far from complete, as an endless variety of probability distributions are

EXHIBIT 4.14 Examples of random variables and dependencies.

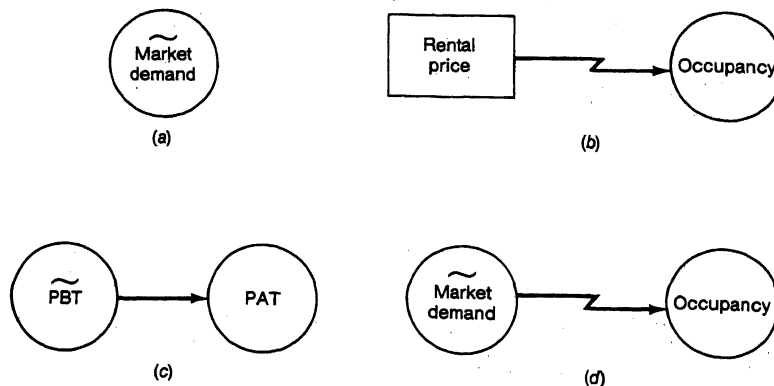
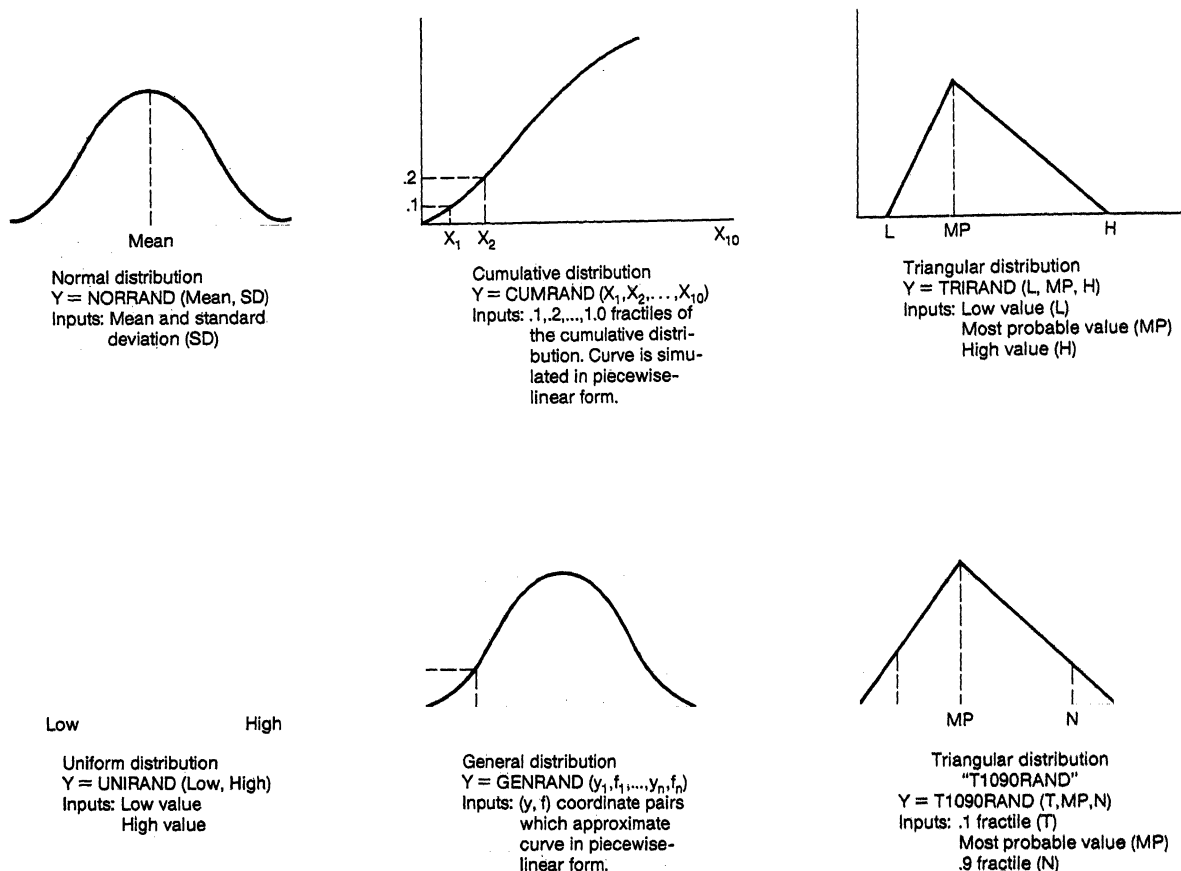



EXHIBIT 4.15 Relative frequency distributions for IFPS. (For more information see *IFPS User's Manual, Release 9.0, Execucom Systems Corporation, Austin, Texas, 1983.*)

possible, but it does include two general forms that may be used to approximate any shape of probability distribution that the modeler might imagine.

The CUMRAND probability function generates a random variable from the deciles of the distribution. (The deciles are the 0 percentile, the 10 percentile, the 20 percentile, and so on, up to the 100 percentile, making in all 11 values.)

The GENRAND probability function uses the frequency distribution curve rather than the cumulative probability. An approximation to the frequency curve is made from points (y_1,f_1), (y_2,f_2), ... Any shape is allowed, and of course the more points used, the closer is the approximation to the true curve.

The other possibilities for a random variable are triangular, uniform, and normal versions of the relative frequency curve. The uniform model assumes that every outcome in the interval for the random variable is equally likely. The triangular assumes (1) that there is a most likely point somewhere in the interval and (2) that likelihood decreases in straightline fashion from that point going in either direction toward the ends of the intervals. In the model on the right of the top row of Exhibit 4.15, the most probable point and the end points are supplied to completely specify the distribution. In the model on the right of the bottom row, the most probable point, the 10 percentile and the 90 percentile of the probability curve are required. The remaining model in the list, the normal or bell-shaped frequency curve, is a common one in statistics. For it, the mean, or average value, and the standard deviation are given to define the whole curve. See any introductory statistics or probability text for more information about these and other probability distributions.

Referring back to Exhibit 4.14*b*, now consider the second type of uncertainty model, where all of the uncertainty is in the relationship rather than in the influencing variable. Perhaps the most useful way to include the uncertainty is simply to add in a variable that contains all of the uncertainty. For example, the influence relationship could be expressed,

Occupancy = 122 − .1 * Rental Price + Uncertainty
Uncertainty = NORRAND(0,1)

Ignoring uncertainty for the moment, the first line is a decreasing linear function. Thus if Rental Price were $250 per month, for example, then Occupany will be 97 percent [122 − .1 * 250]. For each dollar that Rental Price is raised, Occupancy will drop by .1 percent (at least in a reasonable range of Rental Price). But Occupancy cannot be known for certain, not because Rental Price is uncertain (it is set in advance), but because the effect of Rental Price on Occupancy can only be estimated. This uncertainty is modeled by adding a normally distributed term with a zero mean and a standard deviation of 1 (in units of percent).

This model is precisely the form of relationship used in a regression equation. Data may have been collected on the relationship between Rental Price and Occupancy for other apartment buildings and the regression line estimated. The standard error from the regression would be the standard deviation of the normal Uncertainty term that has been added in here.

Nothing requires that the uncertainty must be *added* in the case of Exhibit 4.14*b* or that the dependence be linear in the influencing variable. An uncertain variable might multiply the influencing variable or possibly interact with it in some other mathematical form.

Exhibit 4.14*c* where a certain influence relationship runs between two random variables, is perhaps the easiest relationship of the four to handle. It is not really treated any differently than a relationship involving no random variables. For example, a relationship between Profit Before Tax (PBT) and Profit After Tax (PAT) may be written

PAT = .54 * PBT

even though both PAT and, therefore, PBT, are random variables.

The final case, Exhibit 4.14*d*, on the other hand, is probably the most complicated. In that case, both the influencing variable and the relationship have uncertainty. It is therefore, in a sense, the composite of the cases in Exhibit 4.14*a* and *b*. As such, it is tricky since it may be difficult to decide how to separate the uncertainty in the dependent variable into the two separate uncertainties. Consider the following example:

Market Demand = TRIRAND(1000,1400,2000)
Our Demand = Market Demand * UNIRAND(.2,.4)
Occupancy = MINIMUM(1.0,Our Demand/250).

Here Market Demand is modeled as a triangular random variable with 1000 and 2000 units being the limits on the number of units and 1400 being the most probable. Our Demand is uncertain, but it is modeled to be between 20 and 40 percent of the market and to be uniformly distributed. Finally, Occupancy is the smaller of 100 percent and the ratio of Our Demand to the 250 units we have available. This is expressed through the MINIMUM function.

This example illustrates an uncertain relationship that is a product rather than a summation. Multiplying random variables is common when the random variables are items such as percent share or percent success in an endeavor.

4.2.1 Correlation

The type of relationship in Exhibit 4.14*d* may be more simply expressed using correlation. Recall that the correlation coefficient is a number between −1 and 1 that indicates the statistical

relationship between two variables. If the correlation is 0, then there is no discernible relationship. In other words, knowing X does not help you predict Y. If the correlation is 1, then by supplying X, you can predict Y perfectly. If the correlation is −1, you can still predict Y perfectly, but as X increases, Y decreases. For other values of correlation, the degree of association between X and Y is measured by the absolute value of the correlation.

As an illustration of the use of correlation to model a dependency, suppose that you believe the correlation of Market Share with Advertising Share to be about .50. You believe that both of the variables are normally distributed with means of .40 and .60 and standard deviations of .03 and .01, respectively. The following IFPS lines would express the relationship:

```
Market Share = .40 + Zms * .03
Zms = CORREL(Zas,stdnorm,.50)
Zas = (Advertising Share − .60)/.01
stdnorm = NORRAND(0,1)
```

The first line computes Market Share from its Z score, Zms, which is the number of standard deviations that a variable falls away from its mean. The quantity Zms is needed because of the way in which the IFPS function CORREL is defined. CORREL (the second line) gives a quantity Zms with a mean of zero and a standard deviation of 1. The first line uses Zms to obtain a random variable with a mean of .40 and a standard deviation of .03. The CORREL function also requires inputs that are standard normal (mean 0 and standard deviation 1). The quantity Zas is the Z score for Advertising Share and stdnorm is an intermediate standard normal variable that is created only to carry out the computations in the CORREL function.

Another kind of correlation that is important in modeling is serial correlation: the correlation of a single variable with its level in the previous period. In IFPS, if you use one of the probability distributions for a variable that has a stream of outcomes over time, you can specify whether the variable is set once for all periods or is set once each period. To illustrate, if you wrote

```
Annual Operating Cost = TRIRAND(5000,6000,10000)
```

then the Annual Operating Cost that is simulated in the first period holds through all periods. It is like having a single spin of a wheel. On the other hand, if an R were added to the name of the probability distribution, i.e., TRIRANDR (5000,6000,10000), then there would be a new value each time period. It is as though a probability wheel were spun each period to determine the cost. The first expression assumes a perfect serial correlation (the serial correlation coefficient equals 1), whereas the TRIRANDR signifies a serial correlation of 0.

4.3 SUMMARY

This chapter started with the assumption that the modeler had already completed an influence diagram and was at the point of defining the form of the relationships between the dependent variables and those variables that influence them. Using the menu of functions described in this chapter as a guide, the modeler will discover that a line model naturally develops from the influence diagram. Adding uncertainty is another natural step. The modeler simply makes some of the variables and relationships random, using appropriate probability distributions.

The final two steps in the modeling process are to generate the necessary numbers required to fill in the relationships defined by the procedures described in this note and to analyze the completed model and the information it provides. There are many ways to "solve" models with uncertainties in them. The usual method in IFPS and other modeling languages is through Monte Carlo simulation. The testing and use of a model, including Monte Carlo simulation, will be topics in the next chapter.

TERMS

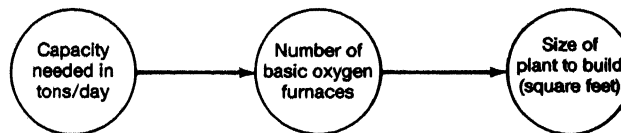
form of relationship: function
estimation
linear
quadratic
power curves
fractional power curves; nth root
exponential
logarithmic curve
logistic curve
interpolate
step function
IF . . . THEN logic
MAXIMUM, MINIMUM functions
many-variable functions: sum and product forms
probabilistic dependence
probability distribution: NORRAND, CUMRAND, TRIRAND, UNIRAND, GENRAND
fractile
random variable
correlation

EXERCISES

4.1 Choose an expression for the relationship that you feel is the most appropriate for each situation below. You may just state the function, draw a graph of the relationship, or (preferably) write an IFPS line for each part. Only deal with the function or the nature of the relationship, not with the estimation of numbers.

a A perishable product spoils spontaneously at a rate that depends on the amount of good product. That is, a given fraction of the remaining good product becomes unusable each day. But the rate of spoilage varies from product to product. What form of relationship should apply for Amount of Usable Product as a function of Time?

b You wish to model the dependence of the Demand for Travel (assume round-trip travel) between any pair of cities based solely on three variables: Population of City 1 and Population of City 2 and Distance Apart. Choose a simple way to express this relationship and state your reasons for it.

c One part of an influence diagram for a capacity planning model for a steel plant is given below. What dependency relationship would you use for each arrow in the diagram?

d Assuming that the number of workers in a job shop is constant and that there will always be more work than the workers can handle, what relationship would exist between Amount of Production and Amount of Equipment in the shop?

4.2 Consider a situation in which a company is trying to decide whether to add a new product to its product line. Based on the following variables, write an expression to calculate the break-even volume needed for the new product: Fixed Costs, Unit Price, Unit Variable Cost.

4.3 Write expressions for each influence expressed in the influence diagram of Exhibit 3.8. Note that expressions for many of these influences are given as examples in this chapter.

4.4 Suppose a new piece of equipment is to be purchased. The variables listed below have been estimated. Write expressions for a model that might be used to determine how long the machine ought to be kept. The variables available are: Initial Investment, Initial Revenue, Annual Decrease in Revenue, Maintenance Expense, Salvage Value, Discount Rate.

4.5 Modeling Small Group Dynamics
Extend the work you did for Exercise 3.4 by completing the following tasks:

a Write (or sketch the form of) functions for each influence relationship in your influence diagram of group dynamics.
b Based on the way you have chosen to model group interactions, can you guess how the variables will change over time? Draw on one sheet of paper curves showing the progression of each variable over time. What would you learn from filling in the numbers for the model and running it?

4.6 What type of frequency distribution would be most appropriate for the following variables? Draw the relative frequency or cumulative distribution curve, or choose an IFPS expression for each variable (do not worry about specific numbers).
a The Consumer Price Index 5 years hence.
b Projected market share for a new product in a new market that is expected to have n competitors of roughly equal size.
c The day of first availability of a new product from the competition that has been announced to come out in the next calendar year.
d The time until completion of an overdue project that is expected to be completed "any day now."

4.7 You have already selected a probability distribution to model the random variable for year 1 in each of the following situations. Now you wish to express what the variable will be in years 2 through 5. Among your choices are (1) to use the same distribution you used in year 1 but to sample independently to set the level of the random variable in each of years 2 through 5 (use the repeat, R, in the expression of the probability), (2) to use the exact same number for each of years 2 through 5 that will be used in year 1, or (3) to correlate the outcome in period 2 through 5 with that of the previous year. Which would you be inclined to do in the following situations:
a The variable is the rate of return from a very well diversified portfolio of Standard & Poor's 500 stocks.
b The variable is the Consumer Price Index.
c The variable is the recoverable amount of bauxite in a tract you may purchase. A complete and accurate survey of mineral resources will be available in 1 year after purchase.

CASE ASSIGNMENTS

4.1 Unicron (A), page 197
a Assuming the role of Greg Willet, build a financial model focusing on five-year net income and return on sales in preparation for your presentation to the New Product Review Committee.
b Investigate, using your model, the "what-if" scenarios Greg is likely to need for his presentation, including at least variations in the price and growth rate of the market.
c Are you convinced that the relationships you have built into your model are appropriate?

4.2 Green Valley Foods, page 257
Write expressions using IFPS or other modeling language for each relationship of the influence diagram you prepared for Case Assignment 3.1.

4.3 John Denison, page 183
Write a model for analyzing the two investment opportunities available to John. Assume that inflation will average 6 percent over the next 25 years. Further assume that Mr. Denison will choose 90 percent debt financing and sell after 7 years if he buys the real estate. Should he take either opportunity? Does the decision change for other inflation rates or debt/equity mixes?

CHAPTER **5**

USING RELIABLE MODELS

This chapter discusses how to put the finishing touches on a model and how to use it. The process of moving from first-cut analysis to final model is described, including how to validate the model and how to carry out what-if analysis, sensitivity analysis, goal seeking, and risk analysis systematically. Risk analysis is considered as a natural, but not always necessary, extension of sensitivity analysis. Since Monte Carlo simulation may be a new concept to some, its basic concepts are introduced (or reviewed for those who have seen it before).

5.1 THE MODEL IN IFPS AND OTHER SOFTWARE

Each expression relating a dependent variable to one or more independent variables, as described in the previous chapter, is a line in an IFPS model. It is convenient to think of the model as a matrix. Associated with each row of the matrix is a variable. Columns usually correspond to time periods, but they may index virtually any other dimension. For example, in Chapter 9, Group Decisions, columns refer to outcomes to specific individuals. Columns may also relate to different alternatives.

Exhibit 5.1*a* shows the definition of rows and columns for a typical model. Exhibit 5.1*b* shows the model itself, complete with all of its relationships. In IFPS, the model may be displayed and edited in line form as in Exhibit 5.1*b*. When the model is solved the results are displayed as in Exhibit 5.1*a* with all of the cells filled in.

The typical spreadsheet software available on microcomputers displays on the screen a picture like Exhibit 5.1*a* with the numbers filled in. The screen shows recalculation of all variables when changes are made in any variables (thus the origin of the name VisiCalc). For nearly all of these programs (Encore and IFPS/Personal being exceptions), it is very difficult to see the whole model in readable form as in Exhibit 5.1*b*. You can display the formula defining any cell in the matrix, or in some cases maybe a row of the matrix, but you cannot usually read the whole model. It is a great boon to the process of validating, extending, and communicating a model to be able to see it written down in one place. Thus in the material that follows in this chapter we will refer to models as the complete listing of expressions that relate cells of a spreadsheet.

5.2 A PROCESS FOR MODEL DEVELOPMENT

5.2.1 Validating the Model

When dependencies have been written for all variables that are related, the model is born, but it is not yet fully developed. In the usual excitement of having something that can be used, the modeler may not thoroughly validate the model. It is tempting to go directly to results; sometimes the analysis is complete and the final report is being written when the model is tested for accuracy and is found to be faulty. A good modeler will validate the model's integrity, improve it, and expand or shrink it in the course of developing it to the point of producing reliable results.

There are a number of ways to check the accuracy of a model you have prepared. The following list is by no means exhaustive, but by going through it you will be able to spot most imperfections in a model.

1 Read through the lines of the model to see that they reflect the mathematical relationships you intended. Examine the logic of any conditional statements. Postulate what will happen to outcome attributes when decision variables take on various values. In IFPS, the command ANALYZE may be helpful at this stage. This command gives the expression for a dependent variable and the levels of all variables appearing as independent variables in the expression. With this command you can work backward through the model from outcome attributes through

EXHIBIT 5.1 Illustrative model.

(a)

```
SALES = 100, PREVIOUS * 1.08
COST OF SALES = .67 * SALES
GROSS PROFIT = SALES - COST OF SALES
```

(b)

the lines of influence to decision variables, examining the influences and dependencies for correctness along the way.

2 Make sure that critical quantities are conserved or that there is the appropriate balance between quantities. For example, do Assets = Liabilities? Are expressions relating variables in units of thousands to variables in actual units scaled properly?

3 Verify that the variables have the right signs and behave appropriately when other variables take on extreme values. A model would be wrong, for example, if a relationship between Price and Sales would give a negative level of Sales for a very high, but plausible, Price. A new product model would be erroneous if you try a Market Share number near the top of its possible range and find it produces an unbelievably high Rate of Return. The model must be faithful in extrapolation over at least the range of possible values for variables.

If you are unsure about the faithfulness of the model, make a first-cut run of the model. Solve it and see if the results are what you expected. Change the values of the decision variables and exogenous variables over a range you have decided is appropriate—such as ± 15 percent. As a variable increases, do the outcome attributes change appropriately? Do the intermediate variables behave as you expected?

4 Examine the fit of projections of the model with data you have collected. Presumably this would have been checked in expressing each individual relationship among variables. But the whole may be more than the sum of the parts, or so it may seem. Even though we may be confident in the relationship between Price and Sales and between Sales and Profit, it is still wise to check the relationship of Price and Profit against historical data, for example. The model may reveal nonintuitive behavior, so now you find out whether erratic behavior is nonintuitive or simply a mistake in the model.

At this stage, the model works. It is limited, perhaps, because you have included only the major influences. You should have confidence in your model and probably have new understanding of the problem that is under study.

5.2.2 What-If and Sensitivity Analysis

In what-if analysis, the value of a variable is changed and the model is re-solved. Sensitivity analysis is just an extended form of what-if analysis, where the level of a variable or variables is automatically incremented by a given amount over a specified range, and the effect on a specified variable is determined.

Find which variables most affect the outcome attributes of the model. Trace them through the influence diagram to find the path of greatest influence. It may even be useful in presenting sensitivity analysis to make the size of circles in the influence diagram proportional to the sensitivity of key attributes to intermediate variables.

The SENSITIVITY command of IFPS may be used to see what changes in other specified variables will result from a change in a given variable. IMPACT is the opposite of SENSITIVITY. It shows the effects of many variables on one variable.

5.2.3 Extend and Refine the Model

Start out with the simplest model you can imagine. Then improve the model as needed, but only as needed. Resist the impulse to add everything you think of. If another variable or influence is to be added, each one introduced should be the next most important.

Iteratively extend the model, adding one variable or influence at a time to it. Introduce other variables or minor influences into the model that link to the path of greatest influence. Or add a variable or influence that is related to a variable that SENSITIVITY shows to have a significant effect on many other variables in the model.

Despite the ease with which this advice is given, it is not so easily taken. I have never seen a student group in their first major modeling effort work this way. The result is a giant unwieldy

model, and students stay up most of the night before a presentation trying to understand and validate it. A good rule of thumb is to have a working model at a point in time where you have spent no more than 60 percent of the total time you will devote to the modeling effort.

Before adding another variable or influence, ask yourself: Would it possibly change any decision that may be made with the aid of the model? If the answer is no, do not make the addition. Refinement for its own sake is no virtue.

As each new variable or influence is added, the sensitivity analysis must be redone. Each influence can dampen or magnify the effect of another. So at each step you need to find again the path of greatest influence through the influence diagram so you know what most affects attributes. Your knowledge of the problem is growing rapidly. With experience, this process is very quick and you proceed confidently.

5.2.4 Goal Seeking

When you have sufficient confidence in the model, you may wish to exercise it to explore not just "what if" but "how to." GOAL SEEKING (a command in IFPS) is the determination of what level of a particular variable (e.g., a decision variable) is necessary to achieve a particular target level in some other variable (e.g., an attribute).

One can think of any number of uses of GOAL SEEKING. It may be important, for example, to calculate the price that will bring a specific rate of return for a new product. Or you may wish to know the sales level which will bring break-even profitability. You may wish to calculate the total amount you need to borrow in order to have sufficient funds for working capital for future periods.

Goal seeking is not the same as optimization! In optimization, some quantity is being maximized or minimized. Usually several decision variables may be changed simultaneously to achieve the optimum. Goal seeking merely calculates how to hit a target goal right on, rather than how to get the most possible or least possible of something. Optimization, which is typically a much more difficult computational problem, will be discussed in Chapter 6.

5.3 ADDING UNCERTAINTY—RISK ANALYSIS

In the influence diagram, you may have indicated that some of the variables in the model are uncertain. It is best in the first-cut run of a model to treat all variables as certain, then add uncertainty as needed and as understanding of the deterministic model permits. If what-if or sensitivity analysis reveals that attributes are not significantly affected by variation of a variable within the plausible range of uncertainty, fine. You normally would not need to treat it as uncertain in a risk analysis. It may turn out this way more often than you might guess, because the effects of variables are dampened by others. It would still be necessary to treat the variable as uncertain, however, if, in *combination* with other uncertain variables, the uncertainty in a variable may swing a decision.

If the attributes remain sensitive to a variable that is ill-understood, or in other words that you deem is uncertain, you have identified a topic for further study. If time does not permit any study immediately, you know where to put your research efforts to the extent that you have time in the future.

It may help to find break-even levels of attributes where a decision is reversed using GOAL SEEKING. You need not spend time refining the estimates of variables and influences for the various "good" outcomes, compared to understanding what causes "bad" outcomes. The most difficult models (which give you the most satisfaction when you do them right) are ones where variation of a variable(s) over the range of its uncertainty switches the decision. Then the uncertainty in the model needs to be treated in detail.

You stop refining the model when your understanding of the situation is sufficient to make the decision. The temptation to go on elaborating a nicely running model is a trap for the model builder. After all, most of your time has been devoted to refinement and extension because the

first form of the model was quick to build and check. Recall that the decision is the goal; the model is the means to get you to it, not away from it or beyond it.

5.3.1 Monte Carlo Simulation

What-if and sensitivity analysis are excellent for checking the effect of variations in one variable at a time while the others are held constant. But there are many occasions where we would like to know the combined effect of variations in two or more variables, or we may foresee numerous possible values for a variable and would like to see quickly the effect of this range on the outcome attributes without performing a what-if analysis on every single possible value. Monte Carlo simulation gives us the power to do both of these important tasks.

Monte Carlo simulation is merely a way of evaluating a large number of possible *scenarios*. A scenario represents one possible set of levels for each of the random variables and the calculation of the levels of all variables that depend on the random variables. We would like to look at many possible scenarios and to select them in accordance with the likelihoods that the decision maker or modeler would put on random variables. The term "Monte Carlo" reflects the fact that random numbers are used to generate the scenarios. The computer, however, not the roulette wheel, produces the random numbers.

In a Monte Carlo simulation, the computer solves the model over and over again, each solution being a *trial* of the simulation, which is a randomly sampled scenario. After running a large number of trials the results are collected in the form of a frequency distribution (or histogram) for one or more selected variables (or attributes).

In each trial of the simulation the value given to a random variable is sampled randomly. The chances for a particular level of the random variable are set by the distribution chosen for that variable in the model. Thus if a variable is represented in the model by UNIRAND(10,20), for example, the separate trials of the simulation will use values that are selected independently in the range 10 to 20 with each value in the range having equal likelihood.

The means for expressing random variables in IFPS and ways to relate these random variables to the other variables of a model were discussed in Chapter 4. When a model that has random variables in it is solved, there will be a range of possible values for attributes or variables that depend on one or more random variables. We use Monte Carlo simulation to evaluate the range and relative likelihood of possible values for any chosen variable or attribute.

As an introductory example of Monte Carlo simulation, consider the following very simple problem.

> A game of dice between two players, A and B, has its own special rules. A rolls *three* times and B rolls *twice*. The highest of A's three dice rolls is compared to the highest of B's two rolls. A wins only if his highest roll is *greater than* the highest roll of B. B wins all ties on highest roll.

An IFPS model of this simple game is shown in Exhibit 5.2. The possible outcomes for the roll of a die are, of course, the numbers 1 through 6, with each outcome being equally likely. Thus line 2 of the model expresses outcomes for player A's roll using UNIRAND (uniform likelihood) in the range .5 to 6.5. The ROUND function then converts the continuous variable to equally likely integers 1, 2, 3, 4, 5, and 6.

EXHIBIT 5.2 IFPS MODEL OF DICE GAME

```
MODEL DICE VERSION OF 04/07/83 11:15
1  COLUMNS 1-3
2  A ROLLS = ROUND(UNIRANDR(.5,6.5))
3  B ROLLS = ROUND(UNIRAND(.5,6.5)),ROUND(UNIRAND(.5,6.5)),0
4  HIGHEST A = MAXIMUM(A ROLLS,PREVIOUS A ROLLS,PREVIOUS 2 A ROLLS)
5  HIGHEST B = MAXIMUM(B ROLLS,PREVIOUS B ROLLS,PREVIOUS 2 B ROLLS)
6  A WIN = 0,0,IF HIGHEST A .GT. HIGHEST B THEN 1 ELSE 0
```

The model has three columns representing three rolls of the dice for A. Because an R (for repeat) is placed at the end of UNIRAND, a random value is chosen each of the three times, independent of the random selection in the other columns. Note the 0 that appears for the third column of B Rolls, since B only gets two rolls. The variables Highest A and Highest B use the MAXIMUM function to select the highest tosses for the two players. The last line of the model gives A a score of 1 when A's highest value is greater than B's.

There are many possible scenarios for the game. For example, one would be for A to roll the numbers 2, 5, and 4, and for B to roll the numbers 1 and 5. Then B would win. In Monte Carlo simulation, the computer, by randomly chosing a value for each random variable generates a scenario. Then it generates another and another until the specified number of trials is completed. At the end the frequency of selections for each random variable may be examined, along with the frequency of outcomes for any other variables in the model.

Suppose that 500 Monte Carlo trials were run for MODEL DICE. Conceptually this is the same as sitting down with the dice and playing the game 500 times, only we let the computer play the 500 games and then tell us the results. Can you predict how many of the 500 times A will win?

Exhibit 5.3 shows the results of 500 Monte Carlo trials for the first roll of player A, the two players' highest rolls and for the variable A Win. The trials are compiled in a histogram, or frequency plot. Note that the distribution for A's first roll is not quite uniform; for example, there were more 5's than 6's. This will be the case with random sampling, even though on each trial the chance of 5 equalled the chance of 6.

As the number of trials becomes large, the percentage of times that a 5 is rolled approaches that of a 6. It is better, therefore, to use a higher number of trials. A tradeoff must be made, however, since the cost of computation is nearly proportional to the number of trials.

For player A, the 500 trials indicate that it is most likely for the highest roll to be a 6. The frequency for other outcomes of the highest roll increases in ascending order. On the other hand, for B the most likely highest value appears to be 5. It can be shown from probability theory that this is not right; the most likely value for B Highest should also be 6. The only explanation for this discrepancy is that we did not do enough iterations to wash out the chance effects of the random sampling process. This places in doubt the apparent result that player A wins less than half the time.

EXHIBIT 5.3 Histograms showing results of 500 Monte Carlo trials.

```
  HISTOGRAM FOR COLUMN      3 OF Highest A
183- 195                                      *
170- 182                                      *
157- 169                                      *
144- 156                             *        *
131- 143                             *        *
118- 130                             *        *
105- 117                             *        *
 92- 104                     *       *        *
 79- 91                      *       *        *
 66- 78                      *       *        *
 53- 65                      *       *        *
 40- 52                *     *       *        *
 27- 39                *     *       *        *
 14- 26       *        *     *       *        *
  1- 13 *     *        *     *       *        *
        ---------------------------------------
        1     1     2    3    4     4      5
        .     .     .    .    .     .      .
        1     8     6    4    1     9      7
        3     9     6    2    9     5      2
START     1.0  STOP      6.1 SIZE OF INTERVAL      .26

  HISTOGRAM FOR COLUMN      3 OF Highest B
141- 150                             *
131- 140                             *        *
121- 130                             *        *
111- 120                             *        *
101- 110                             *        *
 91- 100                     *       *        *
 81- 90                      *       *        *
 71- 80                *     *       *        *
 61- 70                *     *       *        *
 51- 60                *     *       *        *
 41- 50       *        *     *       *        *
 31- 40       *        *     *       *        *
 21- 30       *        *     *       *        *
 11- 20       *        *     *       *        *
  1- 10 *     *        *     *       *        *
        ---------------------------------------
        1     1     2    3    4     4      5
        .     .     .    .    .     .      .
        1     8     6    4    1     9      7
        3     9     6    2    9     5      2
START     1.0  STOP      6.1 SIZE OF INTERVAL      .26

  HISTOGRAM FOR COLUMN      3 OF A Win
505- 540 *
469- 504 *
433- 468 *                                    *
397- 432 *                                    *
361- 396 *                                    *
325- 360 *                                    *
289- 324 *                                    *
253- 288 *                                    *
217- 252 *                                    *
181- 216 *                                    *
145- 180 *                                    *
109- 144 *                                    *
 73- 108 *                                    *
 37- 72  *                                    *
  1- 36  *                                    *
        ---------------------------------------
        .     .     .    .    .     .      .
        0     1     3    4    6     7      9
        2     7     2    7    2     7      2
        5     5     5    5    6     6      6
START      .0  STOP      1.0 SIZE OF INTERVAL      .05
ENTER POOL OR MODELING LANGUAGE COMMAND
```

5.3.2 How Many Trials Should You Run?

Since the trials are selected randomly, the results of another 500 Monte Carlo trials would not likely be identical to the results in Exhibit 5.3. On average, however, they will be close, much closer than, say, two different sets of five trials. In order to produce reliable results, that is, results that are not due to sampling chance, we would like to run enough trials. But how many is enough?

This question can be answered by looking at statistics reported with the results of the Monte Carlo run by most modeling software. Decide what is the critical attribute. Then make sure the distribution of the *mean* of this attribute is narrow enough that decisions can be made unambiguously.

Suppose the decision to be made in the dice game was whether to take the side of player A or player B. The results suggest that player B is slightly better off. But did player A lose more than half of the time in the 500 trials because of chance in the selection of variable values in the simulation or because the odds of the game are stacked against A?

Exhibit 5.4 shows sample statistics for the results in Exhibit 5.3. There are columns for many statistical quantities here, but for now concentrate on the leftmost column for the mean and the rightmost two columns. Note first that the means of the rolls for A and B are not exactly 3.5 in each case, even though the average of equally likely numbers from 1 through 6 is 3.5. This is due to the sampling variation in the simulation. From the mean of A Win, it appears that 47.4 percent of the time player A will win the game. Is the fact that this is less than 50 percent due to chance as well, or is it a legitimate characteristic of the game? This question can be answered by referring to the confidence interval on the mean of A Win, which is in the two rightmost columns. These columns state that 80 percent of simulation runs with 500 trials would have a mean for A Win that falls between .4454 and .5026. In other words, the simulation results should lead the modeler to believe with 80 percent confidence that the win chances for player A are between 44.54 and 50.26 percent. Thus this simulation run is not proof enough that player A's win chances are below 50 percent. If a simulation with more trials were run, the 80 percent confidence interval should narrow.

The results of a separate simulation run with 1000 trials are shown in Exhibit 5.5. Note that the 80 percent confidence interval on the mean for A Win is now the range 44.38 to 48.42 percent with the best estimate of the mean being 46.4 percent. This simulation is conclusive

EXHIBIT 5.4 SAMPLE STATISTICS FOR RESULTS OF 500 MONTE CARLO TRIALS

	MEAN	STD DEV	SKEWNESS	KURTOSIS		
					10PC	CONF MEAN 90PC
A ROLLS						
1	3.414	1.697	.0	1.7	3.317	3.511
2	3.502	1.716	.0	1.7	3.404	3.600
3	3.340	1.711	.1	1.8	3.242	3.438
B ROLLS						
1	3.546	1.635	−.1	1.8	3.452	3.640
2	3.526	1.714	.0	1.7	3.428	3.624
3	0	0	.0	.0	0	0
HIGHEST A						
3	4.866	1.155	−.9	3.2	4.800	4.932
HIGHEST B						
3	4.460	1.339	−.6	2.4	4.383	4.537
A WIN						
3	.4740	.4998	.1	1.0	.4454	.5026

```
                    SAMPLE STATISTICS

        MEAN   STD DEV SKEWNESS KURTOSIS  10PC CONF MEAN  90PC

A Win
  3    .4640   .4990      .1       1.0     .4438      .4842

    HISTOGRAM FOR COLUMN      3 OF Highest B
 295- 315
 274- 294
 253- 273
 232- 252
 211- 231
 190- 210
 169- 189
 148- 168
 127- 147
 106- 126
  85- 105
  64- 84
  43- 63
  22- 42  *
   1- 21  *

                          4

          1    8    6    4    1    9    7
          3    9    6    2    9    5    2
START      1.0  STOP     6.1 SIZE OF INTERVAL     .26
```

EXHIBIT 5.5 Sample statistics for 1000 trials.

that player A's win probability is below 50 percent (at least at a 80 percent level of confidence). Also note in Exhibit 5.5 that the histogram for Highest B has increasing probability over the numbers 1 through 6, as expected theoretically. The overall reliability of the model is thus increased with more trials.

The other columns of Exhibit 5.5 give summary measures of the histogram. Standard deviation (obtained by squaring deviations from the mean, averaging them, and computing the square root of this average) measures the dispersion. Skewness is a measure of the symmetry of the data. A zero skew means the histogram is symmetric about the mean. Positive skew means the frequency tails off to the right, and negative skew means it tails off to the left. Kurtosis measures the peakedness of the distribution. A normal distribution has kurtosis of 3. A kurtosis larger than 3 means the data are more peaked than a normal curve and smaller than 3 means it is less peaked.

The number of trials needed to narrow the distribution of the mean of an attribute sufficiently to make a decision depends on the model and the decision to be made. If there are more uncertain variables and more uncertainty in individual variables, then the mean of an attribute will be more uncertain and more trials will be needed. Just look at the confidence interval on the mean and determine if your decision can be made conclusively with the current degree of precision. If the answer is no, run more trials. In IFPS, the POOL command combines into the same histogram any additional trials with those already run.

5.3.3 A Complex Example with Continuous Uncertain Variables

The dice game was a simplified example that does not really require simulation; the probability that A Wins can be found using probability theory. The model in Exhibit 5.6, however, illustrates a situation where Monte Carlo simulation and risk analysis, the process of using simulation for decisions about risky projects, would be needed. It also demonstrates the treatment of continuous random variables in risk analysis.

EXHIBIT 5.6 A MODEL OF AN INVESTMENT IN A NEW PRODUCT

```
MODEL NEWPROD VERSION OF 04/05/83 12:16
1  COLUMNS 1984–1989
2  MARKET SIZE = TRIRAND(120,160,190),PREVIOUS * (1 + MARKET GROWTH)
3  MARKET GROWTH = GENRANDR(.06,.2,.07,.3,.08,.2,.09,.2,.10,.1)
4  SELLING PRICE = NORRAND(75,10)
5  SHARE OF MARKET = .50 + ZMS * .08,PREVIOUS
6        ZMS = CORREL(ZSP,STDNORM,.50)
7        ZSP = (SELLING PRICE - 75)/10
8          STDNORM = NORRAND(0,1)
9  INVESTMENT = UNIRAND(8000,12000),0
10 RESIDUAL VALUE = 0 FOR 5,CUMRAND(4000,4200,4500,4850,5000,5400,'
11                        5600,6000,6500,7000,7600)
12 UNIT PRODUCTION COSTS = T1090RAND(35,38,45)
13 ANNUAL FIXED COSTS = 1000
14 REVENUE = MARKET SIZE * SHARE OF MARKET * SELLING PRICE
15 EXPENSES=MARKET SIZE * SHARE OF MARKET * UNIT PRODUCTION COSTS'
15.5          +ANNUAL FIXED COSTS
16 NET REVENUE = REVENUE - EXPENSES
17 NET PRESENT VALUE = NPVC(NET REVENUE + RESIDUAL VALUE,.12,INVESTMENT)
18 INTERNAL RATE OF RETURN = IRR(NET REVENUE + RESIDUAL VALUE,INVESTMENT)
END OF MODEL
```

Exhibit 5.6 is a model of an investment in a new product. There are seven variables that are uncertain in this model: Market Size, Market Growth, Selling Price, Share of Market, Investment, Residual Value, and Unit Production Costs. The attributes in the model, Net Present Value and Internal Rate of Return, will also be uncertain since they depend on these seven uncertain variables.

The uncertain variables in this model were used by Hertz (1979) in his classic article on a general risk analysis model for capital investment. In addition to these variables, Hertz treats two others as uncertain: Fixed Costs and Useful Life of Facilities.

Most of the risk-modeling features of IFPS are used in this model. Each form of probability distribution is used; the CORREL feature is used for defining Share of Market (Zms, Zsp, and stdnorm are variables used only to define the correlation), and the repeat feature is used for Market Growth (recall that GENRANDR means a different selection for the Market Growth is selected for each period).

This model can be solved using IFPS without Monte Carlo simulation. The mean values for each probability distribution expressed in the model would then be used to solve the model. In Exhibit 5.7, A shows the result when the model is solved in this way.

B and C represent two other possible scenarios. In B IFPS was commanded to put all uncertain variables at their low levels. In other words the variables are set to their lowest possible values; normal random variables, which in theory have no lower bound, are set 2.5 standard deviations below their means. In C the other extreme scenario is evaluated; all uncertain variables are set to their high levels.

In the low scenario, the new product investment is clearly not a good idea, either in terms of Net Present Value or of Internal Rate of Return of the revenue stream. (Note that in IFPS the Net Present Value or Internal Rate of Return calculated in each column includes only the stream of revenues *up to and including that period;* normally we use just the values in the last column.)

There are numerous other scenarios; those for uncertain variables taking high, mean, and low levels in various combinations; and those for combinations of variables anywhere in between high and low levels. Exhibit 5.8 shows the histograms for selected intermediate variables for a Monte Carlo run of 500 trials. Again, because of sampling the results may not match the form of distribution expressed in the model exactly.

EXHIBIT 5.7 SCENARIOS FOR THE NEW PRODUCT MODEL

	1984	1985	1986	1987	1988	1989
A: Mean						
Market size	156.7	168.9	182.0	196.2	211.4	227.9
Market growth	.0778	.0778	.0778	.0778	.0778	.0778
Selling price	75	75	75	75	75	75
Share of market	.5000	.5000	.5000	.5000	.5000	.5000
Investment	10000	0	0	0	0	0
Residual value	0	0	0	0	0	5400
Unit production costs	39.72	39.72	39.72	39.72	39.72	39.72
Annual fixed costs	1000	1000	1000	1000	1000	1000
Revenue	5875	6332	6825	7357	7929	8546
Expenses	4112	4354	4615	4896	5200	5526
Net revenue	1763	1979	2210	2460	2730	3020
Net present value	−8426	−6848	−5275	−3711	−2163	2103
Internal rate of return					.0346	.1752
B: Low						
Market size	120	127.2	134.8	142.9	151.5	160.6
Market growth	.0600	.0600	.0600	.0600	.0600	.0600
Selling price	50	50	50	50	50	50
Share of market	.2268	.2268	.2268	.2268	.2268	.2268
Zms	−3.415	−3.415	−3.415	−3.415	−3.415	−3.415
Zsp	−2.500	−2.500	−2.500	−2.500	−2.500	−2.500
stdnorm	−2.500	−2.500	−2.500	−2.500	−2.500	−2.500
Investment	8000	0	0	0	0	0
Residual value	0	0	0	0	0	4000
Unit production costs	35	35	35	35	35	35
Annual fixed costs	1000	1000	1000	1000	1000	1000
Revenue	1361	1442	1529	1621	1718	1821
Expenses	1953	2010	2070	2134	2203	2275
Net revenue	−591.8	−567.3	−541.3	−513.8	−484.6	−453.7
Net present value	−8528	−8981	−9366	−9692	−9967	−8171
Internal rate of return						−.1986
C: High						
Market size	190	209	229.9	252.9	278.2	306.2
Market growth	.1000	.1000	.1000	.1000	.1000	.1000
Selling price	100	100	100	100	100	100
Share of market	.7732	.7732	.7732	.7732	.7732	.7732
Zms	3.415	3.415	3.415	3.415	3.415	3.415
Zsp	2.500	2.500	2.500	2.500	2.500	2.500
stdnorm	2.500	2.500	2.500	2.500	2.500	2.500
Investment	12000	0	0	0	0	0
Residual value	0	0	0	0	0	7600
Unit production costs	45	45	45	45	45	45
Annual fixed costs	1000	1000	1000	1000	1000	1000
Revenue	14691	16160	17776	19554	21509	23660
Expenses	7611	8272	8999	9799	10679	11647
Net revenue	7080	7888	8777	9754	10830	12013
Net present value	−5679	609.7	6857	13056	19201	29139
Internal rate of return		.1578	.4176	.5437	.6098	.6669

There is no way to see the correlation of Share of Market with Selling Price in the displays of Exhibit 5.8. Yet if we were to trace the specific trials in the pictures one by one, we would find that the dots in the right-hand side of the plot of Selling Price would tend to be associated more with dots in the right-hand side of Share of Market. If the correlation were set to 1.0 instead of .50, then the association of high Selling Price values with high Share of Market would be strict. If the correlation were set to − 1.0, then high Selling Price would be associated with low Share of Market, a more realistic situation.

```
  HISTOGRAM FOR COLUMN     1989 OF Selling Price
53- 56                    * *
49- 52                  * * * * *
45- 48                * * * * * *
41- 44                * * * * * *
37- 40                * * * * * *
33- 36            *   * * * * * * *
29- 32            * * * * * * * * *
25- 28            * * * * * * * * * *
21- 24            * * * * * * * * * *
17- 20            * * * * * * * * * *
13- 16          * * * * * * * * * * * *
 9- 12      * * * * * * * * * * * * * *
 5-  8      * * * * * * * * * * * * * * *
 1-  4  * * * * * * * * * * * * * * * * * *   *
        ---------------------------------------
        4     5     6     7     8     9     9
        8     7     5     4     2     1     9
        .     .     .     .     .     .     .
        4     0     5     1     6     2     7
START    47.0  STOP     104.0 SIZE OF INTERVAL     2.85


  HISTOGRAM FOR COLUMN     1989 OF Share of Market
61- 65
56- 60                      *
51- 55                    * *
46- 50                * * * *
41- 45              * * * * * *   *
36- 40              * * * * * *   *
31- 35              * * * * * * * *
26- 30              * * * * * * * *
21- 25        *   * * * * * * * * * *
16- 20        * * * * * * * * * * * *
11- 15        * * * * * * * * * * * *
 6- 10        * * * * * * * * * * * *
 1-  4  *     * * * * * * * * * * * * * * * * *


        2     3     4     4     5     6     6
        9     5     2     9     6     3     9
        0     8     6     4     2     0     8
START      .3  STOP        .7 SIZE OF INTERVAL      .02


  HISTOGRAM FOR COLUMN     1989 OF Residual Value
49- 52  *
45- 48  *       * *     *
41- 44  *       * *     *
37- 40  *       * *     *
33- 36  *       * *     *
29- 32  * * *   * *   * *
25- 28  * * *   * *   * * *
21- 24  * * * * * *   * * * *
17- 20  * * * * * *   * * * * *         * *
13- 16  * * * * * *   * * * * *         * *
 9- 12  * *     * * * * * * * *         * *   * *
 5-  8  * *     * * * * * * * *     * * * *   * *
 1-  4  * *     * * * * * * * *     * * * *   * *

        4     4     5     5     6     6     7
        0     6     1     7     2     7     3
        9     3     7     1     5     9     3
        0     0     0     0     0     0     0
START  4000.0  STOP    7600.0 SIZE OF INTERVAL   180.00
```

EXHIBIT 5.8 Histograms showing selected variables for a Monte Carlo run of 500 trials.

```
                      FREQUENCY TABLE
        PROBABILITY OF VALUE BEING GREATER THAN INDICATED

           90     80     70      60      50      40      30      20      10
Net Present Value
1989  -4550  -2520   -918     684    1768    3432    5214    6706    9346

Internal Rate of Return
1989  -.008   .053   .096    .137    .166    .206    .251    .296    .238

  HISTOGRAM FOR COLUMN     1989 OF Net Present Value
66- 70                *
61- 65                *
56- 60                *
51- 55              * * *
46- 50            * * * * * *
41- 45          * * * * * * *
36- 40          * * * * * * *
31- 35          * * * * * * *
26- 30          * * * * * * * * *
21- 25        * * * * * * * * * *
16- 20      * * * * * * * * * * *
11- 15      * * * * * * * * * * * *
 6- 10    * * * * * * * * * * * * * *
 1-  5  * * * * * * * * * * * * * * * * *     *
        -
        1     -                       1     1
        0     5     -     4     9     4     9
        1     2     2     6     6     5     5
        7     2     7     7     2     7     2
        5     5     5     5     5     5     5
START  -11000.0     STOP   22000.0 SIZE OF INTERVAL    1650.00

  HISTOGRAM FOR COLUMN     1989 OF Internal Rate of Return
61- 65                  *
56- 60                  *
51- 55                * *
46- 50                * * *   *
41- 45              * * * * * *
36- 40            * * * * * * *
31- 35          * * * * * * * * *
26- 30          * * * * * * * * *
21- 25          * * * * * * * * *
16- 20        * * * * * * * * * *   *
11- 15      * * * * * * * * * * * * *
 6- 10    * * * * * * * * * * * * * * *
 1-  5  * * * * * * * * * * * * * * * * * *   *

        1     0     0     2     3     4     5
        6     3     8     0     2     4     6
        0     8     3     5     6     8     9
START      -.2  STOP         .6 SIZE OF INTERVAL        .04
```

EXHIBIT 5.9 Frequency table and simulation results for NPV and IRR.

Exhibit 5.9 shows the histograms for the attributes Net Present Value and Internal Rate of Return. Included also are summary statistics and a frequency table showing the *deciles* of these two attributes. The 10, 20, . . . , 90 percentiles are the nine deciles shown. Note that the labels in the exhibit are opposite to what they would be for percentiles. They are expressed in terms of *greater than* instead of *less than* numbers. The smallest number for Net Present Value falls under 90 (instead of 10); thus in 90 percent of the trials the Net Present Value exceeds −4550. In this example, it appears that the chances are about 65 percent that the Net Present Value is positive (or equivalently, that Internal Rate of Return is above 12 percent, since 12 percent was the discount rate used in the Net Present Value calculation).

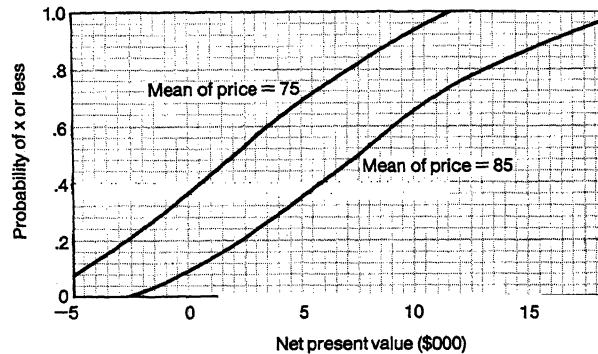



EXHIBIT 5.10 Cumulative distribution curves for new product simulations.

It is sometimes useful to plot the deciles (or other percentiles that may be reported by the simulation run). Exhibit 5.10 shows plots of the cumulative distribution for Net Present Value. One of these curves is faired through the deciles of Exhibit 5.9. The steepness of the curve at any point relates to the relative likelihood that Net Present Value falls in a small interval around that point.

5.3.4 Stochastic Dominance

One carries out a Monte Carlo simulation to test a particular strategy, that is, to see the effects of changing decision variables over their possible values. This amounts to comparing probability distributions for attributes. In certain circumstances comparisons can be made without evaluating the attitude toward risk of the firm or decision maker.

Suppose, for example, that we were looking at pricing strategy for our new product. We are not able to see for sure what we will charge for the product, since it will depend in part on prices set by the competition. Suppose that one strategy we wish to explore, however, would tend to increase selling price. Thus, rather than a mean of 75, this strategy would lead to a mean of 85.

Exhibit 5.11 shows the deciles of Net Present Value and Internal Rate of Return for this strategy. A plot of the cumulative curve for Net Present Value is included in Exhibit 5.10. Note that each decile of Net Present Value is higher for the mean = 85 case than the corresponding decile of the base case pricing strategy. This is called *stochastic dominance:*

> If under choice A the probability of achieving X or better for a specific attribute is greater than under choice B, for any X, then A stochastically dominates B on that attribute.

The cumulative probability curve for strategy A will then be to the right and below that of B as in Exhibit 5.10. If the cumulative probability curves cross, there is no stochastic dominance.

EXHIBIT 5.11 DECILES FOR NEW PRODUCT SIMULATION WITH PRICE = NORRAND(85,10)

FREQUENCY TABLE

PROBABILITY OF VALUE BEING GREATER THAN INDICATED

	90	80	70	60	50	40	30	20	10
NET PRESENT VALUE									
1989	−21	2300	4010	5820	7357	9000	10595	13233	16240
INTERNAL RATE OF RETURN									
1989	.119	.178	.222	.270	.308	.346	.383	.454	.531

5.4 THE VALUE OF INFORMATION

There are numerous occasions in model building and decision making where we ask ourselves Do we have enough information or good enough information or should we collect more? It is not unusual to be concerned as much about the choice of whether to make the decision now or study the matter further as about the choice of which alternative to take. In this section we discuss how to use the model to help decide the value of further information.

The value of information can be determined separately for each random variable that exists in the model. The model is used to determine the dollar outcome (or distribution of outcomes) for better information versus current information. As a benchmark it is useful to first calculate what it would be worth to have information that is perfect. It may be fiction that we could ever purchase perfect information, but it is a useful fiction. The value of perfect information serves as a benchmark, an upper bound on the amount we would be willing to pay for information on that variable. Often the cost of improving the estimate of a variable is higher than this maximum. In such a case, we need clearly not purchase the information, even if it were an infallible estimate.

Usually there are a variety of sources of imperfect information. For example, we might be able to purchase forecasts of a variable (at differing prices, perhaps) from several different sources. It would be necessary to evaluate each one of these sources separately. In Section 5.4.2 we discuss how to carry out this analysis.

5.4.1 Value of Perfect Information

The value of perfect information will be defined simply as the difference between how well we could do with the information versus how well we could do without the information. Suppose, for example, I introduced you to a friend of mine who happens to be clairvoyant. She can give you perfect predictions of variables that might be part of your model, but for each revelation of a variable she has a price. Whether you would pay the price would depend on the dollar value to you of improving your decision making. Let us take an example.

You have a small business engaged in printing directories of local services in your community. Your new specialty directory for business students will be marketed among your fellow MBA students. You must decide how many to print in the face of uncertain demand. You have the following estimates:

Fixed cost = 50
Unit variable cost = 2
Unit sales price = 4
Historical demand is normally distributed with mean of 100 and standard deviation 20.

My clairvoyant friend will sell you perfect information (she has not been wrong yet) about demand for your directory for $50. Is it worth it?

The way to put a price on perfect information will be to use Monte Carlo simulation to evaluate profit with and without perfect information. Then we can compare the mean (or expected) profit. The difference between the two will be called the expected value of perfect information (EVPI),

EVPI = Expected Profit (with perfect information) − Expected Profit (with current information)

Exhibit 5.12 shows an IFPS model that could be used to evaluate the two expected profits. Model PERFECT sets Production to the level of Demand (assuming we have the perfect information). Then profits are calculated using the appropriate costs in that model. Since we do not know what Demand is yet (we still have not decided whether to buy the perfect information), the model treats Demand as a random variable with mean 100 and standard deviation 20.

EXHIBIT 5.12 IFPS MODEL FOR CALCULATING EXPECTED VALUE OF PERFECT INFORMATION

```
MODEL PERFECT VERSION OF 11/05/83 14:30
1 COLUMNS 1
2 DEMAND = NORRAND(100,20)
3 SALES = MINIMUM(DEMAND,PRODUCTION)
4 PRODUCTION = DEMAND
5 PROFIT = 4 * SALES - 2 * PRODUCTION - 50
END OF MODEL
```

EXHIBIT 5.13 Sample statistics and simulation runs with perfect and current information.

```
                              SAMPLE STATISTICS

        MEAN   STD DEV SKEWNESS KURTOSIS    10PC CONF MEAN   90PC

Profit
   1    147.8    38.68      .1       3.1        146.0      149.5

     HISTOGRAM FOR COLUMN         1 OF Profit
 105- 112                   *
  97- 104                 * *     *
  89- 96                  * *     *
  81- 88                  * * * * *
  73- 80                  * * * * *
  65- 72                * * * * * *
  57- 64                * * * * * *
  49- 56              * * * * * * * *
  41- 48              * * * * * * * *
  33- 40              * * * * * * * * *
  25- 32            * * * * * * * * * *
  17- 24          * * * * * * * * * * * *
   9- 16      *   * * * * * * * * * * * * *
   1-  8  * * * * * * * * * * * * * * * * * * * *
          ---------------------------------------
                      1     1     1     2     2
          2     6     0     3     7     1     5
          6     4     1     9     6     4     1

                              SAMPLE STATISTICS

        MEAN   STD DEV SKEWNESS KURTOSIS    10PC CONF MEAN   90PC
Profit
  1    115.8    48.16     -1.6      5.2        113.6      118.0

     HISTOGRAM FOR COLUMN        1 OF Profit
 379- 405                                       *
 352- 378                                       *
 325- 351                                       *
 298- 324                                       *
 271- 297                                       *
 244- 270                                       *
 217- 243                                       *
 190- 216                                       *
 163- 189                                       *
 136- 162                                       *
 109- 135                                       *
  82- 108                                       *
  55- 81                                      * *
  28- 54                        * * * * * * * * *
   1- 27  *   * * * * * * * * * * * * * * * * * *
          ---------------------------------------
          -
          1     -     -                       1
          1     7     2     1     5     9     3
          3     1     9     3     5     7     9
```

Now suppose we want to evaluate how well we do without any additional information about Demand. The best level of Production to use is 100 based on historical information. Then Profit will depend on Demand that is actually observed. Thus to evaluate the expected profit with current information we merely do a what-if analysis on model PERFECT to set Production to 100 and run a Monte Carlo simulation of profit.

Exhibit 5.13 shows the results of running the PERFECT model and the what-if (Production = 100) case through 800 iterations of a Monte Carlo simulation. If we have perfect information, then the mean profit is $147.80, and with current information the mean profit is $115.80. Then we have

$$EVPI = 147.8 - 115.8 = \$32.$$

Since this is less than the $50 price of perfect information, we would turn down the offer from my clairvoyant friend.

The $32 value represents the maximum we would pay for even the best information possible about demand. Thus if there were less-than-perfect sources of information that cost more than the $32, we would clearly decline them. But suppose imperfect information cost less than the $32. Then we have more analysis to do.

5.4.2 The Value of Imperfect Information

In the previous section we saw how to evaluate perfect information, but we recognized that rarely do we have such high-quality sources of information. Suppose instead we wished to evaluate what to pay for an imperfect estimate of a variable. The analysis would be similar to that above. We could calculate the expected value of imperfect information (EVII) as

$$\text{EVII} = \text{Expected Profit (with imperfect information)} - \text{Expected Profit (with current information)}$$

For example, suppose we could purchase a forecast of Demand for your directory for $10 from a marketing research firm. Previous experience with this firm shows that they come close to true demand; the error distribution is normal with mean 0 and standard deviation 10.

If we were to purchase the forecast, then we would use it to set the level of Production for the directories. Then we could model the situation with the model IMPERFECT of Exhibit 5.14. Note that this model is identical to that of Exhibit 5.12, except that line 6, for Forecast, has been added, and Production has been set to this level of Forecast.

The results of a simulation run using this model are shown in Exhibit 5.15. Using the mean results from this run, we have

$$EVII = 132.10 - 115.80 = \$16.30.$$

EXHIBIT 5.14 A MODEL FOR EVALUATING IMPERFECT INFORMATION

```
MODEL IMPERFECT VERSION OF 11/05/83 14:30
1 COLUMNS 1
2 DEMAND = NORRAND(100,20)
3 SALES = MINIMUM(DEMAND,PRODUCTION)
4 PRODUCTION = FORECAST
5 PROFIT = 4 * SALES - 2 * PRODUCTION - 50
6 FORECAST = DEMAND + NORRAND(0,10)
END OF MODEL
```

Since the value is higher than the cost of $10, we would buy the imperfect information.

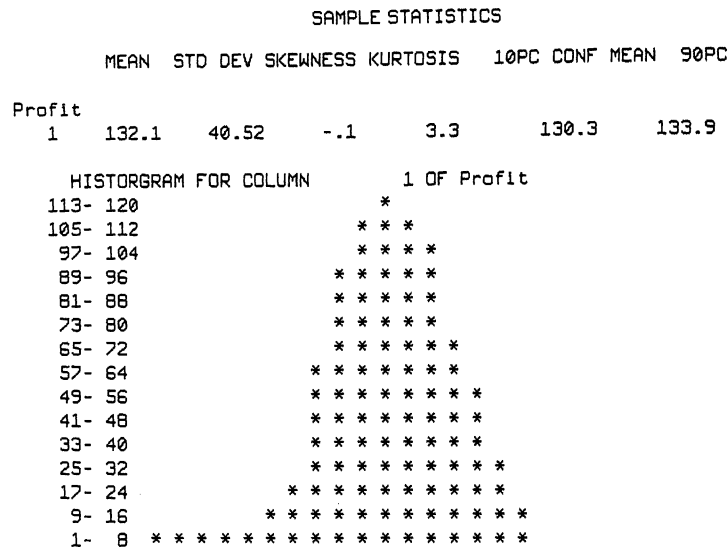
The value of information is a common concept in decision analysis and decision theory. Examples of its use can be found in many standard textbooks [see Vatter, et al. (1978) or Holloway (1979)]. Whereas the calculations of EVPI and EVII are generally done by hand, we find that the model and the decision support system may be used to carry out these calculations quickly and easily.

EXHIBIT 5.15 Sample statistics and simulation run with imperfect information.

```
                          SAMPLE STATISTICS

         MEAN  STD DEV SKEWNESS KURTOSIS   10PC CONF MEAN  90PC

Profit
  1      132.1    40.52     -.1      3.3      130.3     133.9

    HISTORGRAM FOR COLUMN        1 OF Profit
113- 120                     *
105- 112                   * * *
 97- 104                   * * * *
 89- 96                  * * * * *
 81- 88                  * * * * *
 73- 80                  * * * * *
 65- 72                  * * * * * *
 57- 64                * * * * * * *
 49- 56                * * * * * * * *
 41- 48                * * * * * * * *
 33- 40                * * * * * * * *
 25- 32                * * * * * * * * *
 17- 24              * * * * * * * * * *
  9- 16            * * * * * * * * * * * *
  1-  8  * * * * * * * * * * * * * * * * *
```

5.5 CYCLING BACK

If you have gone through the processes in this chapter of testing and improving a model, adding uncertainty and carrying out risk analysis, and determining that no more information is needed, you may be ready to make a decision in many cases. In other cases, the model is still not ready, and you will cycle back through some of the steps of the process. The risk analysis may reveal, for example, that the uncertainty is too much to bear. You look, therefore, for new decision variables that may enable you to reduce risk. At that point you set about planning a research program. Later, you will come back to the model with improved estimates.

Do not be afraid to return to stages of model development that you have already gone through once. You may return to the influence diagram, for example, and revise it. Good models often evolve through several cycles of testing, running, and refining.

A simulation model does not select among decision alternatives. Rather, it only summarizes the probabilistic results of any choice. Sometimes the choice is easy, due to stochastic dominance. At other times the choice is not easy. It may be necessary to evaluate the attitude toward risk of the decision maker. This is the topic of Chapter 7.

KEY TERMS

spreadsheet software	Monte Carlo simulation
model validation	number of trials

what-if	risk analysis
sensitivity	sample statistics
impact	cumulative probability distribution
goal seeking	stochastic dominance

EXERCISES

5.1 Refer back to the expressions developed for Exercises 4.3 to 4.5. Go through the process of validating and running models for these problems. Choose hypothetical numbers for the different variables.

5.2 Pick a decision problem that you currently face, either personal or professional. Go through the process of creating a model for this problem. Use the process described in this book to validate and run the model. Describe how you know when the model is ready for use and can be relied upon.

5.3 You must decide whether or not to grant a 1-year loan of $5000 to a certain customer. The principal plus 20 percent interest is due in 1 year. Your experience with customers of this type has been that the payoff at the end of a year has a triangular distribution with a low of 95 percent, a most likely of 118 percent, and a high of 120 percent of the principal amount. Assume that alternate uses of your money give you a return of 9 percent.

a What is the maximum amount you would pay for perfect information on the amount the customer will pay back (principal and interest) at the end of the year.

b A credit rating service will sell you a forecast of the amount the customer will pay back for $25. is forecast has been known to be in error by a percentage of principal that is normally distributed th mean 0 and standard deviation 3 percent. Would you buy the forecast?

5.4 Use the following steps of analysis with IFPS to understand better and to validate the model NEW-PROD in Exhibit 5.6 on page 64.

a Use the ANALYZE command to understand the influence relationships, starting first with the variable Net Present Value. Draw the complete influence diagram for the model.

b Use the SENSITIVITY command to see the effect of the variable Selling Price on the variables Share of Market, Revenue, and Net Present Value. Based on this analysis, what recommendations can you make for improving the model?

c Use the Impact command to find the degree of influence of Market Size, Market Growth and Investment on Net Present Value. How would you rank these variables on their degree of influence on Net Present Value?

d Use GOAL SEEKING to find a price that gives a Net Present Value of zero.

CASE ASSIGNMENTS

5.1 Green Valley Foods, page 257

The accompanying model is a first-cut attempt at modeling the problem described briefly in the Green Valley Foods case. In this model, the first 15 columns stand for fifteen 48-hour planting periods. Then the next 20 columns stand for twenty 48-hour harvesting periods. The Amount Planted in line 2 is based on the planting schedule planned at the beginning of the modeling project. The variable Plntperiod Ripe keeps track of the latest period of planting of any corn that is ripe.

Describe the process you would go through at this point to validate and use the model. What kinds of data might you want to have to make sure the model is reliable? How would you extend the model? What is your overall appraisal of the model?

```
MODEL       GREEN
1 COLUMNS 1-15,90-109,TOTAL
2 AMOUNT PLANTED = 1033,1033,226,203,179,332,146,145,144,101,109,200,'
3                    132,152,112,0
3.5 CUM ACRES PLANTED = AMOUNT PLANTED,PREVIOUS + AMOUNT PLANTED
4 HEAT UNITS = 10,12,13,15,16,18,19,20,22,24,25,26,28,28,1312,44 FOR 4,'
```

```
5                   43,42,42,40 FOR 13
6 CUM HEAT UNITS = HEAT UNITS, PREVIOUS + HEAT UNITS
7 MATURE ACRES = 0 FOR 15,VMATRIX(CUM ACRES PLANTED,PLNTPERIOD RIPE)'
7.5                     - VMATRIX(CUM ACRES PLANTED,PREVIOUS PLNTPERIOD RIPE)
8 PLNTPERIOD RIPE = 0 FOR 15,'
9.2  IF CUM HEAT UNITS .GE. VMATRIX (CUM HEAT UNITS,PREVIOUS + 3)'
9.5   + 1680 THEN PREVIOUS + 3 ELSE'
9.7  IF CUM HEAT UNITS .GE. VMATRIX(CUM HEAT UNITS,PREVIOUS + 2)'
9.9   + 1680 THEN PREVIOUS + 2 ELSE'
10.2 IF CUM HEAT UNITS .GE. VMATRIX(CUM HEAT UNITS,PREVIOUS + 1)'
10.5  + 1680 THEN PREVIOUS + 1 ELSE PREVIOUS
11 YIELD PER ACRE = 8.176
12 YIELD = MATURE ACRES * YIELD PER ACRE
13 STANDARD = MINIMUM(YIELD,2520)
14 SUBSTANDARD = MINIMUM(PREVIOUS YIELD - PREVIOUS STANDARD,2520 - STANDARD)
15 PASSED POUNDS = YIELD - STANDARD - FUTURE SUBSTANDARD
16 COLUMN TOTAL FOR L13 THRU L15 = SUM(C1 THRU C35)
END OF MODEL
```

5.2 John Denison, page 183

5.3 Stevens and Company, page 267

5.4 DiscoMall, Inc., page 233

INCORPORATING DECISION PREFERENCE INTO MODELS

CHAPTER 6

CONCEPTS IN MULTIOBJECTIVE CHOICE

Those who have wrestled with important decisions (such as what job to take, what company to acquire, what product to develop) realize that few problems involve only one objective. We develop our own methods for "muddling through" a decision with many objectives. Yet we recognize that it does not take many attributes or many alternatives before our intellects are overloaded. Most of us would like our decision making to be more systematic in the face of many objectives than it currently is.

This chapter describes how you can make multiobjective evaluations of alternatives, with or without incorporating your own personal preferences. First, methods are given that do not require tradeoffs among objectives. These methods are simple and minimize subjective judgments. They may be especially useful when the choice must be explained to others. Fewer value judgments mean less to justify to others.

In many instances, however, tradeoffs between objectives must be made and we will not relinquish our decision to a "system" that ignores our personal preferences. The second part of this chapter resolves this difficulty by presenting procedures for assessing personal tradeoffs

among attributes and using these preferences to compare alternatives. As we proceed, illustrative examples will show how DSS aids this process. Generally, most tasks can be carried out using the typical spreadsheet features of decision support software, but for many tasks we may wish to use the superior power and efficiency of optimization.

6.1 THE GENERIC CHOICE PROBLEM

The general problem of this chapter is that of making a choice among alternatives which have consequences measured by more than one attribute. The consequences of an alternative are expressed by a list of quantities or vector, $\mathbf{z} = z_1, z_2, \ldots, z_m$ corresponding to m different attributes. There may be several alternatives, in which case the consequences of the Jth alternative are denoted by the vector $\mathbf{z}^J$. Or there may be an infinite number of alternatives associated with a decision variable x that may be set anywhere in some interval. In this case the consequences are denoted $\mathbf{z}(x)$.

Problems with multiple objectives may involve a variety of attributes. Exhibit 6.1 shows three example problems with diverse attributes. Note that in referring to attributes themselves we use uppercase Z and for actual levels of attributes, lowercase z.

The nature of the attributes may also vary. They may be objectively measured scores, subjective indexes, or assignments of direct preference. See Chapter 2 for a discussion of these terms and the task of identifying attributes in general.

There are simple ways of reducing the number of attractive alternatives which we explore first. Where this does not give a conclusive or a desirable ranking, then a decision maker's tradeoffs among attributes will be used to make the choice. In some cases, we want to be able to write down a way of expressing a numerical valuation for possible vectors of attribute levels, that is a value function, $V(\mathbf{z})$. We end the chapter with some linear additive forms of $V(\cdot)$ that are easy to use in practice.

6.1.1 Illustrative Example

We now will see how one may progress through various stages of analysis of the general multiobjective problem. To give the discussion some life, let us employ a problem familiar to many: selecting a home. Assume that the homebuyer has settled on three objectives for discussion purposes: to minimize the cost of the purchase, to maximize the size of the home, and to maximize the quality of the neighborhood. The relevant attributes of any prospective house are then Price, Size, and Quality of the Neighborhood. Natural objective measures (dollars and square feet of interior space) will be used for the first two attributes, while the last is a direct preference assessment. Thus Quality of Neighborhood is scored from 0 (worst possible neighborhood in the city) to 100 (best possible).

EXHIBIT 6.1 EXAMPLE SETS OF ATTRIBUTES

	Choosing a home	**Promoting a product**	**Medical treatment**
Z_1 =	Price	Cost	Cost
Z_2 =	Location	Market share	Days of discomfort
Z_3 =	Size	Goodwill	Time until relapse
Z_4 =	Architecture		
Z_5 =	Condition		
Z_6 =	Neighborhood		

6.2 FIRST-ROUND ELIMINATION

6.2.1 Elimination by Aspects

Sometimes it is easy to eliminate some alternatives. A process that many of us use to pare down the list of options has been labeled *elimination by aspects*. In this process, alternatives are excluded because of some undesirable aspects. For example, the home buyer may drop a prospective home because of its adobe construction or because it just does not look right on the lot. It usually makes sense to carry out this elimination process before scoring alternatives on the attributes.

6.2.2 Dominance

After alternatives are scored on all attributes, it may be apparent that one (or several) alternatives are inferior to some other alternative with respect to every attribute. Then without question the alternative may be dropped from consideration.

Dominance: If alternative I is at least as desirable as alternative J on all attributes and more desirable on at least one attribute, then alternative J is dominated by I.

Exhibit 6.2 shows the attribute scores for four homes being considered by the home buyer. The home on Antrim Street (A), for example, has a higher price, smaller size, and a worse neighborhood than the Brookmere Road home (B). Thus A is dominated by B and can be dismissed. The home on Canterbury Lane (C), on the other hand, has a better price than (B) and (A), even though it is smaller than the others. Therefore, C is not dominated. Neither is the home on Downfield Avenue.

The nondominated alternatives then are B, C, and D. Nondominated alternatives are often referred to as the *efficient set* or the *admissible set* of alternatives. A scatter plot of all four alternatives for Price and Size (Exhibit 6.3) shows that the efficient alternatives are the upper right-hand frontier. Note that the scale for Price on the horizontal axis of Exhibit 6.3 decreases from left to right, since lower values of Price are preferred. Again, B dominates A because it is both above and to the right of A (and has a higher Quality of Neighborhood, even though it is not shown).

It may be tedious to identify the nondominated set of alternatives when there are very many alternatives and/or attributes. A useful trick for identifying dominance is to generate a line graph of each alternative for the string of attributes, as is done in Exhibit 6.4 using IFPS. The model that generated that line graph is included in Exhibit 6.4. Note that the Price attribute is included as 100 − Price, which has increasing desirability. Since the line for B is always at least as high as that for A, and above it for Price, the graph in Exhibit 6.4 indicates that home B dominates home A.

It is also easy to spot from this graph when an alternative—call it I—comes close to

EXHIBIT 6.2 PROSPECTIVE HOMES FOR PURCHASE

	Price, $	Size, square feet	Quality of neighborhood
Antrim Street (A)	88,000	2800	75
Brookmere Road (B)	82,000	3600	85
Canterbury Lane (C)	70,000	2600	85
Downfield Avenue (D)	95,000	3650	60

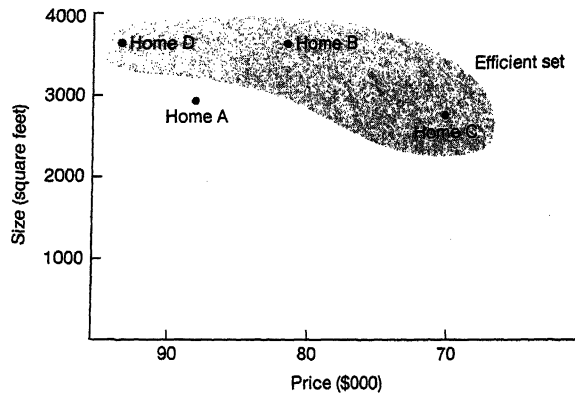
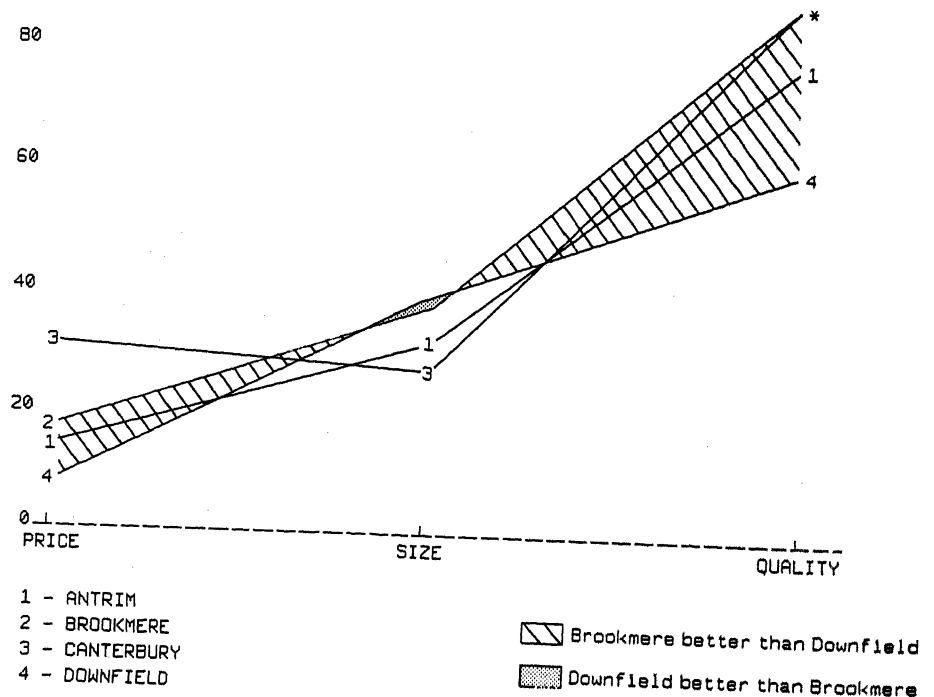



EXHIBIT 6.3 The nondominated set of alternatives.

EXHIBIT 6.4 Nondominated alternatives generated by IFPS.

```
MODEL HOME VERSION OF 04/16/83 17:22
1 COLUMNS PRICE,SIZE,QUALITY
2 ANTRIM = 100-88,28,75
3 BROOKMERE = 100-82,36,85
4 CANTERBURY = 100-70,26,85
5 DOWNFIELD = 100-95,36.5,60
END OF MODEL

ENTER SOLVE OPTIONS
? PLOT ANTRIM,BROOKMERE,CANTERBURY,DOWNFIELD
```

dominating another alternative, J. Simply observe the area under the line for I that is above J and subtract mentally from it the area under J that is above I. If the difference is very large, then J may be marked as an alternative that is likely to be less desirable than I. Notice that home B thus appears superior to home D in this sense, based on the difference between the cross-hatched and shaded areas of Exhibit 6.4. If the difference in these areas is extreme enough, this may be the basis for dropping home D from further consideration. Be aware, however, that in so doing an implicit tradeoff between attributes is being made; the tradeoff judgment is that the attribute on which home D is better could never be so important as to outweigh the greater degree to which home B is better on the other attributes.

6.3 DECISION RULES WITHOUT TRADEOFF JUDGMENTS

Ordinarily, the efficient set of alternatives cannot be pared further without making value judgments concerning the relative importance of the attributes. Sometimes, however, it is appropriate to employ easy-to-use decision rules that do not require apparent tradeoffs. Consider several of these.

6.3.1 The Lexicographic Rule

The lexicographic rule works like a rule for determining alphabetical order, from which it derives its name (a *lexicon* is a dictionary). First, alternatives are rank-ordered according to their scores on a most-important attribute. If alternatives tie on this attribute, they are rank-ordered using a second attribute, then a third, and so on, until all ties are broken.

The decision maker prespecifies the order in which attributes are used to rank the alternatives. As an example, assume that the home buyer first uses Quality of Neighborhood, then Price, then Size to order homes lexicographically. Using Quality of Neighborhood, there is a tie between B and C. This tie is broken using price in favor of C. Thus the lexicographic ranking would be C > B > A > D.

The lexicographic procedure is easy to use since the decision maker specifies only the order in which the attributes are to be considered. Unfortunately, the rule is often inadequate because it does not fully consider every attribute. Only one attribute is used unless there is a tie.

6.3.2 Satisficing

Herbert Simon, Nobel laureate in economics, has suggested that a "satisficing" rule is often used by decision makers. The decision maker searches until finding an alternative that exceeds some aspiration level on each attribute. Like the efficient set, there may be more than one satisficing alternative, but unlike the efficient set, there may be none.

Simon asserts that decision makers seldom exhibit optimizing behavior rather than this satisficing behavior. Even though, perhaps, everyone satisfices in routine decisions, it may not be the best approach for important choices, or for those based on formal analysis. The aspiration levels may not be easy to set explicitly. Conceptually, they may not hold up under careful scrutiny because an infinitesimal decline in some attribute level may change an alternative from acceptable to unacceptable.

The satisficing approach can be used in conjunction with the lexicographic procedure. Thus the decision maker has an aspiration level in mind for the most important attribute. For any alternatives that exceed this aspiration level, the next most important attribute is considered. For any alternatives that are at or above the aspiration level for the second most important attribute, another attribute will be called into consideration, and so on.

As an example of this, suppose that the home buyer considers Quality of Neighborhood most important and has an aspiration level of 70. Price is next most important with an aspiration level of $85,000 (remember lower prices are better). Finally, the aspiration level on Size is

3000 square feet. The lexicographic procedure with aspiration levels would, on the basis of Quality of Neighborhood, give three satisficing alternatives: A, B, and C. A would have an unsatisfactory Price, leaving B and C. Only B could satisfice on Size. Thus B would be the favored home.

Neither the lexicographic nor satisficing approaches allow consideration of the compensating effects of attributes. In other words, a superior performance on one attribute (e.g., Size) may not compensate for a poor performance on another attribute (e.g., Price).

The lexicographic rule, satisficing, and a combination of the two avoid tradeoffs among the attributes. Nonetheless, in setting the order of importance of the attributes in the lexicographic rule, or the aspiration levels in the satisficing rule, it is necessary to make very strong preference statements. Because these rules are simple to use, they have their place in practice. However, more robust methods are needed to capture the compensating effects of one attribute for another.

6.4 RATE AND WEIGHT: LINEAR ADDITIVE SCORING RULES

The simplest decision rule that allows high scores on one attribute to compensate for lower scores on other attributes uses ratings and weightings. *Ratings* are scores for alternatives for the separate attributes; *weightings* are desirability scores for the attributes themselves. The process is outlined as follows:

1 Rate the alternatives on each attribute:

$$r_{ij} = \text{rating of alternative i on attribute j}$$

2 Weight the relative importance of each attribute:

$$w_j = \text{weighting of attribute j}$$

3 Score the alternatives using the sum of the weights multiplied by the rates:

$$V_i = w_1 r_{i1} + w_2 r_{i2} + \cdots + w_m r_{im} \qquad (6.1)$$

where alternatives are ranked according to:

$$V_i = \text{value assigned to alternative i}$$

6.4.1 Rating Alternatives

Chapter 2 discussed the different types of quantities that might be used as attributes. One major type are those that can be objectively measured, such as Price and Size in the home buyer's choice. With these quantities, the units are unambiguous, e.g., dollars and square feet. Indexes, such as the Consumer Price Index, also usually have unambiguous scales.

Subjective measures generally have their own specific scale. For example, an air quality index might be measured on a scale from 1 to 10; a student may be graded on a scale of A, B, C, D, F; an individual's cardiovascular capability might be scored on a scale of I, II, III, IV. When subjective attributes such as these are used, it is necessary to convert ratings into numerical scores. In the rate and weight process, these ratings will be used eventually as measures of desirability; thus the numerical scores should reflect value. Thus if a student grade of A is given a rating of 4.00, B is given a rating of 3.00, and C a rating of 2.00, then the difference between the desirability of A versus B is the same as the difference between the desirability of B over C. The definition of a *unit* for the attribute scale and the *zero* rating is arbitrary. Thus grades F, D, C, B, A may be converted into the numerical ratings 0, 1.0, 2.0, 3.0, 4.0 or into the numerical ratings 20, 40, 60, 80, 100 or into some other scale with a different zero level and definition of a unit. However, the definition of the unit is a key determinant in assigning the weightings to attributes.

Sometimes there is neither an objective measure nor an appropriate scale for a subjective measure. In such cases it is appropriate to rate alternatives by direct preference measurement. Then the attribute is scored on a scale of 0 to 100, as was done for Quality of Neighborhood for the home buyer example. There are two common ways in which the decision maker may define 0 and 100.

1 Predefined range Set upper and lower limits for the attribute such that no alternative would ever be considered that had an attribute rating outside these limits. Assign the rating 0 to the least desirable end of the interval, which may be the upper limit when more of the attribute is less preferred. The most desirable end of the interval is given the rating 100.

2 Range defined by alternatives If the set of alternatives to be considered is complete (no new alternatives are yet to be identified), find the worst and best alternatives with respect to the attribute and give them ratings of 0 and 100, respectively.

In the home buyer example, the first approach was used to define the scale for Quality of Neighborhood, since new homes will be considered as they come on the market. The 0 and 100 levels are defined by the worst and best available in the region in which the home buyer was looking for a home.

6.4.2 Weighting Attributes

The weights associated with the attributes indicate their importance in the decision. Only relative weights matter; weights of 1, 2, and 3 assigned to the attributes Price, Size, and Quality of Neighborhood would give the same rankings of alternatives as the weights 10, 20, 30 or the weights .2, .4, .6. It is usually convenient, but not necessary, to have the weights sum to 1.0.

Weights are often obtained in practice simply by asking the decision maker to assign numbers between 0 and 1.00 for each attribute on the basis of its importance. But this completely ignores the definition of the unit and zero for the attribute. You may as well ask a person for the amount the person is willing to pay for an automobile without bothering to specify whether the price is denominated in dollars or yen! The weights multiply the rates when an alternative is scored. For them to have a consistent meaning they must depend on the units used for each attribute.

The assignment of relative weights is based on tradeoffs. Suppose, for example, that the weight on Quality of Neighborhood is 1.0 and the weight on Price is −.001. This would imply that the home buyer would trade an increase in Price of $1000 to achieve an increase of 1 point in Quality of Neighborhood. Again, be sure to change the weights if the definition of the units changes. Thus if we change the measurement of Price to use units of thousands of dollars, then we would change the weight on Price from −.001 to −1.0. In order to test that the decision maker really believes the weights that have been assigned, ask a number of tradeoff questions, starting at different levels for the attributes. Thus the above weights might be checked by assuring that the home buyer would trade a house with a Price of $80,000 and a Quality of Neighborhood of 70 for one that had a Price of $90,000 and a Quality of Neighborhood of 80. If the answer is no, then the weights must be adjusted.

Assume that the decision maker has settled on the following weights for attributes: −.001 for Price in dollars, .06 for Size in square feet, and 1.0 for Quality of Neighborhood in direct preference units ranging from 0 to 100.

6.4.3 Combining Rates and Weights

The comparison of the home buyer's alternatives by rate and weight can now be completed. The value score for each house could be calculated using

$$V = -.001 * \text{Price} + .06 * \text{Size} + 1.0 * \text{Quality of Neighborhood}$$

which conforms to Equation (6.1) above using the weights that had been assessed.

The results of using Equation (6.1) to evaluate each home (Exhibit 6.5) indicate that the ranking would be B > D > C > A. In the exhibit, separate weight times rate scores are evaluated for each attribute for each alternative.

EXHIBIT 6.5 RATE AND WEIGHT RESULTS FOR POSSIBLE HOMES

	Price, ($000)		Size, sq. ft.		Quality of neighborhood		Total
Home	Amount	w * r	Amount	w * r	Amount	w * r	V score
A	88	−88	2800	168	75	75	155
B	82	−82	3600	216	85	85	219
C	70	−70	2600	156	85	85	171
D	95	−95	3650	219	60	60	184

6.4.4 Comments on the Rate and Weight Procedure

It may not be apparent how serious a matter it is for the weights to reflect the scaling of the attributes. Many believe that the importance of an attribute is given by the weight, and that the weight is absolute—the scaling of the attributes has nothing to do with their interpretation.

Consider, for example, weights used by a professor to calculate grades of students. Suppose that the professor announces at the beginning of the course that there will be three attributes used in grading: participation in class (Part), a final examination (Exam), and other written work (Writn). The professor announces that the weights will be .30, .40, and .30, respectively. How can it be known what these weights mean without knowing the scaling of each individual element? While an assumption may be implicitly made that the weights suggest literally that total performance is based 30 percent on participation, 40 percent on the final examination, and 30 percent on the other written work, such an outcome would only be assured if the range of scores for each element were identical. This outcome is rarely the case, for scores depend on several factors: how hard the exam is, how well the students do, and what rescaling may be made of the data.

Exhibit 6.6 has scores on the three parts of the grade for 10 students. Each part is rated on a 0 to 100 basis. Consider three approaches to using the attribute weightings. The first approach simply uses the value score

$$\text{Total} = .3 * \text{Part} + .4 * \text{Exam} + .3 * \text{Writn}$$

The other two approaches transform the scores according to how the scores turn out for the class.

The first transformation is to standardize scores based on the mean and standard deviation

EXHIBIT 6.6 ATTRIBUTE SCORES FOR GRADING EXAMPLE

Student	Part	Exam	Writn
1	34.	69.	83.
2	39.	73.	82.
3	59.	73.	86.
4	68.	59.	86.
5	45.	59.	84.
6	50.	73.	83.
7	53.	61.	87.
8	58.	80.	85.
9	47.	75.	75.
10	70.	82.	80.

of the scores. Each score is transformed into a so-called Z score, which is the number of standard deviations (SD) that the score differs from the mean. Thus a student's Z score for participation would be

$$Z_{Part} = (Part - Mean_{Part})/SD_{Part}$$

This approach is appropriate if you suspect that scores will be normally distributed and you intend to grade "on the bell-shaped curve." Once the Z-scores for the attributes have been calculated, the value score would be

$$Z_{Total} = .3 * Z_{part} + .4 * Z_{Exam} + .3 * Z_{Writn}$$

The second way to transform the attributes would be to normalize so that the best student on each attribute received a score of 100 and the worst student a score of 0. Thus the normalized score for participation would be

$$PartN = [Part - Minimum(Part)]/[Maximum(Part) - Minimum(Part)]$$

After Exam and Writn were also normalized in this way, a third value score would be

$$TotalN = .3 * PartN + .4 * ExamN + .3 * WritnN$$

The question is Would these three ways of using the same weights give the same ranking of the students in the class? It would simplify life if they did. They do not! For the data in Exhibit 6.6 the scores and rankings of students would be those given in Exhibit 6.7. The rankings of the students (10 is the highest, 1 is the lowest) are given by Rank, ZRank, and RankN for the three scoring expressions, respectively. Notice the difference in rankings.

The moral of the story is to be sure what the units of the attributes are in assessing and interpreting attribute weights. In this example there were three different definitions for the units of the attributes, one the raw scores, one in terms of the standardized Z score, and one where lowest to highest score is transformed to a change from 0 to 100.

6.4.5 Assumptions Built into Rate and Weight

There are two strong assumptions implicit in the rate and weight procedure as it has been described:

1 Linear value in each attribute The desirability of an additional unit of any attribute is constant for any level of that attribute. Thus, an additional 100 square feet in a home is worth the same regardless of whether it is added to a home of 1000 square feet or one of 5000 square feet.

2 Additive attributes There is no interaction between attributes. An interaction might occur, for example, if the value of an additional 100 square feet of home size were higher for a home in a higher-quality neighborhood than for a home in a lower-quality neighborhood.

It is apparent that there are many situations where these two assumptions do not apply. What do you do then? Suppose we did not accept linearity in an attribute. Then it would be appropriate to use a value function to convert the attribute into a value score which is linear in desirability. Chapter 8 presents methods for carrying out this task. It also describes nonadditive forms of value functions for multiple attributes, for example, the multiplicative form, which includes terms that are the products of value scores for individual attributes.

Where applicable, the linear additive value function of the rate and weight procedure is a particularly simple and handy way of treating choices with multiple objectives. So far we have treated only situations where there is a finite set of alternatives that we need to rank-order. We

EXHIBIT 6.7 SCORES AND RANKINGS BY THREE METHODS OF USING THE SAME WEIGHTS

Student	Z_{Exam}	Z_{Part}	Z_{Writn}	PartN	ExamN	WritnN
1	− .17	−1.56	− .03	.00	43.48	66.67
2	.31	−1.14	− .31	13.89	60.87	58.33
3	.31	.57	.82	69.44	60.87	91.67
4	−1.38	1.34	.82	94.44	.00	91.67
5	−1.38	− .62	.25	30.56	.00	75.00
6	.31	− .20	− .03	44.44	60.87	66.67
7	−1.13	.06	1.10	52.78	8.70	100.00
8	1.16	.49	.54	66.67	91.30	83.33
9	.55	− .45	−2.29	36.11	69.57	.00
10	1.40	1.51	− .88	100.00	100.00	41.67

Student	Total	Z_{Total}	TotalN	Rank	Z_{Rank}	RankN
1	62.70	−.55	37.39	2.00	3.00	2.00
2	65.50	−.31	46.01	3.00	4.00	4.00
3	72.70	.54	72.68	8.00	8.00	8.00
4	69.80	.10	55.83	7.00	7.00	6.00
5	62.30	−.66	31.67	1.00	1.00	1.00
6	69.10	.06	57.68	6.00	6.00	7.00
7	66.40	−.11	49.31	4.00	5.00	5.00
8	74.90	.77	81.52	9.00	10.00	9.00
9	66.60	−.60	38.66	5.00	2.00	3.00
10	77.80	.75	82.50	10.00	9.00	10.00

simply compute the value score for each alternative and then rank them on the basis of the score. Other decision problems, however, may require more computation and the use of optimization procedures.

6.5 OPTIMIZATION

The type of problem treated so far in this chapter, exemplified by the home buyer's problem, is one of choosing the best of a finite set of objects. As such, the problem may be characterized by a single decision variable, a discrete variable that indexes the alternatives. By setting this variable to an integer value, one of the alternatives is specified.

There are many decision problems, of course, with more than one decision variable. Computational help may then be needed to set these decision variables optimally.

There are many varieties of software available to assist a manager in finding optimal solutions to models. To be consistent with the other examples, the optimization examples in this chapter are carried out using IFPS/Optimum, which optimizes IFPS models.

The principal task that the manager must carry out prior to using optimization software is to structure the problem. The manager must identify three principal elements:

1 What is to b· maximized (or minimized)
2 What the decision variables are and what kind of variables they are (continuous or discrete)
3 What the constraints are

By developing the influence diagram for a problem, much of the above effort has been accomplished.

Consider the illustrative problem of designing the features of an engine for a new automobile. [This problem was inspired by an exercise in Goicoechea *et al.* (1982), p. 36.] The modeler has completed an influence diagram of the problem, which is shown in Exhibit 6.8. The decision

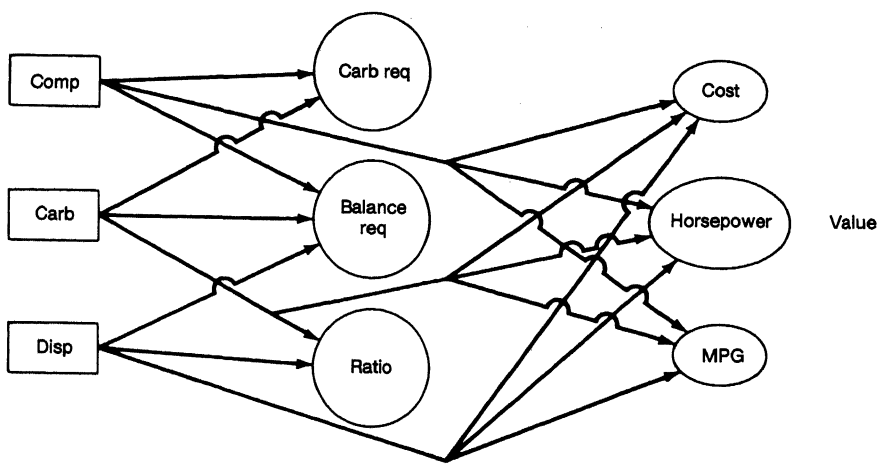



EXHIBIT 6.8 Influence diagram for automobile engine design.

variables are the compression ratio (Comp), the carburetion (Carb) in square inches, and the displacement (Disp) in cubic inches. Attributes are Cost, Horsepower, and Miles Per Gallon (MPG). These attributes are combined into an overall value score. There are three intermediate variables, Carb Req, Balance Req, and Ratio, relating to technical balance requirements for the engine. These variables must be in a certain range for the engine to work.

Exhibit 6.9 shows an IFPS model for the problem and a solution for one set of values for the decision variables. The relationships in the model are based on engineering characteristics of this particular engine and automobile. Note that the attributes are combined using weights set by the designer into the value score.

EXHIBIT 6-9 IFPS MODEL AND SOLUTION FOR AUTOMOBILE ENGINE

```
MODEL CAR VERSION OF 04/16/83 12:16
1  COLUMNS 1
2  COMP = 9
3  CARB = 32
4  DISP = 100
5  COST = 2000 + 150 * XPOWERY(COMP - 8,.9) + 10 * CARB + 3 * DISP
6  HORSEPOWER = 20 * (COMP - 8) + CARB + .5 * DISP
7  MPG = 35 - 1.7 * XPOWERY(COMP - 8,.8) - .2 * CARB - .05 * DISP
8  VALUE = 5000 - COST + 30 * HORSEPOWER - 50000/MPG
9  RATIO = DISP/CARB
10 CARB REQ = 3 * CARB - 20 * COMP + 160
11 BALANCE REQ = 30 * CARB - 50 * COMP - DISP + 400
END OF MODEL
ENTER SOLVE OPTIONS
? ALL
                      1
COMP                  9
CARB                 32
DISP                100
COST               2770
HORSEPOWER          102
MPG               21.90
VALUE              3007
RATIO             3.125
CARB REQ             76
BALANCE REQ         810
```

EXHIBIT 6.10 IFPS/OPTIMUM DIRECTIVE SET FOR ENGINE DESIGN AND OPTIMAL SOLUTION

```
DIRECTIVE CARDIR
1  OBJECTIVE
2      MAXIMIZE VALUE(1)
3  DECISIONS
4      COMP(1) BETWEEN 8 AND 11
5      CARB(1) .LE. 40
6      DISP(1) BETWEEN 80 AND 130
7  CONSTRAINTS
8      RATIO(1) .LE. 20
9      CARB REQ(1) .GE. 0
10     BALANCE REQ(1) .GE. 0
LOCAL OPTIMUM FOUND
                    1
COMP               11
CARB               20
DISP              130
COST             2993
HORSEPOWER        145
MPG             20.41
VALUE            3907
RATIO           6.500
CARB REQ        -.0001
BALANCE REQ     320.0
```

The optimization task is to find the choices for the decision variables that maximize value. Exhibit 6.10 is a directive set in the language of IFPS/Optimum that would be used to solve the problem. There are three sections corresponding to the selection of (1) the IFPS variable to maximize, (2) the IFPS variables to use as decision variables and their bounds, and (3) the IFPS variables to use as constraints and their bounds. Also in the Exhibit is the optimal solution found by IFPS/Optimum. With this software it is not necessary for the modeler to become involved with the algorithm for carrying out the optimization, which is itself a complex and intricate piece of design. The modeler, in general, has enough to do to make sure the problem is structured correctly and to know how to implement the solution.

In this example, the decision variables were continuous. In other situations they may be discrete. It is generally necessary to know which type of decision variables you have in order to select the right piece of software or to inform the software (as in the case of IFPS/Optimum) so that it can solve the problem correctly.

An example of an optimization problem with discrete decision variables would be the design of a factory. There may be a decision variable for the number of assembly lines, the number of loading ports, etc. Of course there must be an integer number of assembly lines and loading ports; therefore decision variables are integers and thus discrete. Computationally, the optimization problem with integer decision variables is solved differently and is in general more difficult for the same size problem than one with continuous variables.

KEY TERMS

tradeoffs	satisficing
elimination by aspects	rate and weight
dominance	optimization
lexicographic	constraints

EXERCISES

6.1 The Pismo Beach Consulting Group (PBCG) was completing a study comparing three microcomputer systems for a not-for-profit client. The ratings of the three systems and the weights on attributes are given below. Complete the rate and weight analysis for PBCG.

	Computer			
Attribute	**HAL**	**MSO**	**HNS**	**Weight**
1. $ cost of system	.7	1.0	.7	1.0
2. Manufacturer's commitment	1.0	.1	.6	1.0
3. # of units sold	1.0	.1	.3	.9
4. On-site maintenance	1.0	1.0	.0	.875
5. Ease in understanding manuals	.5	.4	.4	.75
6. Password protected	.1	1.0	.1	.75
7. Link to national data	1.0	.8	.8	.7
8. Training	.6	1.0	.8	.7
9. Vendor commitment	.5	.7	.1	.7
10. Ease of use	.5	.7	.7	.6
11. Software available now	.6	1.0	.8	.45
12. Software in future	1.0	.5	.3	.4
13. Ease of expansion	1.0	.0	.3	.3

6.2 Take a problem of importance to you, such as the choice of an automobile, house, vacation, position of employment, etc., and work through the multiobjective choice:

a Identify the objectives and develop measures of performance (attributes) for each objective.

b Determine the alternatives available and score each alternative with respect to each attribute.

c Weight the attributes.

d Compute the total score for each alternative using the sum of rate times weight. Which is the best alternative?

6.3 The aggregate scheduling problem in a job shop is that of determining the total resources (labor, raw materials, supplies, facilities) that have to be available for each period of time. The accompanying IFPS model [taken from Gray (1983)] allows you to consider tradeoffs between work force size, overtime, and inventory in meeting demands for a product. (This classic model was developed by Holt, Modigliani, Muth, and Simon—the same Herbert A. Simon mentioned in the chapter.) Formulate an optimization model (that might be written in IFPS/Optimum, for example) to be used with this model. To be more specific:

a Identify what should be maximized.

b Identify decision variables.

c Identify constraints—write them completely out.

d What assumptions are built into your optimization?

```
MODEL HMMS VERSION OF 08/31/82   08:28
10  COLUMNS 1–6
20  DEMAND = 420,360,390,350,420,340
30  PRODUCTION = 380
40  WORK FORCE=PRODUCTION/10
50  CHANGE IN WORK FORCE=0,WORK FORCE - PREVIOUS WORK FORCE
60  CURRENT INVENTORY=PRODUCTION - DEMAND
70  TOTAL INVENTORY=CURRENT INVENTORY,PREVIOUS TOTAL INVENTORY +'
71  CURRENT INVENTORY
80  *
90  PAYROLL = C1 * WORKFORCE
100 HIRING COST=C2 * CHANGE IN WORK FORCE * CHANGE IN WORK FORCE
110 OVERTIME=C3 * (PRODUCTION-C4 * WORKFORCE) * (PRODUCTION'
110 -C4 * WORK FORCE)
120 OVERTIME2=C5 * PRODUCTION
130 OVERTIME3=(-C6 * WORK FORCE)
140 INVENTORY=C7 * (TOTAL INVENTORY-C8-C9 * DEMAND) * (TOTAL
    INVENTORY'
141 -C8-C9 * DEMAND)
150 *
270 COST = SUM(L90,L140)
280 TOTAL COST= PREVIOUS TOTAL COST + COST
```

Note that C1, C2, . . . , C9 are cost constants that depend on the specific problem.

CASE ASSIGNMENTS

6.1 *California Oil Company,* page 203

a Carry out the task described in the case. Which site would you recommend to Mr. Shimer? Is it possible to make a choice among the four alternatives without making tradeoffs between attributes? Why designate a best and worst outcome or consequence for each attribute?

b What are the characteristics of the recommended site? In how many of the attributes was the site the best of the four? The worst of the four?

c Suppose the environmental attribute became top priority. Would your recommendation change?

6.2 *Green Valley Foods* page 257

Structure an optimization for this case. Decide what is to be maximized or minimized, identify the decision variables, and specify any constraints. How would you go about assessing a value or objective function to be used in this case? You needn't solve the resulting statement of the optimization problem.

CHAPTER 7

RISK PREFERENCE AND UTILITY

Many risky opportunities are evaluated solely by the average of the possible financial outcomes, that is, by the expected monetary value (EMV). But in reality very few people are willing to play the averages when making decisions of any importance. For example, most people will gladly pay more than the average potential loss for insurance on any significant piece of property. Most people would sell for $500 cash a lottery ticket giving a 50 percent chance at $1000.

How much above the average loss should one be willing to pay for insurance? Or how much below the average value of $500 should one set the price on the lottery ticket? In this chapter, methods are presented for explicitly incorporating risk into decisions in a consistent, systematic way. The approach is to score the *utility* of various outcomes according to the decision maker's attitude toward risk. The best course of action is the one with highest expected utility. The power of the utility method is that one can represent a rich variety of risk preferences within a single unifying concept and set of procedures.

7.1 ASSIGNING UTILITY SCORES

For starters, consider the use of utility in the everyday problem of whether you carry an umbrella with you when you go out. The decision tree is shown in Exhibit 7.1, where p is the probability of rain (for simplicity, let us leave snow and other forms of precipitation out of this). With

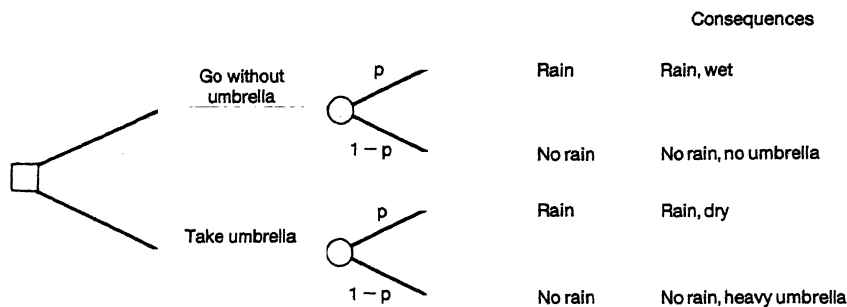



EXHIBIT 7.1 The umbrella decision example.

each choice of action, denoted by a small circle on the diagram, there are two possible outcomes (rain and no rain). Thus there are four possible consequences. To complete the analysis, you must weigh the comparative value of the four consequences. Let us assume your first thoughts about these consequences are that to remain dry when the umbrella is left at home is the best of the lot and to get caught in the rain without the umbrella is the worst. The desirability of the other alternatives is harder to establish. You think for a moment about which is better—to get wet or to carry an umbrella when it does not rain. Let us say that you prefer "rain, dry" to "no rain, heavy umbrella." Having ranked the four consequences, you are still not able to make the decision without deciding exactly how much you prefer one to another. Getting impatient with such a minor problem, you quickly assign a score to each of the four consequences.

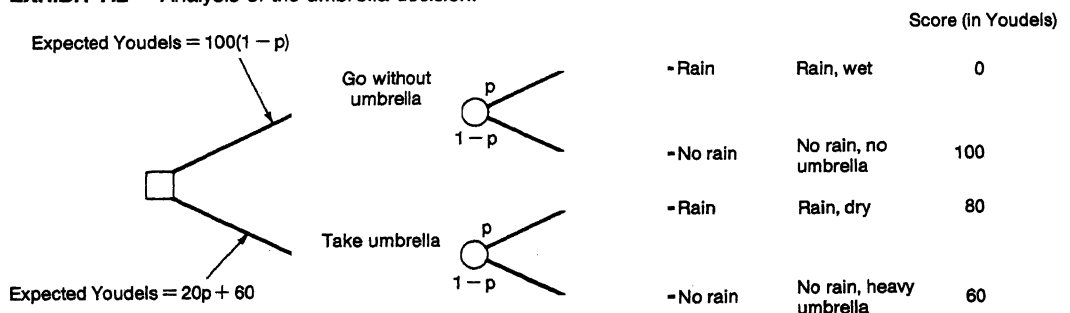
You do this by first making up your own currency—call it Youdels—to express the relative desirability of each consequence. Thus, rain, wet gets 0 Youdels and no rain, no umbrella gets 100 Youdels. Having the umbrella when it rains gets 80 Youdels and carrying the umbrella when it does not rain gets 60 Youdels. You make your decision by maximizing Youdels. When outcomes are uncertain you multiply Youdels by the probability for the outcome and sum over the possible outcomes, giving expected utility. As you can see in Exhibit 7.2, the expected utility (in Youdels) of leaving the umbrella home is

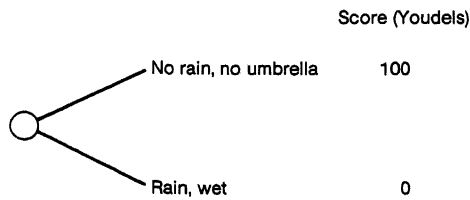
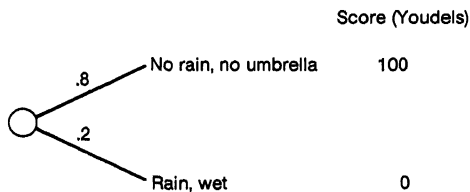
$$0(p) + 100(1 - p) = 100(1 - p)$$

and the expected utility of taking an umbrella is expressed

$$80(p) + 60(1 - p) = 20p + 60$$

When p is near 0, you would leave the umbrella at home, but for high p you would take it. For what p is the decision a toss-up? The answer could be found by setting expected utility

EXHIBIT 7.2 Analysis of the umbrella decision.

equal for the two alternatives and solving for p. This gives p = .33, when the chance of rain is above 1 in 3; take the umbrella, otherwise leave it at home.

What basis can there be for assigning Youdels to the possible consequences? These are subjective judgments made by the decision maker. An appropriate device for assessing the utility of each consequence is to compare them to a *reference lottery*. This lottery will have two possible outcomes. It is most convenient to use the best and the worst of the available outcomes. In the umbrella problem, these outcomes are no rain, no umbrella and rain, wet. The reference lottery which follows uses these outcomes and the Youdel scores assigned to them.

To assign utility scores to the remaining alternatives, the decision maker alters the probabilities of the reference lottery's two outcomes until finding that he or she is indifferent between the outcome being evaluated and the lottery. For example, to evaluate the outcome rain, dry, our decision maker adjusts probabilities of the lottery outcomes to an 80 percent chance of no rain, no umbrella, leaving a 20 percent chance of rain, wet, as shown in the reference lottery below. In other words, our decision maker has decided that an 80 percent chance of no rain, no umbrella is as good as a 100 percent chance of rain, dry.

Expected Youdels = .8(100) + .2(0) = 80

The consequence no rain, heavy umbrella, which was assigned 60 Youdels, would be equally desirable to a lottery with the same outcomes but a probability of the better outcome of .6.

If every consequence in a decision tree is assigned Youdels in this way, then a valid rule for making the decision is to choose the alternative with the highest expected Youdels. Expected utility is based on a principle of substitution. To see how it works, replace the consequence rain, dry by the reference lottery above and the consequence no rain, heavy umbrella by its reference lottery to get a diagram of the take umbrella option, as shown in Exhibit 7.3*a*. Then collapse the two probability nodes into a single equivalent reference lottery, as shown in Exhibit 7.3*b*. The probability of the better outcome in Exhibit 7.3*b* is found by multiplying the probabilities on the branches that lead to this outcome and summing over the two paths to the better outcome giving .8p + .6(1 − p). Once an alternative has been expressed in terms of a reference lottery, it can be compared to the reference lottery of any other alternative simply on the basis of its probability of the better outcome. This is precisely the same as comparing expected utilities of the alternatives; the take umbrella option has 100 [.8p + .6(1 − p)] as its expected utility, which equals 20 p + 60 as found in Exhibit 7.2.

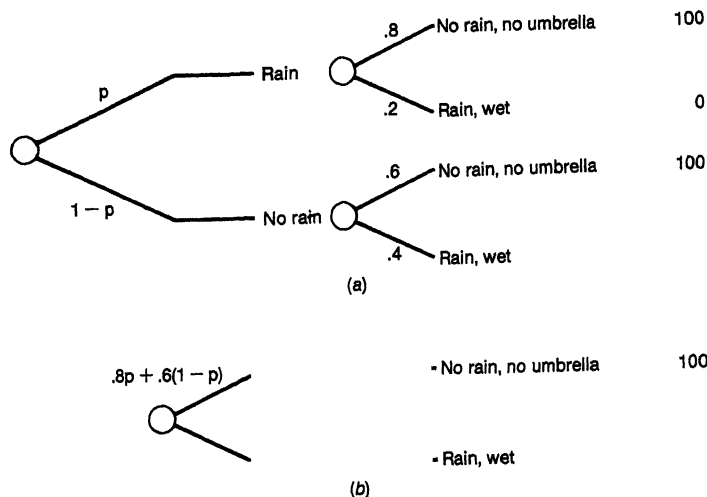



EXHIBIT 7.3 The substitution principle and expected utility. (*a*) Substituting the reference lotteries for consequences of "take umbrella" option; (*b*) collapsing to an equivalent single-reference lottery.

The scaling of the consequences between 0 and 100 is arbitrary; Youdels is one of many equivalent ways of scoring utility. We could instead have used utility values scaled between 0 and 1.00 (which is the usual rating employed) or between any other two values. The important matter is that the scores assigned to all of the consequences are measured in the same currency. A utility scale is like a temperature scale; the zero level and the units assigned to an interval are arbitrary. But as in temperature, it is important to know what scale is being used (whether Youdels or utiles); we would not know, for example, if 30° is cold or warm unless we knew whether it was measured in Fahrenheit or Celsius.

7.2 ASSESSING UTILITY OF MONETARY CONSEQUENCES

In the umbrella problem, a decision was made under uncertainty where the consequences were nonmonetary. When the consequences are monetary, we face the same fundamental question of how to value the end points of a decision tree. You may be thinking that the monetary value itself can be used to evaluate an end point and that it is sufficient to simply average the monetary values. However, this does not account for the effects of some of the truly bad outcomes. For example, consider the 50-50 lottery between \$−5000 and \$10,000. The expected monetary value is .5(10,000) + .5(−5000) or \$2500. Would you pay \$2500 to purchase this lottery? Most people would not. We all use something besides monetary value and probability to evaluate uncertain prospects. Our evaluation is usually based as well on a third factor: our willingness to face the risk of loss. Utility is a way to score monetary values using the decision maker's own subjective risk preferences.

The arithmetic of utility is simple, as we have discussed. Suppose you wished to evaluate the lottery just described—a 50-50 chance of making \$10,000 or losing \$5000. If the utility of \$10,000 is labeled u(10000) and the utility of losing \$5000 is labeled u(−5000), then the utility of the lottery is expressed as follows:

$$\text{Expected utility} = .5u(10000) + .5u(-5000)$$

The value of the lottery to you depends on how much risk you want to assume. Suppose that you decide that if you owned the lottery and were offered \$1000 for it, you would be indifferent

as to selling it or keeping it. Thus, for you the utility of \$1000 is equal to that of the lottery,

$$u(1000) = .5u(10000) + .5u(-5000)$$

We calculated the EMV of this lottery above as \$2500. In your case, a certain \$1000 is as valuable as this uncertain EMV of \$2500. The \$1000 is termed the *certainty equivalent* (CE) of the gamble. The difference between the EMV of the gamble (\$2500) and the CE (\$1000) is called the *risk premium* (RP) (\$1500 in this case). The RP is the amount of money you are willing to give up to avoid the risk of loss. By definition,

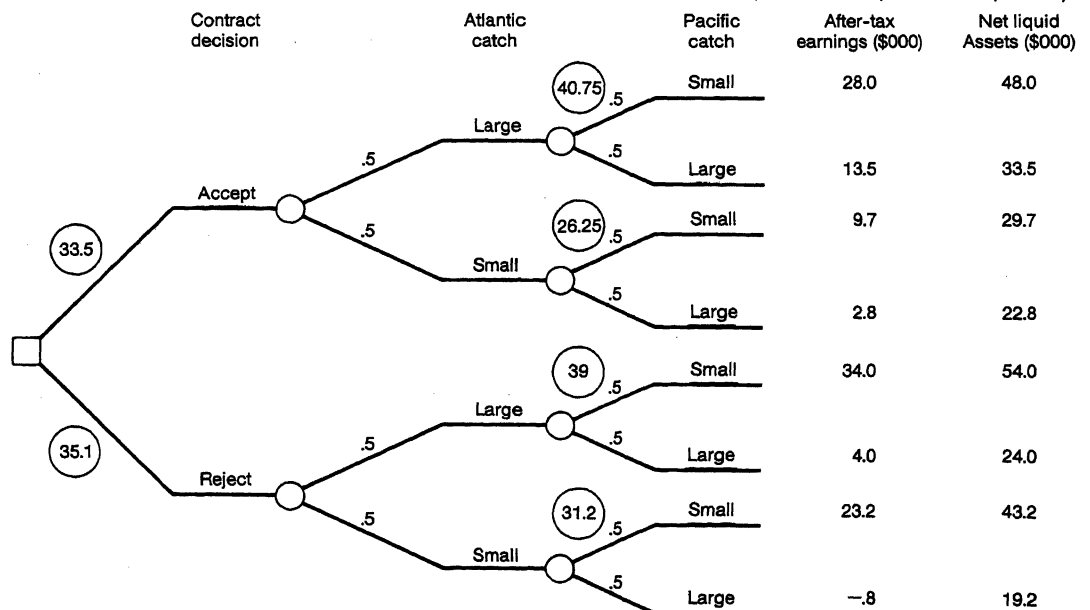
$$RP = EMV - CE$$

Determining the CE and RP in this example was fairly simple, since there were only two possible outcomes in the lottery. In problems having many possible outcomes of varying probabilities, it is helpful first to assess the decision maker's preferences, then to apply these preferences to the problem.

Consider the problem of Lars Dyson (taken from the case "Edgartown Fisheries," Harvard Case Services). He must decide whether to accept or reject a contract under which his earnings will depend on the sizes of the fish catches in the Atlantic and Pacific oceans. His decision tree is shown in Exhibit 7.4. The figures in circles represent the expected net liquid assets. According to this analysis, accepting the offer is worth \$33,500 and rejecting it is worth \$35,100. The obvious solution is to reject the contract.

But this method does not account for Dyson's risk preferences. How can we assess those preferences in this more complex situation? What follows is a dialogue that you might use to determine the utility to Dyson of the various monetary consequences.

You: In order to begin to determine your risk preferences for liquid assets Mr. Dyson, we first need to establish a range of outcomes which encompasses all possible consequences of your decision. What are the worst and best possible outcomes.

EXHIBIT 7.4 Dyson's decision with probabilities and consequences and expected net liquid assets (circled).

Dyson: The decision tree indicates a range from $19,200 to $54,000. But I'd like to use this preference analysis for some other decisions I will be making. So let's assume that liquid assets could be anywhere between $10,000 and $60,000.

You: Fine. I will ask you a series of questions about gambles, all having outcomes within this range. These will each be 50-50 gambles with two outcomes. First, consider a 50-50 gamble with the outcomes at the extremes of your range: $10,000 and $60,000. If this gamble represents an investment opportunity you now own where the outcome is to be determined momentarily, say by the flip of a coin, would you trade that opportunity for $33,000?

Dyson: Let's see, the average outcome for the gamble you described is $35,000. Yes, of course I'd take $33,000. I'd give up $2000 to avoid the prospect of ending up with only $10,000 of liquid assets.

You: O.K.

Would you trade that gamble for $15,000?

Dyson: No, that's giving up too much to avoid the risk.

You: Do you have any idea what is the least amount you would take in exchange for that gamble?

Dyson: I'll have to think about that for a minute. Hmm. I believe I would give up about $11,000 to avoid the risk. Then the least I would take would be $24,000.

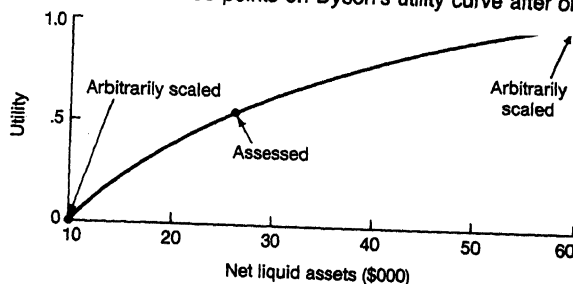
You: The $11,000 figure you mentioned is called a *risk premium*. It constitutes the difference between the expected outcome of the gamble, $35,000, and the $24,000 you chose, which is called the *certainty equivalent* of the gamble. I know it's hard to evaluate these hypothetical lotteries and you may wish to adjust your certainty equivalents up or down as we go on. But I want you to think of it as a decision you make, not an estimate; there is no right or wrong answer for the certainty equivalent. It's only an expression of your preference. I am working toward drawing a utility curve for you and this response gives me one point on the curve. I have arbitrarily assigned to the end points of your range the utility values 0 and 1.0. Since you are indifferent between $24,000 and a 50-50 gamble between the end points, $24,000 has a utility value halfway between 0 and 1.0, that is, .5. That gives me three points on your utility curve: the end points and this certainty equivalent (Exhibit 7.5). Let me proceed with some more questions. Suppose that you now own the right to a 50-50 gamble with outcomes $10,000 and $24,000. I want you to think about a certainty equivalent for this gamble. What is the least amount you would take in exchange for this gamble?

Dyson: Again, I like to think of this in terms of the average for the gamble, which is $17,000. I'd put the certainty equivalent at, let me see, $16,000.

You: Then your $16,000 has a utility of .25. Now let's look at the upper region. What is your certainty equivalent for a 50-50 gamble between $24,000 and $60,000?

Dyson: Well, since both of those outcomes are fairly good ones, I would have a low risk premium for that gamble. I'll say $37,000.

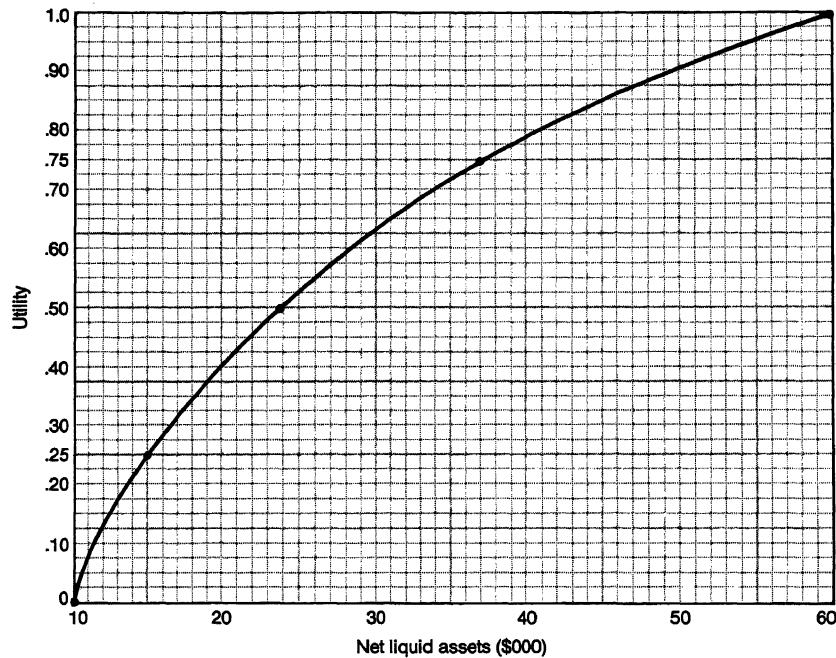
You: Just to be sure, I want to ask you a question to check your consistency. Are you indifferent between a certain amount of $24,000 and a 50-50 gamble with outcomes of $16,000 and $37,000?

EXHIBIT 7.5 Three points on Dyson's utility curve after one assessment question.

Dyson: Not really, I'd take the gamble.

You: Something's wrong, then. You have set the utility of $16,000 at .25, the utility of $37,000 at .75, and the utility of $24,000 at .5. Then $24,000 ought to be halfway between $16,000 and $37,000 on the utility scale, showing that it is equally desirable to a 50-50 lottery between $16,000 and $37,000. What number should I change?

Dyson: I think the low number $16,000 was a little hasty. I'm going to move it down a bit to $15,000. That will make $24,000 just as desirable as a 50-50 gamble between $15,000 and $37,000. Now what do we have?

You: Well the $15,000 and $37,000 are two new points on your utility curve. We could continue the assessment process further to get more points on the curve, but if we stop here and I draw a line through the points, we have the curve I've drawn here (Exhibit 7.6).

EXHIBIT 7.6 Dyson's utility curve.

To summarize the assessment process, the respondent was asked for CEs to 50-50 lotteries. Starting with a lottery involving the two end points of the assessment region, we used the CE to divide the region into two pieces. Then CEs were found for the 50-50 lotteries involving the end points of the two pieces. The utility of each CE was plotted. The process could have been continued to find the CEs which further divide the four pieces into eight pieces and so on until enough points in the assessment region exist so the utility curve can be faired through their respective utility values.

7.3 USING THE UTILITY CURVE

Having assessed Lars Dyson's preferences, let us see how they may be used to resolve his choice problem. Using a utility curve to analyze a decision tree involves the following steps, which have been completed in Exhibit 7.7.

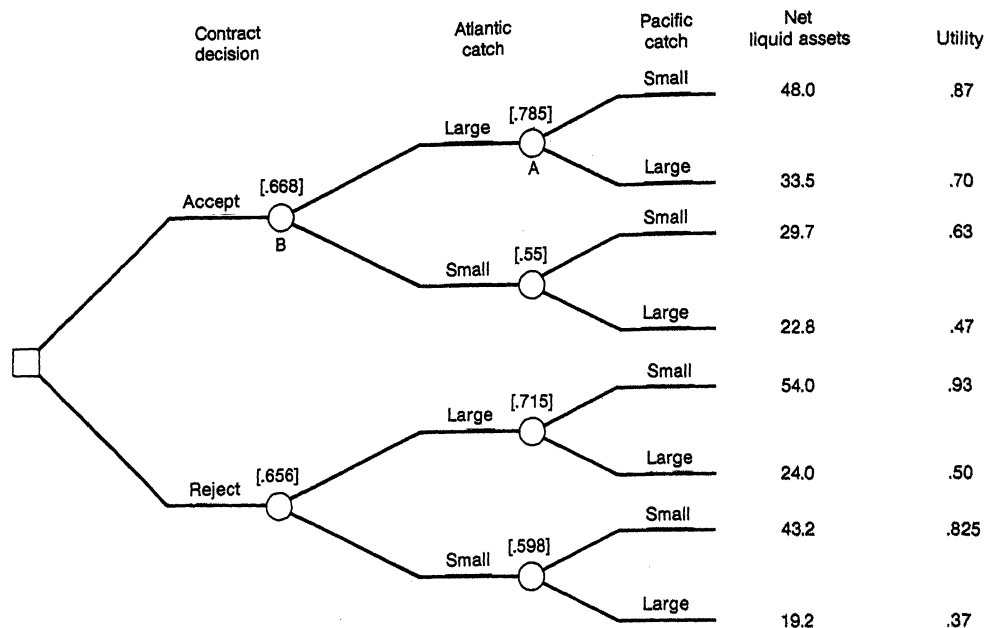



EXHIBIT 7.7 Analysis of Dyson's problem using utility.

1 Using the curve, find the utility for every end point of the decision tree For example, reading up from 48 on the curve, we find a utility of .87 which is assigned to the topmost terminal point of the tree in Exhibit 7.7.

2 Fold back the tree, using expected utility Starting at the right and working left, compute expected utility for each event node. Expected utility is the sum of the probabilities multiplied by the utilities. Thus at the event marked A in Exhibit 7.7, the expected utility is .5[.87] + .5[.70] = [.785]. Expected utilities are enclosed in brackets. Once all are computed move left; expected utility at event B is computed as .5[.785] + .5[.55] = [.668].

3 Choose the action with the highest expected utility At a decision node, marked by a small square on the tree, take the branch with the highest expected utility. In this case the best action is to accept the contract which has expected utility of [.668]. The CE for any position in the tree is found by entering the utility curve on the vertical scale for the given expected utility value and reading the asset position on the horizontal scale. Thus, the CE for the entire tree is 31.7 (corresponding to an expected utility of [.668]). This number represents the least amount Dyson would take in cash to sell out of the deal. The RP in this case is $33,500 − $31,700 = $1800.

The utility analysis suggests the opposite choice from the EMV analysis. This is due to Lars's risk aversion and to the greater amount of risk in the reject choice. Note that even though utility numbers appear to be very close, the CEs for the two alternatives are several hundred dollars apart. It is often necessary to keep several digits of accuracy in using utility because of the choice to scale it between 0.0 and 1.0.

These steps can be used for any decision tree with discrete probabilities, no matter how complex. The utility curve may also be used to evaluate decisions involving continuous random variables. Suppose, for example, that Lars Dyson was offered another investment opportunity. This opportunity was best modeled by a triangular distribution on Net Liquid Assets, with low, most likely, and high values of 10, 25, and 55, respectively. IFPS could be used to evaluate this prospect by including the following lines in a model:

```
NET LIQUID ASSETS = TRIRAND(10,25,55)
  Utility = INTERPOLATION ON(NET LIQUID ASSETS, 10,0,15,.25,20,.39,24,.50,'
            30,.63,37,.75,45,.84,51,.90,60,1.0)
```

The first line models the triangular probability distribution. The second line expresses utility using points from the assessed utility function for Dyson, approximated by a piecewise-linear function using the INTERPOLATION ON function of IFPS. Since we want to obtain expected utility, we would run a Monte Carlo simulation for this model and use the *mean* of utility reported by IFPS to evaluate the prospect. The result of a 1000-trial simulation was a mean utility of .5292. Goal seeking was used to find the Net Liquid Assets that has this utility, which is 28.65. This is the CE, and thus the price Lars Dyson would put on this prospect.

Once assessed, a utility function can be used for any number of decision analyses. The preference curve may be viewed as a statement of standard operating procedure. It is in effect until it is changed; it can be adjusted at any time. Using this function, decisions in the face of uncertainty will be made consistently and efficiently. Differences of opinion between those involved in a decision can be divided into arguments over beliefs (probabilities) and preferences (utility).

7.4 RISK AVERSION

If a person's RP for any money gamble is always positive, then we refer to the individual as being "risk averse." The larger the RP for a given gamble, the greater is the person's risk aversion. On the other hand, if the CE exceeds the EMV (and the RP is therefore negative) the person is risk-prone. If the CE always equals the EMV, then the person is risk-neutral. In this case it is unnecessary to assess a utility curve, since an EMV evaluation will always rank alternatives the same as expected utility. The utility function in this case would be a straight line.

Lars Dyson is risk-averse for the range of net liquid assets shown in Exhibit 7.6. The fact that his utility curve is always above a straight line drawn between its end points (the dish opens down) indicates aversion to risk. A risk-prone curve would open upward.

There is no reason that one cannot be risk-averse in one region and risk-prone in another. For example, an S-shaped curve would indicate risk proneness for low outcomes (i.e., a willingness to gamble when things are bad) and risk aversion when outcomes are high. Most individuals, however, tend to be risk-averse for all significant monetary levels. Managers in firms are even more likely to be risk-averse regarding company money. There is a good reason for this. If a firm consistently pays more than the EMV for risky opportunities, then statistical laws predict economic ruin in the long run.

For gambles with many outcomes, the procedure for finding CEs and RPs is to compute expected utility first, then convert to the CE in order to find the RP. However, when the decision maker exhibits certain characteristics, it is possible to simplify the process of finding CEs or RPs. One such characteristic is constant risk aversion.

7.4.1 Constant Risk Aversion: Negative Exponential Utility

Decision makers show *constant risk aversion* if they have the same positive RP for any two gambles that are identical, except that each successive outcome of one differs from the corresponding outcome of the other by a constant. The series of gambles and RPs in Exhibit 7.8 illustrates constant risk aversion.

When a decision maker is constantly risk-averse, we do not need to draw the utility curve

EXHIBIT 7.8 EXAMPLE OF CONSTANT RISK AVERSION

50-50 gamble	EMV	CE	RP
$-10,000, 0	$-5,000	$-7,169	$2,169
0, 10,000	5,000	2,831	2,169
10,000, 20,000	15,000	12,831	2,169
20,000, 30,000	25,000	22,831	2,169

or read utilities off the curve. Rather we can express the curve mathematically and use this equation to compute CEs. The utility of any outcome is given by the following negative exponential expression:

$$u(\$) = (1/r)\,(1 - e^{-r\$}) \tag{7.1}$$

where r = a constant expressing the degree of risk aversion and e = 2.718. (Note that $e^{-r\$}$ is the constant e taken to the power $-r\$$, which is equivalent to $1/e^{r\$}$.)

To assess a utility curve for a decision maker who is constantly risk-averse, it is necessary to determine a single constant r which establishes the degree of risk aversion. Consider two ways of determining r. One method is to ask for the decision maker's CE to a 50-50 lottery between 0 and x, where x may be any positive value. The ratio of this CE to x has a fixed relationship with the value rx (shown in Exhibit 7.9), as long as the decision maker is constantly risk-averse. For example, suppose that you set x at $10,000 and that the decision maker is indifferent between receiving $3000 and having a 50-50 lottery between 0 and $10,000. The CE is $3000. Using CE/x = 30 percent, look up rx, which is 1.8 in Exhibit 7.9. Then r(10000) is 1.8, and r = .00018.

As another example, recall that in Exhibit 7.8, the decision maker's CE for a 50-50 lottery between 0 and $10,000 was $2831. Thus CE/x was 28.31 percent, which is associated on the curve of Exhibit 7.9 with rx = 2.0, and thus r = .0002. Thus the utility function for this decision maker is given by

EXHIBIT 7.9 Relationship between the CE ratio of a standard reference gamble and the risk aversion constant.

$$
\begin{aligned}
u(\$) &= (1/.0002)\,(1 - e^{-.0002\$}) \\
&= 5000\,(1 - e^{-.0002\$})
\end{aligned}
$$

This utility curve is graphed in Exhibit 7.10.

A second way to evaluate the constant r is to offer the decision maker the following 50-50 gamble, increasing the value of y in the gamble until the decision maker is indifferent between the gamble and zero.

The risk-aversion constant r is *approximately* the reciprocal of whatever y gives indifference. In other words $r = 1/y$.

This question might be used as a consistency check on the r assessed by the CE ratio method. Thus the person whose $r = .0002$ should be indifferent between

It is always a good idea to assess preferences using more than one kind of question to assure that the utility curve adequately represents the feelings of the decision maker.

Another means of double checking the r is to use Bernoulli's suggested rough rule of thumb. In 1738, Daniel Bernoulli, perhaps the first writer on risk preferences, suggested that risk aversion be inversely proportional to wealth ($r = 1/\text{wealth}$). This implies that one would be willing to take a 50-50 gamble either to double one's wealth or to cut it in half. Clearly, not every person or every institution with equivalent wealth will have the same aversion to risk. Yet Bernoulli's guideline may serve as a *very* rough rule of thumb to indicate the general neighborhood for r.

With the utility curve written mathematically as in Equation (7.1), we can compute the CE of any lottery directly without drawing the curve and then reading values from it. Any lottery having outcomes $x_1, x_2, \ldots, x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$ has a CE given by the following:

$$
CE = -(1/r)\ln(p_1e^{-rx_1} + p_2e^{-rx_2} + \cdots + p_ne^{-rx_n}) \qquad (7.2)
$$

where ln is the natural logarithm (ln x is the number which as the power of e gives x). If we

EXHIBIT 7.10 Constantly risk-averse utility curve

were finding the CE for a 50-50 lottery with outcomes 0 and $10,000 for r = .0002, then the calculations would be

$$\begin{aligned} CE &= -5000 \ln(.5e^{-.0002(0)} + .5e^{-.0002(10,000)}) \\ &= -5000 \ln[.5(1) + .5(.1353)] \\ &= -5000 \ln(.5677) = -5000(-.5662) = 2831 \end{aligned}$$

7.4.2 Decreasing Risk Aversion

Most of us become more tolerant to risk as we become richer. For example, in our poorer student days we chose to have no deductible on our auto insurance. At that time it may have seemed impossible to cover a deductible in the event of an accident. As our asset positions improved, the deductible looked more attractive, until eventually we looked for the highest deductible available. What has happened? The probability of an accident may not have changed, but in evaluating the alternative losses, we have decreased our RP. When the RP drops sufficiently, we self-insure. If our asset position were to increase enough, say to the point that it was comparable to that of Prudential or Traveler's, then we would probably be willing to sell insurance to others. Decreasing RPs explain why companies with huge assets may self-insure or sell insurance to others. In fact, if risk aversion did not decrease over a very wide increase in wealth, the insurance industry might not exist. Why else would a large company take on the risks of a smaller company?

If the RP decreases for gambles that are identical, except for adding the same constant to each possible payoff, then the decision maker has *decreasing risk aversion*. Exhibit 7.11 shows the risk premiums of such a person.

Given the apparent importance of decreasing risk aversion, it would be nice to have some easy-to-use utility curves that exhibit this property. Many utility curves show this property, but a particularly simple one is the generalized logarithmic utility model. This model has been labeled the "premier model of financial markets" [Rubenstein (1976)] because of its superior ability to model investor behavior. This model may be written

$$u(x) = \ln(x + A) \text{ for } x > -A$$

EXHIBIT 7.11 EXAMPLE OF DECREASING RISK AVERSION

50-50 gamble	EMV	CE	RP
$-10,000, 0	$-5,000	$-6,339	$1339
0, 10,000	5,000	4,365	635
10,000, 20,000	15,000	14,580	420
20,000, 30,000	25,000	24,686	314

where x may be any of a number of monetary quantities and A sets the degree of risk aversion (higher A is lower risk aversion). Typically, x is an incremental after-tax cash flow and A is the decision maker's (or institution's) net worth prior to realizing benefit due to the decision at hand. Then x + A is ending net worth after realizing benefit from the current decision.

A larger A means less risk aversion. That is, if one is richer, one worries less about solvency and thus is less averse to uncertainty. Note that as x approaches $-A$ (that is, you lose an amount approaching your net worth), utility approaches negative infinity. Thus A may be interpreted as your distance from economic ruin. Using this interpretation, you may wish to set A to some other number than your current net worth. For example, making A greater than your net worth simply means that you allow yourself to go to a negative net worth before your utility goes to negative infinity. Or conversely, A may be less than your net worth if there is some positive level of reserve for future consumption below which you will not let your wealth drop. Note that as x becomes large, risk aversion approaches 0. Thus, the utility curve approaches a straight line for large x (Exhibit 7.12 shows an example log utility function).

If constant risk aversion applies, it does not matter whether an investment opportunity is evaluated in terms of incremental cash flows or ending net worth—the difference between the two is a constant—namely, beginning net worth. When constant risk aversion does not apply, however, then the decision maker's attitude toward risk in incremental cash flows will not be the same as that toward net liquid assets. Thus, in the logarithmic utility model, if x is incremental after-tax cash flow, then utility is a function of x + A, which may be ending net worth, ending net liquid assets (as was used in Lars Dyson's problem), or some other measure of total financial position or adjusted financial position (depending on how A is set).

Total measures provide for more consistent evaluation than do incremental measures when utility is assessed directly. If preferences are assessed over net worth or net liquid assets, then if asset position changes, it is not necessary to reassess preferences; just move to the right on the curve. When using an incremental measure, however, it may be necessary to reassess preferences when asset position changes, since risk aversion may have changed.

A useful property of the logarithmic utility model (like the constant-risk-aversion model) is that CEs can be calculated directly. For the log utility model they are computed using

$$CE = (x_1 + A)^{p_1} (x_2 + A)^{p_2} \cdots (x_n + A)^{p_n} - A$$

Again, p_i represents the probability of outcome x_i.

For instance, let us calculate the CEs of Exhibit 7.11. They were obtained from a utility function

$$u(x) = \ln(x + 15000)$$

EXHIBIT 7.12 Decreasingly risk-averse logarithmic utility curve.

which is shown in Exhibit 7.12. To find the CE for the 50-50 lottery between 0 and $10,000, the calculations are the following:

$$\begin{aligned} CE &= (0 + 15000)^{.5} * (10000 + 15000)^{.5} - 15000 \\ &= 19365 - 15000 = 4365 \end{aligned}$$

The log utility model is easy to use. The only judgment necessary is the existing level of net worth (or alternatively the point of economic ruin). All calculations are straightforward on a calculator, with no need to graph the curve. The model may be generalized somewhat by adjusting A to reflect the desired degree of risk aversion.

The log utility model may be generally useful as an approximate model of decreasing risk aversion. However, it will be exact only when this condition applies: The decision maker is indifferent between receiving x (ending with net worth x + A) and the following lottery.

for any $x > -A$ and $p > 0$. The decision maker would be equally happy to take a given

ending net worth (x + A) or to face equally likely chances of multiplying the ending net worth by p or dividing it by p. If p is set to 2, for example, the decision maker would be just willing to flip a coin to decide whether ending net worth is doubled or halved.

It is useful to see the effect of alternative levels of A on the evaluation of lotteries on incremental dollars. Setting A to $180,000 means the decision maker is indifferent betweeen

The lottery in this instance has a mean of $70,000 and the RP is $5000. On the other hand if A is set to 0, the following are equally desirable:

This lottery has a mean of 110,909 and a RP of $90,909. Thus, it may be observed that changing the A can have a profound effect on the degree of risk aversion.

7.4.3 Mean-Variance Analysis

A simplified way to treat risk is to base the RP strictly on the variance (standard deviation squared) of an uncertain outcome. Thus the CE is computed using only the EMV, or the mean, and the variance. Such an approach is based on the approximation

$$\text{RP} = \frac{r * \text{variance}}{2}$$

where r measures the degree of risk aversion. This approximation, while often assumed in general, holds exactly only in certain circumstances, the most important of which are as follows:

1 Utility is negative exponential (constant risk aversion)
2 The risky quantity being evaluated is normally distributed

When these conditions apply, the CE is easily computed as

$$\text{CE} = \text{mean} - \frac{r * \text{variance}}{2}$$

where r is the risk-aversion coefficient defined in a previous section.

There is a catch to using the approximation at any other time. The only other condition for which the approximation is exact is a quadratic utility function (of the form $a + bx - cx^2$, with a, b, c positive). Unfortunately, the quadratic utility function has the undesirable property that its risk aversion *increases*. The value of r would not be a constant, but instead increases with the mean of the gamble. In the previous section, it was argued that risk aversion would tend to decrease not increase! Thus, use of mean-variance analysis, except in the conditions above, is either inaccurate or accurate because of an assumption of increasing risk aversion.

7.4.4 Target-Semivariance Utility

A problem with mean-variance analysis is that no distinction is made between upside variation and downside variation. For example, consider the following gambles:

These two gambles have the same mean of $11,000 and variance of $9 million (standard deviation = $3000). Thus they would have identical CEs based on mean-variance analysis. But clearly, the downside loss is more significant with the gamble on the right—it is possible to end up $9000 below the mean, while in the left lottery the worst that can happen is $1000 below the mean. The left gamble, however, has more upside potential. Most people feel differently about upside uncertainty (how *well* one might do) versus downside uncertainty (how *badly* one might do) and would prefer the lottery on the left. Another way to think of this is in terms of the probability distribution—the left gamble is skewed to the right, and the right gamble is skewed to the left. A way to account for this phenomenon is a model that treats downside variations as risky, but not upside variations. A general, yet relatively simple, utility curve is the target-semivariance utility curve,

$$u(x) = \begin{cases} x & \text{when x exceeds a target} \\ x - c * (\text{target} - x)^2 & \text{when x is below the target} \end{cases}$$

A graph of this type of curve showing several values for the constant c is given in Exhibit 7.13. The target is an aspiration level. Above the target the utility is a straight line, exhibiting no risk aversion. Below the target the curve drops off at a rate depending on how the constant c is set. The value of c corresponds very roughly to r/2, where r is the risk-aversion coefficient in the constant-risk-aversion model.

Suppose we used the target-semivariance utility model with c = .0001 to compare the two gambles of the previous paragraph. The expected utility for the gamble on the left would be calculated

$$\text{Expected utility} = .1[20{,}000] + .9[10{,}000] = [11{,}000]$$

while that for the gamble on the right would be

EXHIBIT 7.13 Target semivariance utility for different values of a constant.

$$\text{Expected utility} = .9[12{,}000] + .1[2000 - .0001\,(+8000)^2] = [10{,}360]$$

Thus we see that the gamble on the left is favored using this model of utility. This we would expect due to its upside potential compared to the downside risk (below the target of 10,000) of the right gamble.

7.5 SIMPLIFYING THE USE OF A UTILITY CURVE

There are two problems faced by someone who intends to use any utility curve: (1) assessing the utility function and (2) carrying out the computations of expected utility and/or CEs. Both problems are alleviated somewhat by using one of the several simpler utility models (exponential, logarithmic, target semivariance). Even with these forms it is necessary to make judgments: to set the constants r or A, or the target and c in the target-semivariance model. Often it is very difficult to set these constants precisely. There may be no single decision maker to make the judgments, but instead several people involved in the decision. Whose preferences does one use?

7.5.1 Simplifying Judgments About Risk Preference

One can avoid direct assessment of the constants yet still account for risk in a scientific way. The approach is as follows: First discover over what range of values for the constant of the utility function each alternative is most desirable, then show this to the decision maker(s) to decide in which range the risk aversion falls. In many cases it will turn out that the decision is obvious without making any strong preference statements. Or one could evaluate the implied risk aversion in other decisions made by the decision maker or firm in order to pin down the risk-aversion constant.

Consider an example. Suppose there are two projects, A and B. With project A there is a 30 percent chance that the net contribution will be $15,000, a 40 percent chance that it will be $5000, and a 30 percent chance that it will be $−5000. On the other hand, project B has

five possible outcomes for net contribution: \$10,000, \$4500, \$1500, \$ − 1000, \$ − 2000. The probabilities of these outcomes are, respectively, .2, .2, .1, .2, .3. Exhibit 7.14 is an IFPS model that computes expected utility for the two projects using the logarithmic utility curve with several values of A. The plot in the exhibit was made by IFPS and shows the expected utility profile of the two projects for values of A ranging from 9000 to 6000. Had the expected utility curves not crossed, the decision could be made easily: Take the project with the higher expected utility curve. But the curves do cross. Project A has a higher curve and greater utility if the constant A exceeds \$6740. Project B has greater utility if A is less than \$6740, in other words, if the firm is more risk-averse. To choose between these projects, management must decide whether the effective prior net worth (and implied risk tolerance) is greater or less than \$6740.

This approach of plotting the expected utility curve for various levels of risk aversion can be used with any general utility curve that can be expressed in terms of a single constant. The idea works just as well with CEs. One may convert an expected utility to a CE and plot the CE profile.

7.5.2 Simplifying Computations

The second practical problem with using utility curves is that computations become complex. This problem can be alleviated by using a modeling language like IFPS as illustrated in Exhibit

```
MODEL PROFILE VERSION OF 02/08/82  17:03
2  COLUMNS 1-24
3  A OUTCOMES = 15000,5000,-5000,0,0,PREVIOUS 5
4  B OUTCOMES = 10000,4500,1500,-1000,-2000,PREVIOUS 5
5  A PROBABILITY = .3,.4,.3,0,0,PREVIOUS 5
6  B PROBABILITY = .2,.2,.1,.2,.3,PREVIOUS 5
7  A = 9000 FOR 5, PREVIOUS 5 - 1000
8  A UTILITY = NATLOG(A OUTCOMES + A)
9  B UTILITY = NATLOG(B OUTCOMES + A)
10 A EXP UTILITY = A PROBABILITY*A UTILITY
11 B EXP UTILITY = B PROBABILITY*B UTILITY
12 COLUMN 21 FOR A EXP UTILITY THRU B EXP UTILITY = SUM(C1 THRU C5)
13 COLUMN 22 FOR A EXP UTILITY THRU B EXP UTILITY = SUM(C6 THRU C10)
14 COLUMN 23 FOR A EXP UTILITY THRU B EXP UTILITY = SUM(C11 THRU C15)
15 COLUMN 24 FOR A EXP UTILITY THRU B EXP UTILITY = SUM(C16 THRU C20)
END OF MODEL
```


EXHIBIT 7.14 IFPS model with logarithmic utility curve.

7.14. While that example considers only discrete outcomes, IFPS may be used to incorporate preferences for continuous variables as well. To deal with continuous probability distributions, however, it is more convenient to use Monte Carlo simulation. To do this, express random variables in a Monte Carlo model using the menu of available probability distributions, then add a line expressing utility. An expression for utility might be written directly using one of the standard forms discussed in this note, or a more general curve might be specified using the INTERPOLATION ON approximation feature of IFPS. Exhibit 7.15 is a model for choosing between self-insurance, a deductible insurance policy, and an insurance policy with no deductible. The amount of loss that may be incurred is expressed by the cumulative probability distribution in line 15 (the numbers in parentheses are deciles). The deductible amount is $3000 and deductible insurance costs $4500. Without the deductible, the insurance costs $7300. The constant-risk-aversion exponential utility curve is used in lines 50, 55, and 65, to evaluate the three options.

```
MODEL INSURANCE VERSION OF 02/09/82  11:07
5  COLUMNS 1–5
10 *
15 LOSS = CUMRAND(0,500,1000,2000,3000,4700,7500,9300,11000,'
20          12000,15000)
25 NET WORTH = 15000
30 NODEDUCT COST = 7300
35 DEDUCT COST = IF LOSS .GT.3000 THEN 7500 ELSE 4500 + LOSS
40 *
45 R = .0001, PREVIOUS R + .0004
50 SELF INSURE UTILITY = (1/R) * (1-(NATEXP(-R * (NET WORTH - LOSS))))
55 NO DEDUCTIBLE UTILITY = (1/R) * (1 - (NATEXP(-R * 7700)))
60 DEDUCTIBLE UTILITY = (1/R) * (1 - (NATEXP(-R * (NET WORTH - DEDUCT COST))))
END OF MODEL
```

EXHIBIT 7.15 IFPS model for evaluating insurance deductibles.

The three insurance alternatives may be compared by running a Monte Carlo simulation of the model including the utility lines, then ranking alternatives according to the expected (or mean) utility. (Although the simulation will give a distribution for utility, only the mean utility is relevant for decision purposes.) The simulation may be run for several levels of risk aversion to obtain an expected utility profile, as shown in Exhibit 7.15. The results shown there indicate that, using a constant-risk-aversion utility curve, it is best to self-insure for r less than .00015. Above that level of risk aversion the deductible insurance is better. The no-deductible option never becomes the favored alternative (because of the combination of premiums and loss distribution used in this example).

The approach we have just described would allow one to build adjustments for risk into any IFPS simulation model. All that is needed is to add a line expressing the utility curve, then use it in evaluating alternatives.

7.6 CORPORATE RISK POLICY

A person's attitude toward risk for money depends on whose money it is—the person's own money or the firm's money for which the person is responsible. Because of the level of loss that can be taken by a firm relative to an individual, the manager usually is less risk-averse with the company's money than with personal money. This is not being profligate; if a firm's RPs are instead set as high as those for the individual, then profitable projects that have riskiness the firm could reasonably accept may be lost to other firms or even other individuals. Thus it would be desirable for the manager to consistently adopt a treatment of risk that fits the firm rather than the individual.

In addition, managers in separate parts of a firm ought to be consistent with one another in their treatment of risk. It is inefficient, for example, for one division to readily accept investments that would be avoided by another division. The company would be better served if both divisions took opportunities with riskiness halfway between two such projects. Using a utility function would provide for such consistency; it would also help assure that risk judgments are in the firm's best interests rather than accounting for a manager's private level of risk aversion.

Realistically, few companies are going to go through the formality of using a utility function for every risky decision. Nonetheless, it would be to the company's advantage at least on occasion to test decision behavior in the firm against a utility model. It would identify (1) inconsistencies in the use of probabilistic information, (2) overly conservative risk aversion, and (3) similarity of treatment of risk aversion in various parts of the firm. At the very least it would enhance communication in the firm about corporate policies toward risk.

KEY TERMS

utility
reference lottery
expected monetary value (EMV)
certainty equivalent (CE)
risk premium (RP)
expected utility
substitution principle
utility assessment
tree fold-back
corporate risk policy
risk aversion
risk neutral
constant risk aversion
negative exponential utility
risk-aversion coefficient
decreasing risk aversion
logarithmic utility
mean-variance analysis
target-semivariance utility

EXERCISES

7.1. Using procedures similar to those in the assessment dialogue with Mr. Dyson, assess someone's risk preferences (perhaps your own) for personal net worth. Sketch the utility curve on the accompanying graph.

7.2. A friend confidentially mentions to you that he is constantly risk-averse in personal wealth. Trying to impress him with your recently developed skills you assess his CE for a 50-50 gamble between $0 and $10,000 to be $2500. You learn that he owns an investment that will leave him with personal wealth of either $21,000 or $11,000 in today's dollars with equal probabilities. He can sell out of the investment for $14,000. He has asked you for advice. Should he sell out? Do you wish to buy it (refer to Exercise 7.1)?

7.3. Your friend in Exercise 7.2 suggests that he keep half of his investment and find someone to buy the other half for $7000. Is this a better idea for him than what you suggested?

CASE ASSIGNMENTS

7.1. Horse Sense, page 261.

7.2. Hedging, page 262.

CHAPTER **8**

MULTIPLE-ATTRIBUTE PREFERENCE MODELS

Chapter 6 introduced ways to treat problems having more than one objective. A method of analysis involving a linear and additive value function was described. Chapter 7 treated the use of utility functions for incorporating attitude toward risk into decisions that involve a single attribute. This chapter builds on the concepts in those chapters and extends them in a number of ways.

First, this chapter deals with the assessment and use of a value function in one dimension. Such a function is similar to the utility function for risky situations but applies instead to conditions of certainty. Second, we describe various forms for both value (in the certainty case) and utility (in the uncertainty case) for more than one attribute. We start with cases where independence properties allow the use of additive or multiplicative forms of these functions. Methods of assessing these forms from decision makers are given. Finally, we treat cases where there are preference dependencies among attributes, that is, where the level of a variable or attribute may influence the desirability of another attribute.

8.1 VALUE IN ONE DIMENSION

There are many occasions when the desirability of an attribute is not linear. For example, an attribute, such as the size of your home, may have decreasing marginal value; the value of an additional 100 square feet of home size decreases for larger home size. In this section, we

introduce a method for assessing a value curve for an attribute that will account for this or any other nonlinearity of value.

In Chapter 7, utility curves, which were normally nonlinear, were used to make risky decisions. The utility curve does, of course, capture the strength of preference for various levels of an attribute, but it also incorporates something else, namely attitude toward risk. Here we want to capture just strength of preference for situations where the attribute level is assumed to be known with certainty. Thus lottery-type questions will not be used in the assessment process. Both rankings of alternatives and the degree to which one alternative is better than another will be given by the value score.

8.1.1 Assessing a Value Curve by the Midvalue Splitting Technique

The assessment method that we will use is called the *midvalue splitting technique*. It is analogous to the 50-50 lottery technique for assessing a utility function. The difference between the methods is in the nature of the question being asked; the scaling of the curve and the sequence of questions is the same. Suppose we are assessing someone's value curve for total years of life (YOL), which may be needed for a medical treatment decision. The steps of assessment are as follows:

1 Decide the range over which the value curve is to be assessed. Assume for YOL that this is 10 to 80 years (the relevant outcome range for the decisions for which the value curve may be used). Assign the values 0.0 and 1.0 to these end points, respectively.

2 Find the midvalue point between these end points. This can be accomplished by the following: Consider the two outcomes for YOL, 10 years and 80 years. There is some number of years that has value midway between the value you place on these two outcomes. This midvalue point is such that the increase in value to you from changing YOL from 10 years to the midvalue point is equal to the increase in value to you of changing YOL from the midvalue point to 80 years. What is your midvalue point?

Some prompting may be needed to help the decision maker respond to this question, which is not an easy one. For example, he or she could be asked whether the value of years 10 through 30 is more or less than the value of years 30 through 80. If the answer is "less," then the midvalue point will be greater than 30 years.

Suppose after some thought the decision maker responds with a midvalue point of 38 years. Given the scaling of value between 0.0 and 1.0, this midvalue point has a value of .50. We can now begin to construct the value curve by putting the point labeled 3 on the value curve shown in Exhibit 8.1.

EXHIBIT 8.1 A possible value curve for years of life.

The midvalue point in conditions of assumed certainty is analogous to the CE for 50-50 lotteries in conditions of uncertainty. Both have preference value halfway between the outcomes used to assess them.

3 Find the midvalue point between 10 and 38 and the midvalue point between 38 and 80. The first of these has value equal to .25 and the second has value equal to .75. Add these two points to the value curve [points 4,21 years and 5,52 years on Exhibit 8.1].

4 Continue by finding the midvalue points between the outcomes already assessed and adding these points to the value curve until as many points are obtained as needed for the desired accuracy level of the curve (see points 6 through 9 on Exhibit 8.1).

5 Draw the value curve through the assessed points.

6 Ask the decision maker other questions to assure consistency. For example, ask to verify that the midvalue point between 21 and 52 years is 38. If not, the decision maker should adjust one or more of his or her responses.

In some cases, it is necessary to work with attributes that have decreasing value, such as a pollution level. It may be easiest simply to turn the scale around and assess the curve against a decreasing horizontal scale.

If the value is not steadily increasing or decreasing, for example it has a single peak in it, the midvalue technique can still be used. Simply divide the entire range for the attribute into intervals in which the attribute is always increasing or always decreasing. Then assess the value function separately for each interval.

It might be convenient for an experienced individual to simply draw an appropriate value function. It could even be entered directly into the computer using a digitizer (e.g., a graphics tablet or a mouse).

The value curve may be incorporated into an IFPS model via the INTERPOLATION ON function using a few points from the curve. Or the curve may be entered as an analytical expression (see Chapter 4 for possible shapes). For example, the value function may fit the shape of the functions used in Chapter 6 (e.g., negative exponential or logarithmic) for utility.

8.2 VALUE IN TWO OR MORE DIMENSIONS

There are situations when it is desirable to express a value function or a utility function in more than one dimension. The task is to define value, $v(\mathbf{z})$, or utility, $u(\mathbf{z})$, where $\mathbf{z} = (z_1, z_2, \ldots, z_n)$ is a vector of attribute levels. In what follows, we develop forms for preference in multiple attributes in the context of $v(\mathbf{z})$, but keep in mind that there is an analogous form in each instance for $u(\mathbf{z})$. In general, similar restrictions are required to give certain forms of both $v(\mathbf{z})$ and $u(\mathbf{z})$, although utility is based on comparisons of *lotteries* on attributes and value is not. Where important distinctions exist in the construction of utility, as compared to value, they will be mentioned.

It is easiest to write $v(\mathbf{z})$ [or $u(\mathbf{z})$] in terms of separate one-dimensional value functions $v_i(z_i)$. This can be done only when preferences for the attributes exhibit particular independence properties.

8.2.1 Preference Models with Independence

There are several types of independence, each of which implies a different form of relationship between v, the overall value, and v_i, the single attribute value. The differences among these kinds of independence are subtle and sometimes confusing. To simplify the explanation of the independence concepts, let us consider just two dimensions and start with the simplest preference model that has the most restrictive linearity and independence properties. Then we will remove the restrictions one by one and discuss how the form of the value function changes.

Additive Value The simplest two-attribute preference model would be the linear additive model

$$v(z_1, z_2) = w_1 z_1 + w_2 z_2$$

This is the linear additive rate and weight model used in Chapter 6. For example, suppose that you are considering job alternatives which differ in salary and in the amount of work required of you. You might evaluate the job opportunities with the following value function:

$$\text{Value} = \text{Salary} - 35 * (\text{Hours Worked}) \qquad (\text{units are \$})$$

Here $w_1 = 1$ and $w_2 = -35$. The tradeoff between time and salary is such that a one-hour reduction in Hours Worked is worth a reduction of $35 in Salary.

This model assumes that every incremental change in Hours Worked is valued the same, regardless of whether you are now working 20 hours a week or 70 hours a week. This is called the *linearity* assumption. The model above also assumes two kinds of *independence:*

1 Additive independence Total Value is simply the sum of value for Salary (measured in $) and the value (negative) of Hours Worked.

2 Difference (or utility) independence The value differences for alternatives differing in Hours Worked that have the same Salary level are unaffected by the level of Salary and vice versa. If we were talking about uncertainty in the attributes and therefore utility independence, this assumption would be that decisions made about lotteries on Hours Worked assuming a constant Salary would be unaffected by the level of Salary, and vice versa.

Let us first deal with the linearity assumption and then worry about the two independence assumptions. If you decide that the linearity assumption is invalid, the value function for each attribute could be separately assessed using the midvalue technique of the previous section. For Salary, assume you place marginal value on an additional dollar in the precise form shown in Exhibit 8.2*a*. For Hours Worked, you would rather work less if you can, and you feel more work becomes increasingly less desirable when it exceeds 40 hours per week. Let us assume the value curve shown in Exhibit 8.2*b* has been assessed for you. Note that the horizontal scale is reversed (running from 100 hours worked per week to 0) since value is decreasing in Hours Worked.

Using the curves in Exhibit 8.2, one additive value expression in the two attributes is

$$v(\text{Salary, Hours Worked}) = .6v_S(\text{Salary}) + .4v_H(\text{Hours Worked}) \qquad (8.1)$$

Given the scaling of the individual value curves and the fact that the weights on individual value sum to 1.0, it can be verified that overall value v falls between 0 and 1.0. Thus, if Salary = $100 and Hours Worked = 82, the overall v = 0. If Salary = $2500 and Hours Worked = 0, then v = 1.

EXHIBIT 8.2 Value curves for Salary and Hours Worked.

The general form for the additive value function in n attributes is

$$v(z_1, z_2, \ldots, z_n) = w_1v_1(z_1) + w_2v_2(z_2) + \cdots + w_nv_n(z_n)$$

Additive $u(\mathbf{z})$ would be written similarly in terms of $u_i(z_i)$, $i = 1, 2, \ldots, n$.

While this model no longer relies on the linearity assumption, it still rests on the two independence assumptions. The additive independence assumption that value is simply the weighted sum of one-dimensional values for each attribute may be appropriate for some uses, but let us consider cases in which it might not be acceptable. Consider the marginal value of an incremental dollar of Salary. This is given by the slope of the value curve for Salary in Exhibit 8.1 multiplied by the weight of .6 in Equation (8.1). Similarly, the marginal value of a unit reduction in Hours Worked is the slope of its value curve multiplied by .4. Now suppose that a one-unit change is made in both attributes simultaneously. The additive value form implies that the marginal value is the sum of the marginal values just computed.

Multiplicative Value However, you may feel that you gain extra value from changing both attributes together, i.e., that there is some interaction between the attributes. This interdependence could be accommodated in the model by inserting an interaction term, such as the final term in the following model:

$$\text{Value} = .45v_S(S) + .3v_H(H) + .25v_S(S)v_H(H) \qquad (8.2)$$

where S stands for Salary and H for Hours Worked. Value is scaled arbitrarily, so that even though the weight on Salary in Equation (8.2) has changed to .45 from the .6 value in Equation (8.1), the ratio of the weight on Salary relative to the weight of Hours Worked remains 1.5. This constant ratio signifies that the attributes have not changed in their relative importance. As before, the weights on the terms in the expression sum to 1.0.

The fact that the weight on the interaction term is positive implies that the attributes in this model are complementary, in the same sense that bread and butter or right shoe and left shoe are complementary goods. If the attributes were substitutable (as are butter and margarine), then the weight on the product term would be negative instead of positive.

The form of value in Equation (8.2) is multiplicative, which in general is written

$$v(z_1, z_2) = w_1v_1(z_1) + w_2v_2(z_2) + ww_1w_2v_1(z_1)v_2(z_2) \qquad (8.3)$$

for two attributes. The sign of the interaction constant w determines whether the interaction is complementary [positive w as in Equation (8.2)] or substitutable (negative w).

When there are more than two attributes, the multiplicative form includes interactions among all attributes. Since it is lengthy to write each of these terms, a shorthand expression is better. A little algebra converts Equation (8.3) into the following form:

$$v(z_1, z_2) = \frac{\{[1 + ww_1\, v_1(z_1)][1 + ww_2\, v_2(z_2)] - 1\}}{w}$$

The n-attribute generalization is then

$$v(z_1, z_2, \ldots, z_n) = \frac{\{\prod_{i=1}^{n} [1 + ww_i\, v_i(z_i)] - 1\}}{w}$$

where $\prod_{i=1}^{n}$ signifies that the product of the term that follows should be taken, varying i from 1 to n.

This product form can be used to conveniently express a multiplicative value function in

IFPS, if columns are related to attributes. If the model has a variable W_i expressing the weights column by column and a variable V_i, then v can be given by

$$v = (1 + w * W_i * V_i), \text{PREVIOUS} * (1 + w * W_i * V_i) \text{ FOR } 4, \\ (\text{PREVIOUS} * (1 + w * W_i * V_i) - 1)/w$$

In this example there are columns for six attributes and value for all attributes is given in column 6 of v.

For the multiplicative form to be valid in n dimensions, it must be the case that every *pair* of attributes is *difference- (or utility) independent* of the rest of the attributes. Thus if there were three attributes Salary, Hours Worked, and Difficulty of Work, the value of a pair of attribute levels, say Salary and Hours Worked, would be difference-independent of Difficulty of Work. Salary and Difficulty of Work must also be difference-independent of Hours Worked, and Hours Worked and Difficulty of Work must be difference-independent of Salary.

There are more complex forms of value than the additive and multiplicative forms discussed here for situations where there are three or more attributes. For example, the multilinear form in Keeney and Raiffa (1976) has an interaction term with a separate interaction weight for every possible combination of attributes. We will not discuss its assessment here due to the large number of weights that must be assessed. See Dyer and Sarin (1979) for a more rigorous treatment of independence properties for value functions and Keeney and Raiffa for a corresponding treatment of utility functions.

8.2.2 Assessment of Weights

There are two principal ways to assess the weights of a value function in multiple attributes with nonlinear one-dimensional value. One involves indifference pairs and simultaneous equations; the other requires the comparison of extreme points.

Indifference Pairs If the decision maker considers two different attribute vectors equally desirable, they will have the same level of v. Thus if $v(z^A) = v(z^B)$, then z^A and z^B are an indifference pair of alternatives. The weights are found by writing an equation for each indifference pair and solving the simultaneous equations for the w's. An additional equation is needed. For an additive value function it is

$$w_1 + w_2 + \cdots + w_n = 1$$

and for a multiplicative function it is

$$w = \left[\prod^{n} (1 + ww_i) \right] - 1 \tag{8.4}$$

A search procedure is required to solve for w, but this can be done in IFPS as shown in the next section.

The weights in an additive value function with n attributes can be set by specifying $n - 1$ indifference pairs. For a multiplicative form, n indifference pairs are needed since there is an additional interaction parameter w.

For example, suppose the decision maker provides the following two indifference pairs (S stands for Salary and H for Hours Worked per week):

	S	H		S	H
Alternative 1A:	\$600	60	is indifferent to 1B:	\$500	36
Alternative 2A:	\$113	50	is indifferent to 2B:	\$1000	72

Then, using the multiplicative form, an equation may be written relating 1A to 1B and another relating 2A to 2B. Then adding Equation (8.4) above, the following three equations would result:

$$w_S v_S(600) + w_H v_H(60) + w w_S w_H v_S(600) v_H(60) = w_S v_S(500) + w_H v_H(36) + w w_S w_H v_S(500) v_H(36)$$

$$w_S v_S(113) + w_H v_H(50) + w w_S w_H v_S(113) v_H(50) = w_S v_S(1000) + w_H v_H(72) + w w_S w_H v_S(1000) v_H(72)$$

$$w = (1 + w w_S)(1 + w w_H) - 1$$

The weights w_S, w_H, and w can be found by solving these simultaneous equations, which would give .45, .30, and 1.852, respectively.

Observe that in this case the sum of the weights on each attribute is less than 1 and w, the weight on the interaction parameter, is positive. There is only one other possibility given the way that we have chosen to scale v and the v_i's. If the attribute weights are substitutable, then the sum of the attribute weights will be greater than 1.0 and the interaction parameter w will be negative. The interaction parameter in the multiplicative form will range from -1 to infinity but cannot be 0 ($w = 0$ corresponds to an additive form).

8.2.3 Extreme Points

Another way to assess the weights does not require the solution of simultaneous equations. But it does require responses to more difficult, and perhaps hypothetical, assessment questions.

The decision maker is asked to compare alternatives where one attribute is set at its most desirable level (that level which has a one-dimensional value of 1.0) and all other attributes are set at their least desirable levels (with one-dimensional value 0). For example with the attributes Salary and Hours Worked the alternatives would be

Alternative	S	H
S-Best	\$2500	82
H-Best	100	0

These alternatives, which may not be available in reality, represent corner points of the space defined by the range of the two attributes (see Exhibit 8.3). The other two corner points of that space represent the situations where both attributes are at their worst levels (which is assigned $v = 0$) and where both attributes are at their best levels (which is assigned $v = 1.0$). It can be verified from the expression for v that $v = w_S$ at the corner point S-Best and that $v = w_H$ at the corner point H-Best.

The following procedure is used to obtain the weights:

1 The decision maker ranks the alternatives. This ranking sets the ranking of attribute weights. Suppose the ranking is S-Best is preferred to H-Best.

2 The decision maker specifies the weights of the other attributes relative to the weight of the attribute ranked first. This is done by identifying a point of the axis for the most important attribute that is indifferent to the corner point for each other attribute. Thus to find the relative weight for Hours Worked, the decision maker fills in the blank below to make the following two alternatives equally desirable:

H-Best
An alternative with Hours Worked = 82 hours and Salary = _____

Suppose that the decision maker fills in the blank with Salary = \$675. The alternative is marked on Exhibit 8.3, and H-Best then constitutes an indifference pair. Since the value of H-

EXHIBIT 8.3 Attribute space and corner points.

Best is w_H, thus $w_H = v(\$675, 82) = w_S v_S(\$675)$. Reading from the one-dimensional value curve for S, we have $v_S(\$675) = .67$. Thus $w_H = .67 w_S$. Where there are more attributes, a similar question would be asked to obtain the weight of each other attribute relative to the most important attribute. For the additive form this is all that is needed; for the multiplicative form it is necessary to establish the absolute level of the most important attribute. This may be done with the following procedure:

> It is now necessary to place the value of the alternative S-Best relative to the worst possible case Salary = \$100, Hours Worked = 82 hours, and the best possible case Salary = \$2500, Hours Worked = 0 hours. Think about how far between these two cases S-Best lies. Is it 30 percent of the way between the worst possible and best possible? 50 percent? Specify a percentage.

The percentage given here is the weight on Salary, the most important attribute. Assume 45 percent for the example which implies $w_S = .45$.

3 Find all the weights such that

a For the additive form they sum to one, thus $w_S + w_H = 1.0$, which substituting the relative weight above gives $w_S + .67 w_S = 1.0$ or $w_S = .6$, $w_H = .4$.

b For the multiplicative form the weights satisfy Equation (8.4).

For the example, it would be necessary to solve

$$w = [(1 + .45w)(1 + .3w)] - 1$$

This can be solved directly for w with only two dimensions, but for more than two, an iterative search is required to find w for Equation (8.4). Exhibit 8.4 shows an IFPS model that carries out this search and gives the solution.

With either assessment procedure it is critical that the assessed model represent the decision maker's true preferences. Test it with a few other comparisons. Give the decision maker an alternative, and some other alternative that has all but one of the attributes specified. Ask the decision maker to set that other attribute so that the alternative is equally desirable to the first alternative. Then compute the value for the two alternatives and check to see that they are the same or close. If they are not close, probe for the reason. This will help the decision maker to learn more about his or her values for the situation and, perhaps, he or she will go back and make adjustments either to the one-dimensional value function or to the weights.

Remember that the weights only apply to that specific scaling of the one-dimensional value functions. If the scaling is changed, for example if the range of Salary were changed to be from \$500 to \$1000 and v_S is rescaled from 0 to 1.00 over this new range, then the weights should change also. Salary may no longer be the most important attribute (at least as suggested by the weights) for this new scaling, since a unit of v_S is associated with a much less significant change in dollars.

EXHIBIT 8.4 SOLVING FOR THE INTERACTION PARAMETER

```
MODEL DBLU   VERSION OF   04/22/83   20:20
1 COLUMNS 1
2 ws = .45
3 wh = .3
4 w = 1.0
5 zero = (1 + w * ws) * (1 + w * wh) - 1 - w
END OF MODEL
? SOLVE

ENTER SOLVE OPTIONS
? GOAL SEEK

GOAL SEEKING CASE 1
ENTER NAME OF VARIABLE TO BE ADJUSTED TO ACHIEVE PERFORMANCE
? w (1)

ENTER COMPUTATIONAL STATEMENT FOR PERFORMANCE
? zero (1) = 0


***** GOAL SEEKING CASE 1 *****

                              1

w                         1.852

ENTER SOLVE OPTIONS
? all


                              1

ws                        .4500
wh                        .3000
w                         1.852
zero                          0
```

It is possible to mix utility and value in the additive or multiplicative form. Some attributes may be treated as certain and have one-dimensional value functions and other attributes treated as uncertain with one-dimensional utility functions. There are two differences between a value function and a utility function: (1) The utility function is assessed using lotteries and the value function is assessed using certainty and (2) value is assigned directly to outcomes, but when using a utility function in uncertainty, expected utility is the criterion. The process of assessing a utility function in multiple attributes is detailed in the Whirlpool Research and Engineering Division (B) case.

The weights to use in combining one-dimensional value curves with one-dimensional utility curves would be assessed as discussed above. In using the function that results (let us call it a *preference function*), probability distributions would be used for the uncertain attributes and a single level for the certain attributes. In IFPS, the preference function would be written as a line of the model and a Monte Carlo simulation would be run, but, of course, only the uncertain variables would be treated as random variables in the simulation. The mean of the preference function resulting from the simulation would be used to compare alternatives.

The multiplicative form of multiattribute value still employs a kind of independence, even though it does not require additive independence. It still assumes that value curves in each of

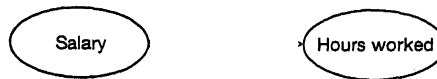
the attributes can be drawn without dependence on the level of the other attribute. This we have called *difference independence*. Without difference independence any value curve drawn for an attribute only applies when the other attribute is at a stated level. In the next section we discuss some of the simpler cases of dependent value.

8.3 PREFERENCE MODELS WITH DEPENDENT ONE-DIMENSIONAL VALUES

Suppose now that you have thought more deeply about compensation and effort and have found that you cannot assume that Salary and Hours Worked are difference-independent. Rather, a higher Salary gives you a greater tolerance for working more hours. It would be clear, then, that the following influence diagram applies, showing a preference influence from Salary to Hours Worked:

This effect may be modeled in many ways, but a model that may make sense here is one in which Salary level determines a target for how many hours you wish to work. When Hours Worked exceeds this target, then the value curve falls off rapidly. One way to model this relationship is with a target-semivariance value model (introduced in Chapter 7) such as the following one.

$$\text{v(HOURS WORKED|SALARY)} = \begin{cases} .01 * (100 - H) & \text{for } (100 - H) \geq (100 - T) \\ .01 * [(100 - H) - .01 * (H - T)^2] & \text{for } (100 - H) < (100 - T) \end{cases}$$

$$T = \text{TARGET} = \text{SALARY}/50$$

The vertical bar in the term v(Hours Worked|Salary) is read as "given the level of," to indicate that the preference value given to Hours Worked is conditional upon Salary. The working hours is expressed as $100 - H$, thereby reversing the scale on this attribute to express the decreasing value of each incremental hour worked. In this model, Salary divided by 50 determines the target number of work hours; when Hours Worked exceeds this target (i.e., $100 - H$ is less than $100 - T$), value falls off. It turns out that if Salary is set to \$2000, then the value curve for Hours Worked is that shown in Exhibit 8.2*b*.

There are many other ways in which an attribute may condition one's preference for another attribute, whether the attributes are treated as certain or uncertain or a combination of certain and uncertain. Suppose, for example, that the decision maker is constantly risk-averse in an attribute (such as cash flow in year n), but that the degree of risk aversion is influenced by another attribute (e.g., the time horizon of the investment which may be treated as certain). This is expressed in the following two IFPS lines:

```
UTILITY = (1/R)(1 - NATEXP(-R * CASH FLOW))
      R = .0003 + .00005 * N
```

In this case the influencing attribute affects the shape of the utility curve in the other attribute. This type of dependence has been labeled *parametric dependence*, since the influence on the utility curve for cash flow is strictly through one or more parameters. In this case the risk-aversion coefficient R is the parameter.

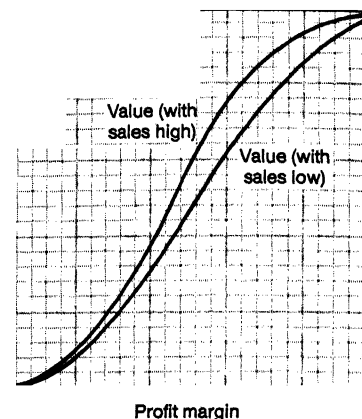



EXHIBIT 8.5 Value curves for profit margin for two sales levels.

Of course, it is possible to have conditional preference for an attribute where the expression for value or utility does not fit one of the convenient analytical forms (like target semivariance or constant risk aversion). In these cases one cannot express the preference dependence through a parameter like target or risk aversion. But if the value or utility curve can be drawn for two or more levels of the influencing attribute, then it can be obtained for other levels through *value interpolation*. For example, Exhibit 8.5 shows two value curves for Profit Margin that apply when Sales is high ($10 million) or low ($5 million). If we want a conditional value curve in Profit Margin for other levels of Sales, we could write it as follows:

VALUE = FRACTION * VALUE WITH HIGH SALES
+ (1 − FRACTION) * VALUE WITH LOW SALES
FRACTION = (SALES − 5)/5

The first equation specifies Value as a weighted sum of the two curves in Exhibit 8.5, with Fraction being the percentage weight placed on the conditional value curve with high sales. The second equation defines Fraction as 1.00 when Sales equals 10, as 0.0 when Sales equals 5, and, in general, as the fraction of the distance that Sales falls between $5 million and $10 million. Thus, Value is a linear interpolation (on Sales level) between the two curves of Exhibit 8.5.

EXHIBIT 8.6 An example of several variables influencing preference for IRR.

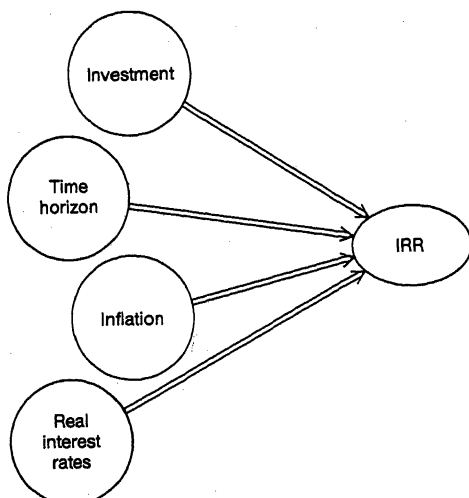
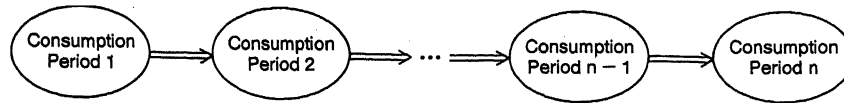



EXHIBIT 8.7 Preference influences for consumption.

The examples we have given of conditional value and utility curves all assume that one variable influences the preference for an attribute. Another more complex model would be one with preference influences running in both directions between two attributes. For example, suppose in the previous example we added that Hours Worked in turn affects feelings for Salary. This is a subtlety which is beyond our purposes; we leave it to more advanced work.

We can, however, model situations where more than one variable or attribute influences preference for an attribute. For example, suppose that ultimately the internal rate of return (IRR) is the attribute of importance, but that the desirability of IRR is influenced by a number of other variables, as indicated in the influence diagram of Exhibit 8.6.

A value function for IRR for this example, v(IRR|INV, TH, I, RIR) might use the influencing variables to set the target for IRR in a target-semivariance model. A variety of influences on IRR value can, of course, be imagined here.

Multiple-preference influences in which there is a chain of influence can be handled with relative simplicity. Exhibit 8.7 illustrates this form of conditional preference for consumption attributes that might be used in personal investment and consumption decisions.

Similar chain models of preference influence may be developed for general attributes not associated with time. The possibilities for structuring the conditional preferences are enormous. In the next section an illustrative example with five attributes shows some of the ways preference modeling can aid decision making.

8.4 ILLUSTRATIVE EXAMPLE: THE ACQUISITION DECISION

George Jepperson, owner and chairman of the board of Jepperson Enterprises, intends to use excess cash to purchase a smaller company. He has several objectives for the acquisition, chief among them: (1) to achieve maximum immediate profitability from the acquisition; (2) to obtain maximum long-run efficiency, given the existing corporate strategy and competitive advantages of Jepperson Enterprises; and (3) to acquire the most valuable assets.

It would truly be good fortune if an acquisition that was expected to achieve the first objective would also be expected to best achieve the other two objectives. Such a dominating alternative will not usually be available, however. Instead, it becomes necessary to make tradeoffs among attributes associated with each objective. Careful thought by Mr. Jepperson and his staff has identified the following attributes as the best measures of the degree of achievement of the objectives: (1) return on equity (ROE); (2) three separate efficiency measures subjectively scored between 0 and 100, complementary management (CM), operating efficiencies (OE), and marketing efficiencies (ME); and (3) book value divided by price (BV/P). The next step is to model the preferences for these attributes.

Deliberations on the part of Mr. Jepperson and his staff reveal several other factors that may affect the attributes singled out as most important. One is whether new technological innovations will be found in the proprietary holdings of the company after it is acquired. Another is whether the management of the company can be persuaded to stay. Another is the degree to which the channels of distribution of the two firms can be merged. Finally, a major decision to be made is whether to alter the new company's corporate strategy in fitting the firm into Jepperson enterprises. Mr. Jepperson has worked these factors into an influence diagram (in Exhibit 8.8), showing their effects on the attributes already mentioned. (Recall that squares represent decisions, circles are intermediate variables, and ovals are attributes.)

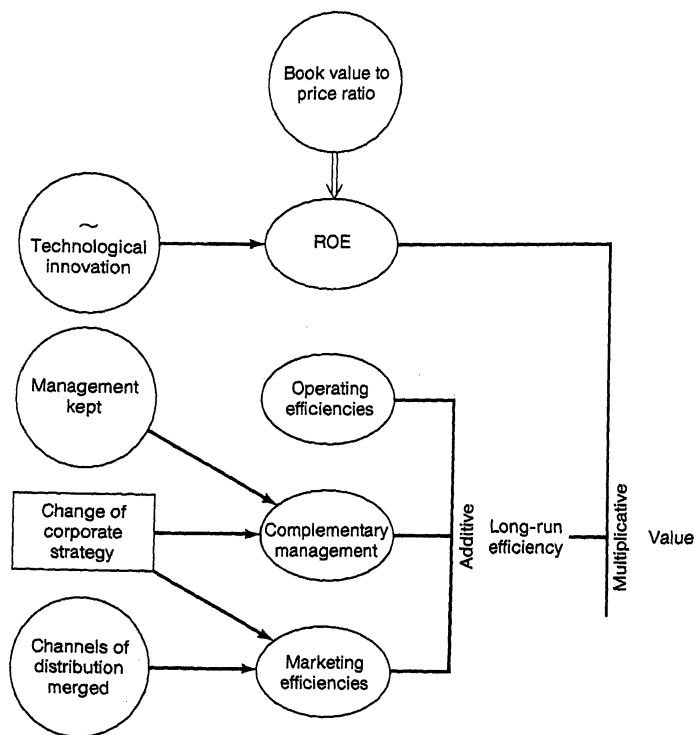



EXHIBIT 8.8 Influence diagram for an acquisition evaluation model.

The diagram shows something more than other influence diagrams we have seen so far. It includes the way in which the attributes combine to form value. There are two layers in the hierarchy of aggregating value for the attributes. The three attributes combine additively into a value number that has been given its own attribute label, Long-Run Efficiency. Then ROE and Long-Run Efficiency combine multiplicatively to give overall Value. We see that the attribute Book Value to Price Ratio (BV/P) influences the value curve for ROE, but otherwise BV/P does not affect overall valuation and therefore is modeled as an intermediate variable.

Note that only Technological Innovation is treated as a random variable (indicated by ~ in the influence diagram). Thus, only the attribute ROE is random. It is appropriate to use a utility curve for evaluating that attribute. The use of utility for this attribute means simply that lotteries are used to assess this preference curve, and in solving the model Monte Carlo simulation is used and the mean utility is the basis for comparing alternatives.

Now that relationships among variables and attributes have been structured, the next step is to model the preferences in more detail—that is, to specify the value and utility curves for each attribute and the weights needed to combine them into overall value.

Exhibit 8.9 is a completely specified preference model written in the IFPS language for evaluating potential acquisitions on the basis of the structure of Exhibit 8.8. The utility curve for ROE is constantly risk-averse, and the risk-aversion coefficient is linearly and negatively related to BV/P. The three subattributes of Long-Range Efficiency are assumed to be scored by direct preference assessment, so that no value curve is needed. Line 40 aggregates these three attributes into Long-Range Efficiency using the weights .30, .25, and .45. Then the last line multiplicatively combines the utility for ROE with the value for Long-Range Efficiency. The weights on the attributes are .41 and .24, respectively, and the interaction parameter is 3.55. The positive interaction parameter indicates that the attributes are complementary.

In order to use the model to evaluate acquisition candidates, each of the attributes must be

EXHIBIT 8.9 IFPS MODEL OF MULTIATTRIBUTE PREFERENCES FOR ACQUISITION EVALUATION

```
10 ROE UTILITY = (1 - NATEXP(-1 * R * ROE))/(1 - NATEXP(-1 * R * .50)
20 R = .0001 - .00004 * BOOK VALUE TO PRICE RATIO
30 EFFICIENCY VALUE = (.30 * OPERATING EFFICIENCY + .25 * COMPLEMNTRY MANGMNT'
35                      + .45 * MARKETING EFFICIENCY
40 PREFERENCE = .41 * ROE UTILITY + .24 * EFFICIENCY VALUE'
45                 + 3.55 * .41 * ROE UTILITY * .24 * EFFICIENCY VALUE
```

estimated for each candidate. These estimates could be entered into a model with a line for each attribute (and also lines for the other variables in the influence diagram) and a column for each alternative. Then when the model is run, the last line, Preference, is the bottom-line criterion. Of course, various sensitivity and what-if analyses could be performed to observe the effect of altered preference assumptions on the evaluation of alternatives.

8.5 DISCUSSION

Too often decision modelers assume that their work is done when they have linked decision variables to outcome variables. But the hardest job may still remain, namely, to choose the preferred alternative or pick the best strategy. The modeler may structure the preferences of the decision maker directly into the model itself. By including these preferences, the model will allow decision makers to balance judgments about their own or the firm's best interests with judgments about the way the world works. If the modeler does not structure such preferences into the model, the decision maker may allow unacknowledged factors to play too great a role in the decision making.

The fact that many preference judgments are highly subjective is no problem. This chapter described ways to incorporate a wide range of subjective preferences. Thus the model will reflect more accurately the managerial perspective, especially in the more strategic decisions where lack of structure forces subjective concerns to play a larger role in decision making.

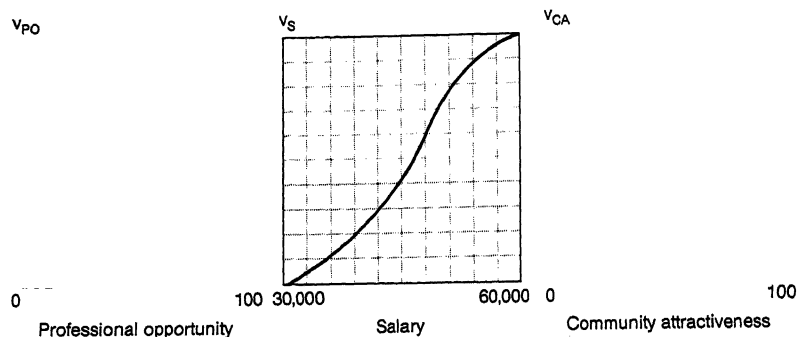
KEY TERMS

value function
strength of preference
midvalue splitting technique
additive independence
difference independence
(analogous to) utility independence
additive value
multiplicative value
interaction weight
complementary or substitutable
assessment of weights
indifference pairs
extreme points
dependent (or conditional) value
parametric dependence
value interpolation

EXERCISES

8.1 Assess your value curve for square feet of interior space in your personal residence (for your family). First establish a reasonable range over which square feet for your residence might vary. Then use the midvalue splitting technique to assess value for the various possible levels of square feet within this interval.

8.2 Using the value curve below for the attributes Professional Opportunity, Salary, and Community Attractiveness, write an expression (in one or more lines) for the total value of a job offer that has been scored on these three attributes. You may use conventions of IFPS or other software. It has been established that the attributes combine multiplicatively with the following weights:

Professional Opportunity:	.50
Salary:	.40
Community Attractiveness:	.30
Interaction	−.4516

Use your expression to evaluate the following two job alternatives:

	Professional Opportunity	Salary	Community Attractiveness
A	60	$53,000	70
B	80	$40,000	90

8.3 In order to evaluate some alternatives in the design of a transportation system, it is necessary to develop a value model for the attributes Convenience and Cost. It has been established that these two attributes are difference-independent. In addition, the overall values of several alternatives having different levels of Convenience and Cost have been assessed. These values are indicated on the plot below, where the dimensions of the plot are the levels of Convenience and Cost. Using this information, answer these questions.

a What are the values of the weights placed on the two attributes?

b Is the value model complementary or substitutable (that is, is the sign of the interaction term positive or negative)?

c Write an expression for value in terms of separate one-dimensional value for the two attributes, that is, in terms of $v_{\text{Convenience}}$ and v_{Cost}. What is the value of the interaction term?

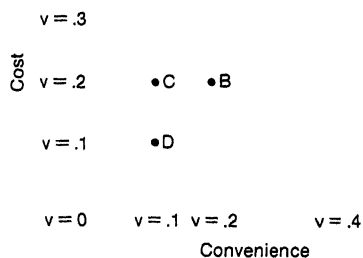
d What are values of the points A, B, C, and D?

v = .3

Cost v = .2 •C •B

v = .1 •D

v = 0 v = .1 v = .2 v = .4

Convenience

8.4 In a study of multiobjective management of the small firm in Denmark, Johnsen (1978) sought to define a process of development by small-firm managers. Managerial objectives were (1) the desire to *control* operations under stable conditions, (2) the desire to *adapt* the firm's structure to changes in the market, and (3) the desire to *grow*. Putting yourself in the shoes of the small-firm manager, define your own value function over three attributes associated with these objectives. Use your own subjective preferences to specify this function completely.

CASE ASSIGNMENTS

8.1 Data Electronics Laboratories, page 161.

a Draw influence diagrams to show the relationships among NPV, SOM, and growth implicit in the models suggested by the two analysts. Are there other preference models you might suggest, which may include, perhaps other attributes as well?

b Use one of the preference models in the case to evaluate the three strategic alternatives faced by DEL's corporate planning committee. Which of the three is best for the company? Carry out the type of sensitivity analysis of the preference model that you think is appropriate. How is the decision affected? Evaluate also what decision would be suggested by preference models of your own design.

c What preference model would be appropriate for the continuing planning process at DEL? In what ways it is useful to seek to quantify values in the context of strategic planning?

8.2 Universal Products, page 170.

(Note: This is a longer preparation that has served as the basis for a term project for students.)

a Create an IFPS model (or more than one) and use it to treat Mr. Gomez's problems as described on page 179 of the case. Use IFPS/Optimum or other optimization program to help in selecting projects.

b What options from the list of recommendations would you suggest that Universal Products take? If you had taken the perspective of either Mr. Knowles or Ms. Baker, how would that have affected your analysis?

c What decision process would you suggest that Universal use on a continuing basis?

8.3 Whirlpool Research and Engineering Division (B) & (C), pages 211 and 229.

a Outline what Keefer was doing in each of the five meetings described in the case. How did he use the information he obtained?

b What does Keefer's analysis in Whirlpool case B tell you? How do his suggestions compare with the current plan?

c Use Whirlpool case C to size up the situation after the passage of some time. Keefer's process was used only once, despite the interest at Whirlpool in having a decision aid. What kind of decision aid would you suggest for Whirlpool?

TREATING SPECIAL PROBLEMS

CHAPTER 9

GROUP DECISION

Many important decisions are based on the preferences of groups of people. Elections and committee decisions are but two of many everyday situations where the subjective desires of individuals are aggregated. Most of us are aware of only a few means of resolving conflict in these situations, majority voting being the principal method. But there are a number of other ways to help a group choose, many of which are preferable in certain situations to simple majority rule. Particularly when the consequences of alternatives are complicated and are projected by computer models, the methods of voting that are most familiar are also most deficient.

In this chapter, a menu of methods for making group decisions is given, starting with the simpler forms and going on to more sophisticated forms that are best implemented with computer support. It will be seen how some straightforward additions to decision models can incorporate preferences of n individuals and thereby accommodate the diverse interests of groups.

There are very many different kinds of group decisions. Volumes could be written about the general problem, and they have been. So it will be necessary to focus our discussion on specific types of problems. Even at that, we will only scratch the surface of what are some difficult theoretical problems. Our purposes are to give a feel for the kinds of problems that arise in group decision, learn the general approaches available, and give guidelines on what to do in practice.

9.1 INTRODUCTORY CONCEPTS

9.1.1 Pareto Optimality

A requirement that most people would want to put on the choice taken by a group is that it not be dominated by some other alternative. An alternative is dominated if there exists some other alternative that is at least as desirable to every individual and more desirable for at least one individual. The alternatives that are not dominated are referred to as the *Pareto optimal set*.

The Pareto optimal set is the northeast frontier of the available alternatives when plotted according to their desirability to any pair of individuals. Exhibit 9.1 shows the available alternatives and the Pareto set for a problem involving just two individuals. Chapter 5 discusses how the nondominated set may be found and graphed for more than two individuals using decision support systems.

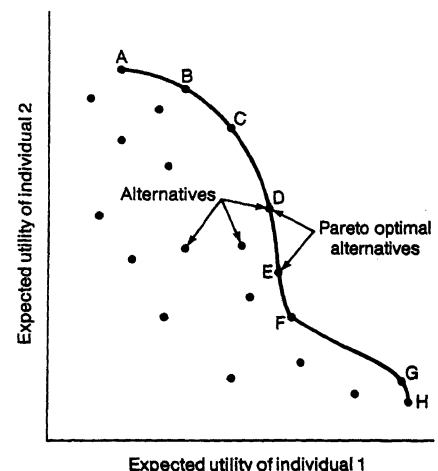
While there seems to be little reason to object to the Pareto rule as a way of eliminating alternatives, it has shortcomings as a general procedure for group decision making. One defect is aptly stated by Sen [1970]:

> An economy can be optimal in this sense [Pareto optimality] even when some people are rolling in luxury and others are near starvation as long as the starvers cannot be made better off without cutting into the pleasure of the rich.

Thus the Pareto rule does not necessarily result in egalitarian alternatives. For example, note the inequity inherent in alternatives A and H in Exhibit 9.1.

Another shortcoming of the Pareto rule is that it does not provide a "complete" ranking of alternatives; there may be many Pareto optimal alternatives. Every individual is benefited by moving from a non-Pareto to a Pareto optimal alternative. Unfortunately, the difficult group choice problem begins only when the Pareto set has been identified.

A reasonable approach is the following. First identify the Pareto optimal alternatives. If there is only one in the Pareto set, then go no further. But if there are several Pareto alternatives, some other method will be needed to select one. The Pareto rule may then be used as one test of the quality of that choice. Other tests will be applied as well, to test such things as the equity of the choice.

EXHIBIT 9.1 Pareto optimal boundary of alternatives for two individuals.

9.1.2 Voting

Suppose we ask ourselves how a group of individuals, each with their own ranking of alternatives, might jointly come up with a group ranking of the alternatives. The first idea to come to the minds of most of us would be to vote. To compare two alternatives, we could use majority rule. Several alternatives could be compared by plurality, that is, a simple count.

How well will this work? Take the case of three individuals who have the following preferences among three alternatives:

Individual 1: $A > B > C$
Individual 2: $B > C > A$
Individual 3: $C > A > B$

where $A > B$ is read "A is preferred to B." A majority vote taken for any pair of the alternatives will result in two-thirds majority for the following group rankings:

$$A > B, \; B > C, \; C > A$$

The dilemma here is that even though the individuals were all transitive in their preferences (that is, $X > Y$ and $Y > Z$ implies that $X > Z$), the group is not transitive.

One way that voting is often conducted is to have a runoff election. Pick a pair of candidates and vote on them. Then take the winner and pit it against the remaining alternative. It is not difficult to see that one could manipulate the election in the case of the preferences above simply by holding out the alternative you want to win until the second round of voting.

The voting rules we use so often are inadequate, at least in some instances. A correctable problem with voting is that it is not responsive to all of the preference information that is often available. Suppose, for example, that two individuals rank five alternatives as follows:

Frank: $A > C > D > E > B$
Earnest: $B > A > C > D > E$

Now consider what would happen in a majority vote between A and B. It would be a tie between A and B. Yet one might argue that it would make sense for Frank and Earnest jointly to prefer A. The reasoning would be that for Frank A is very much preferred to B (note that C, D, and E have intermediate desirability), while for Earnest A is right behind B.

9.1.3 The Borda Rule

The above example suggests that perhaps *strength of preference* may be used by groups to resolve conflicts. Perhaps the simplest way to do this is to utilize the place ranking held by alternatives. The Borda rule is one way to do this. If each individual ranks m alternatives, then first place gets m points, second place $m - 1$, and so on down to last place, which gets 1 point. Thus the rankings above would produce Borda scores of $5 + 4 = 9$ points for A and $1 + 5 = 6$ points for B.

The Borda rule is used to combine rankings of college football and basketball teams made by coaches or sportswriters into the familiar published rankings in the sports pages of the newspaper. For many cases the Borda rule makes more sense than a simple counting of votes and majority rule, yet it also does not use all of the strength of preference information potentially available. For example, suppose it were revealed Frank and Earnest had scaled the alternatives against each other with the following results:

Knowing this, Frank and Earnest may wish to jointly choose B. The interval scale reveals that combined utility is much higher for B than A.

9.1.4 Impossibility Results

One potential problem with the Borda rule is that it violates so-called independence of irrelevant alternatives. That is, it is not in accord with the desirable property that if an alternative is eliminated from consideration for whatever reason, the new group ordering for the remaining alternatives should be identical to the original group ordering for those same alternatives. To observe this, consider an example of ranking three basketball teams: Virginia (V), North Carolina (NC), and Wichita State (WS). The preferences of three sportscasters will be used; their rankings are as follows:

1 WS > V > NC
2 WS > V > NC
3 NC > WS > V

The Borda rule would give the following scores: V, 5; NC, 5; WS, 8. Now suppose Wichita State is placed on probation by the NCAA and thus is ineligible for ranking. The new Borda scores with WS out of the running would be V, 5; NC, 4. Thus the situation between Virginia and North Carolina changes from a tie to Virginia receiving one point more even though no probationary judgment about them is made by the NCAA. There are many who would consider this to be undesirable behavior in a decision rule.

Arrow (1963) showed that there is no way to combine individual rankings of alternatives to give a group ranking of alternatives which satisfies five reasonable axioms of choice. More formally, if R_i is individual i's rank ordering of alternatives and R_g the group ranking of alternatives, there is no decision rule f defined by

$$R_g = f(R_1, R_2, \ldots, R_n)$$

that satisfies Arrow's axioms of choice. We will not review the axioms [see Keeney (1976)], but merely indicate they deal with principles of nondictatorship, positive association of group and individual orderings, individual's sovereignty, independence of irrelevant alternatives, and complete domain of the decision rule. Transitivity of individual and group choices is implicit in the work. Following his work there have been many other "impossibility" results showing the difficulties of aggregating individual choice into group choice.

9.1.5 Cardinal Utility

Arrow treated the group decision problem in great generality, requiring only that rankings of alternatives (called *ordinal comparisons*) be known. Of course, his result holds even if alternatives are given precise numbers that represent scaled (cardinal) comparisons of alternatives, provided that only ranking information is used by the function f to get a group ranking.

By now the reader has probably observed that a big part of what choice a group will make is determined by what preference information is used in the analysis. If only votes between two alternatives are considered, the result may be different than if strength of preference information is used, such as in the Borda rule.

If we make use of all of the information in cardinal utilities, it is possible to make some headway with aggregating individual preferences, despite the result of Arrow. It will be necessary, however, to make interpersonal comparisons of preference. This means literally to weigh one individual's utility against another. This is a difficulty that Arrow and others sought to avoid.

Of course, if it were possible to develop good group decision rules without trying to compare

utility values across individuals, that would be better. But, as Arrow showed, it does not appear to be possible to do that, at least under his set of reasonable, relatively innocuous conditions.

A problem laid open by using cardinal utility and making interpersonal comparisons of utility is manipulation by group members. The true preferences of people are known only to themselves. Any process of eliciting these preferences and scaling them against each other is fraught with opportunities for individuals to misrepresent their preferences for personal gain. We must be careful, therefore, that the process for the interpersonal comparison of scaled preferences is not vulnerable to manipulation. Other considerations are ease of use and decisiveness of the decision rule. It is clear, again, that no method will be perfect on all of these dimensions.

9.2 THE NASH BARGAINING SOLUTION

One way to look at a group decision problem is as a bargaining problem. Nash (1950) formulated the two-person bargaining problem and derived a solution. His result was that the players should maximize the product of their separate utilities of gain from the bargain.

Luce and Raiffa (1957) informally extended the approach to the n-person case. Then Harsanyi (1963) wrote in simplified form a set of postulates that give the Nash solution. It is beyond our purposes to review these in detail. Briefly stated, the postulates relate to principles of individual expected utility maximization, symmetry, and Pareto optimality of the solution.

The Nash solution computes the gains from a bargain relative to a base value that would apply if no agreement were struck. Call the payoff to individual i, z_i, and the base value, t_i, which is so labeled because it may depend on the *t*hreat that can be made by each party in the event of disagreement. The level of t_i reflects the status quo for participant i. The individual with the least to lose if the bargain falls through poses the greatest threat to striking the bargain and has the highest level of t. The Nash group decision rule is to

$$\text{Maximize } u_1(z_1 - t_1)\, u_2(z_2 - t_2) \ldots u_n(z_n - t_n) \tag{9.1}$$

where u_i (.) is the utility function for the ith participant, defined so that $u_i(0) = 0$. In other words, the zero of i's utility applies when $z_i = t_i$.

As an illustrative example of a Nash solution, consider a simple joint business opportunity. The question is how to split up the investment and payoffs among five people who had the idea for the business over lunch one day. The project would not work unless all were involved, although the shares in the project could be unequal. They have already decided that the payoff to each person would be proportional to the share of the $10,000 investment needed for the business. The business would return $12,000 at the end of six weeks, making the total profit $2000. But the idea generated during that lunch also had value to each person, if they could not agree on a joint venture. These status quo, or threat values, were, respectively $100, $200, $300, $400, $500 for the five people. Assume that the five people are each linear in their utility for money. How should they divide up the business?

Exhibit 9.2 shows an IFPS model that captures the essence of the above problem. The five

EXHIBIT 9.2 IFPS MODEL OF EXAMPLE BARGAINING PROBLEM

```
MODEL NASH VERSION OF 03/04/83   17:21
1 COLUMNS 1-5
2 INVESTMENT = 1000
3 PROFIT = INVESTMENT * .2
4 THREAT = 100,200,300,400,500
5 G = PROFIT - THREAT
6 UTILITY = G * .05
7 TOTAL UTILITY = UTILITY, PREVIOUS * UTILITY
8 TOTAL INVESTMENT = INVESTMENT, PREVIOUS + INVESTMENT
```

columns relate to the five individuals. The Nash utility, which is the product of each individual utility, is column five of the Total Utility variable.

Exhibit 9.3 shows the Nash solution along with an IFPS/Optimum directive set that was used with the IFPS model to find the Nash solution. The Nash solution shows that the amount of participation in the venture goes from low to high as the threat level goes from low to high. Thus the person who has least to lose if no bargain is struck gains the most from the Nash solution.

It is easy to use the model and IFPS/Optimum to explore how the solution may change for different threat values and different utility functions for each individual. Check your intuition. If the threat levels were changed, would the Nash solution change? The answer is yes. If the threat levels were all set equal to zero what would the Nash solution be? The answer is that each participant would share equally in the venture. Now suppose we change the definition of the units of utility for some of the individuals. Suppose we multiplied the utility for one of the individuals by 20. Would his share change? No. Intuitively this makes sense, since multiplying any individual's utility by 20 changes the overall Nash utility in the same way—by multiplying it by 20. Multiplying overall utility by a constant will make no change in the bargaining solution.

We have learned some things about the kind of interpersonal scaling of utility that is assumed by the Nash solution. The following points have been proved in general for the Nash solution:

1 The definition of zero utility does affect the solution. In other words, the setting of an individual's t value will affect the solution.

2 The solution is invariant to a positive linear transformation in one or more of the utility functions.

These properties of the Nash solution make the interpersonal comparison of utility simple. No one has to weigh a unit of one person's utility against that of another, because the definition

EXHIBIT 9.3 NASH SOLUTION AND IFPS OPTIMUM DIRECTIVES USED TO GET IT

```
2  OBJECTIVE
3          MAXIMIZE TOTAL UTILITY (5)
4  DECISIONS
5          INVESTMENT (1) .GE. 0
6          INVESTMENT (2) .GE. 0
7          INVESTMENT (3) .GE. 0
8          INVESTMENT (4) .GE. 0
9          INVESTMENT (5) .GE. 0
10 CONSTRAINTS
11         TOTAL INVESTMENT (5) .LE. 10000
12         G (1) .GE. 0
13         G (2) .GE. 0
14         G (3) .GE. 0
15         G (4) .GE. 0
16         G (5) .GE. 0
```

INVESTMENT	1000	1500	2000	2500	3000
PROFIT	200.0	300.0	400.0	500.0	600.0
THREAT	100	200	300	400	500
G	100.0	100.0	100.0	100.0	100.0
UTILITY	5	5	5	5	5
TOTAL UTILITY	5	25.00	125.0	625.0	3125
TOTAL INVESTMENT	1000	2500	4500	7000	10000

EXHIBIT 9.4 NASH SOLUTION AFTER A CHANGE IN THE SHAPE OF PERSONAL UTILITY CURVES

	1	2	3	4	5
INVESTMENT	1208	1708	1694	2536	2854
PROFIT	241.7	341.7	338.7	507.1	570.8
THREAT	100	200	300	400	500
G	141.7	141.7	38.74	107.1	70.83
UTILITY	7.083	7.083	3.657	82.93	8.416
TOTAL UTILITY	7.083	50.17	183.5	15214	128046
TOTAL INVESTMENT	1208	2917	4610	7146	10000

New Personal Utilities:
6 UTILITY = G * .05 FOR 2,NATLOG(G), 200 * (1-NATEXP(-.005 * G)),XPOWERY (G,.5)

of personal utility units does not matter. The definition of personal zero utility does matter, however. But that is set in a convenient way as the status quo point or threat point.

What happens to the Nash solution when the shape of the personal utility function changes? If your intuition tells you it will affect the result, you are right. Exhibit 9.4 shows the Nash solution for the example problem where the utility functions for individuals 3 through 5 have been changed. Individual 3 now has utility that takes the form a natural logarithm, individual 4 has negative exponential utility, and individual 5 has utility which takes a square root form. As can be seen in Exhibit 9.4, the Nash solution is quite different. The first two individuals, for example, have higher shares now after the change in the shapes of personal utility.

The Nash approach is based on a competitive notion of group decision making and an equilibrium solution to the bargaining problem. The effects of threats of each participant are taken into consideration. Each individual is seeking his or her own good and the group is valuable to him or her only in that it may further this self-interest. This is only one way to look at the problem; other approaches may ascribe to the group a more congenial and cooperative nature.

9.3 ADDITIVE UTILITY

Suppose we look at the group as working together to achieve the collective good. Suppose the group has structured its problem as a decision diagram with choice nodes and uncertain events. Further, assume the members agree on all aspects of the problem, including probabilities but not utilities. How would you advise them to act? Would you suggest that they behave collectively according to expected utility maximization as they would for their individual decisions? If so, then if you agree to only one more assumption, your advice to them would be consistent with maximizing a group utility function which is a weighted sum of individual utility,

$$U(z) = a_1u_1(z_1) + a_2u_2(z_2) + \cdots + a_nu_n(z_n) \qquad (9.2)$$

The uppercase U reflects group utility, $z = (z_1, z_2, \ldots, z_n)$ is the vector or ordered sequence of payoffs to the group members, and a_i is the weight given to individual i's utility.

The assumptions that imply this group decision rule are simple enough to state here [Harsanyi (1955)]:

1 Social (group) preferences satisfy postulates for expected utility maximization (see Chapter 7).

2 Individual preferences satisfy postulates for expected utility maximization.

3 If two prospects P and Q are equally desirable from the standpoint of every individual, they are also equally desirable from a social (group) standpoint.

Keeney (1976) showed that this decision rule is the only one that satisfies some Arrow-like conditions on group choice.

Surprisingly, the third of the assumptions above, which appears quite innocuous, may be the one most objectionable. For example, two risky opportunities may be equally desirable to each individual, yet one may be more acceptable to the group due to the group's greater capacity to take risk. But, for good or ill, the philosophy of this approach is to treat group decisions in the same fashion as individual decisions.

The use of the additive utility approach requires not only assessment of utility for each individual and agreement on probabilities, but also the setting of weights. There are several ways that the weights may be set.

The simplest method would be for one individual to set the weights. In a committee that has a chairperson, the chairperson would be the logical choice to carry out this task. But the group may have no such person or may not wish to invest such power in one person. It may be that the groups can jointly set the weights in a participatory manner. This task may be very time consuming or it may create horse-trading and tensions that the group members would prefer to avoid. Thus the group would welcome some amicable way to set the weights.

9.3.1 A Delegation Process for Achieving Consensus on Weights

When people have differing preferences and a joint decision to make, a common approach is to delegate the problem of selecting a fair compromise to some unbiased outsider. This principle of delegation is exploited in a procedure developed to achieve consensus on utility weights [see Bodily (1979)]. The process is based on three postulates:

1 *Delegation* Each committee member designates voting weights in a delegation subcommittee made up of other group members. Individual i assigns weight w to member j, such that $0 \leq w_{ij} \leq 1, j = 1, 2, \ldots, n$; $w_{ii} = 0$; and $\Sigma\, w_{ij} = 1$ for all i.

2 *Decision Rule* The delegation subcommittee employs a group utility function that is the sum of its members' utility functions weighted by the delegation weights specified above.

3 *Substitution* The utility function of a committee member's delegation subcommittee substitutes for his or her own. This substitution continues repetitively, each substitution constituting a *step of delegation*.

If members of the group knew the utility functions of the other group members, they would use this information to set weights to reflect their own preferences. But they do not know the weights used by others. Thus w_{ij} reflects how individual i would allow the utilities of others to be used to make decisions in his or her own absence.

Individual i delegates part of the decision to j who delegates it on to k, and so forth. The process continues through steps of delegation. The delegation steps have the effect of paring down the set of Pareto optimal alternatives. As the number of steps of delegation gets large, the group utility functions used by the delegation subcommittees converge to one additive utility function, under suitable conditions. A sufficient, but not necessary, condition is that each set of individuals in the group delegates some portion of the problem to everyone else. Person i may delegate to person j directly through $w_{ij} > 0$ or indirectly through a higher-order chain (e.g., $w_{ik} > 0$ and $w_{kj} > 0$).

If the delegation process converges, the weights are found by solving the following system of equations:

$$
\begin{aligned}
a_1 + a_2 + \cdots + a_n &= 1 \\
a_1 &= w_{11}\, a_1 + w_{21}\, a_2 + \cdots + w_{n1}\, a_n \\
a_2 &= w_{12}\, a_1 + w_{22}\, a_2 + \cdots + w_{n2}\, a_n \\
\\
a_n &= w_{1n}\, a_1 + w_{2n}\, a_2 + \cdots + w_{nn}\, a_n
\end{aligned}
$$

EXHIBIT 9.5 EXAMPLE CENSENSUS PROBLEM WITH SOLUTION MODEL

0	.2	.4	.4		w_{11}	w_{12}	w_{13}	w_{14}
.4	0	.5	.1		w_{21}	w_{22}	w_{23}	w_{24}
.25	.25	0	.5		w_{31}	w_{32}	w_{33}	w_{34}
.1	.2	.7	0		w_{41}	w_{42}	w_{43}	w_{44}

```
MODEL CONSENSUS  VERSION OF  02/25/83  15.57
1 COLUMNS 1
2 A1 = .4 * A2 + .25 * A3 + .1 * A4
3 A2 = .2 * A1 + .25 * A3 + .2 * A4
4 A3 = .4 * A1 + .50 * A2 + .7 * A4
5 A4 = 1 - A1 - A2 - A3
6 SIMULTANEOUS A1 THRU A4
END OF MODEL
? SOLVE
MODEL CONSENSUS  VERSION OF  02/25/83  15:57—1 COLUMNS 4 VARIABLES
ENTER SOLVE OPTIONS
? ALL

                                   1

A1                               .1891
A2                               .1815
A3                               .3570
A4                               .2723
```

Remember that $w_{ii} = 0$ for all i, so that the diagonal terms above would be zero. Also, there are $n + 1$ equations above in n unknowns, and one of the equations is redundant. Therefore one of the equations, say the last one, may be dropped. The equations may be solved using IFPS, as is done in Exhibit 9.5. The weighting data for an example problem is shown in Exhibit 9.5 along with an IFPS model that solves the simultaneous equations for the a's.

The consensus model has incentives built in for group members to honestly give their true preferences and weights on others' utilities. First, the process will give them what they state they want. Secondly, members will be kept in check by the threat that their preferences may be given a very low weight by others when the process is used again in the future.

9.4 COMPARISON OF NASH AND HARSANYI APPROACHES

There are many differences between the purposes and implications of the Nash bargaining solution and the Harsanyi additive utility. To clarify how they are distinct, recall

1 Nash was considering a bargaining (competitive) situation.

2 Harsanyi looked at decision making under *uncertainty* for a group that presumably were already committed to making some joint choice.

Thus, the methods were intended for different situations and the circumstances of a group decision would suggest which method might be most appealing. Since it is often not clear what kind of group decision problem one really has, it is still conceivable that a choice can be made between the two methods. The Nash model has been used by people for uncertain situations, merely by taking the product of expected utilities. And, of course, the additive utility approach can be used in the case of certainty. A certainty may be represented by a probability distribution with a single event having a probability of 1, so the expected utility rule reduces to using utility directly.

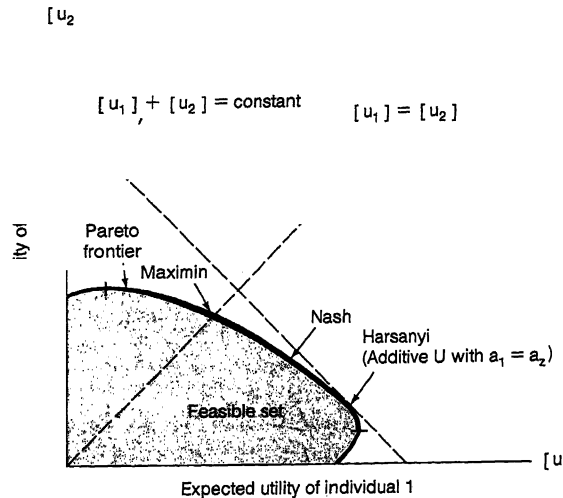



EXHIBIT 9.6 Comparison of group decision rules.

Exhibit 9.6 illustrates which of a set of feasible alternatives would be picked by the Nash product of utility approach and the Harsanyi additive utility approach. The feasible alternatives are in the shaded region, which in this case is continuous. These alternatives are plotted in the space of expected utility for two individuals. The zero point for utility is taken to be that defined by the threat or status quo levels of the individuals.

The northeast frontier, between the hash marks, is the Pareto optimal set of alternatives. Harsanyi's additive utility with equal weights is the point where the line $.5[u_1] + .5[u_2] = \text{constant}$ (remember [ ] denotes taking expectation), just touches the boundary of the feasible set of alternatives. By selecting the appropriate weights, any point on the Pareto frontier could be selected by additive utility. The weights would be such that the ratio a_1/a_2 would equal the negative of the slope of the Pareto frontier.

The Nash solution is on the Pareto frontier at the point where the ratio of expected utilities, $[u_2]/[u_1]$, is the negative of the slope of the Pareto frontier. This corresponds to an additive utility solution where the ratio of weights is equal to the reciprocal of the ratio of expected utilities.

Brock (1980) argues that the Nash solution is just a way of setting weights which might be used in an additive utility form. And, he suggests, the notion behind the particular set of weights indicated by the Nash solution is one of *relative need.* His concept of relative need may be translated as relative intensity of desire. Specifically, his postulate is that u_j/u_i equals the negative of du_j/du_i, the intensity of desire. This intensity of desire is nothing more than the slope of the boundary of the feasible alternatives.

Brock would conclude that the Nash solution gives a more *equitable* solution than the additive utility with equal weights. It is interesting to compare the two of them to another decision rule, designed specifically with equity in mind.

9.4.1 Maxmin

Rawls (1971) proposed an alternative to utilitarianism as a rational basis for collective choice. He defines the concept of an "original position," a state prior to knowledge of which particular role each individual might play in society. He argues that the justice of any society should be judged from this position and that in so doing we should seek to maximize the welfare of the individual who is worst off in that society. Hence, a decision rule based on these notions would be a *maxmin* rule, that is maximize the welfare of the person with minimum welfare.

Even though Rawls was dealing with more global issues of societal organization in his work, the maxmin rule may still serve as a benchmark of equity for group decision problems. It can be demonstrated that maximizing the worst outcome would give a solution on the diagonal as shown in Exhibit 9.6. Note that the Nash solution is between the maxmin solution and the additive utility solution with equal weights on the Pareto optimal frontier. This can be shown to be true in general [Bodily (1976)].

There are a variety of other decision rules that may be, and have been, employed for group decision. The multiplicative form of multiattribute utility (Chapter 7) is easily adapted for use in the group context. This too differs from the additive utility in that it provides for a greater degree of equity in choice. For a complete discussion of this and other group utility forms see Chapter 10 of Keeney and Raiffa (1976).

Both the Nash product utility form and the additive utility form are easily added to an IFPS model in order to evaluate alternatives for a group. Thus the IFPS model is a potentially powerful tool for resolving differences of opinion in committee decision problems. Very often, once individuals realize the points on which they disagree a compromise or consensus decision can quickly be reached. In other situations it is apparent why people disagree. Generally there is enough room for negotiation that with the help of the model a suitable bargain can be struck.

KEY TERMS

Pareto optimality
majority voting
Borda rule
independence of irrelevant alternatives
Arrow's impossibility theorem
ordinal utility
cardinal utility
interpersonal comparison of utility
Nash bargaining
threat value
additive utility
consensus
maxmin

EXERCISES

9.1 (Due to Ralph Keeney) there are three individuals with preferences for alternatives A, B, C, and D as follows:

1. A > B > C > D
2. C > A > D > B
3. B > D > C > A

Suppose votes are to be taken comparing two alternatives at a time. The losing alternative is thrown out and the winning alternative is compared with a new alternative.

a If individual 2 gets to choose the sequence in which new alternatives are brought into the comparison process, which order should be chosen to best serve his or her preferences?

b If individuals 1 and 3 recognize that individual 2 is controlling the vote, what possible collaboration can they follow for their mutual advantage?

c Who would benefit the most from this collaboration?

d In the original problem, should the order of voting be different if individual 1 got to specify it?

9.2. Three neighbors have decided to jointly purchase some open space near their homes. They differ in their preferences for the features they will design into the landscaping of the land. They have decided, however, to use the consensus process in the chapter to determine how to weight their separate utility functions and the weights that they have assigned are given below:

$$w = \begin{bmatrix} 0 & .4 & .6 \\ .35 & 0 & .65 \\ .5 & .5 & 0 \end{bmatrix}$$

What weighting of each individual's utility function will result from the consensus process? (You may wish to use IFPS to solve this.)

9.3 Two investors have formed an investment club and are faced with the problem of jointly picking a portfolio of investments. Their utility functions are based strictly on the mean and the variance of Rate of Return as follows:

u_1 = mean − .1 (variance)
u_2 = mean − .4 (variance)

The club has a wide range of possible investments that they may put together in many ways. Having eliminated combinations that are dominated by other possible combinations (that have more mean return for the same variance or less variance for the same mean), the choice boils down to choosing from the efficient set of investment opportunities. This efficient set of opportunities has a mean given by the equation

$$\text{Mean} = .10 + .2 * (\text{variance})^{.6}$$

for variance above .05 (the minimum variance portfolio).
The first investor suggests that if he takes his money out of the investment club he can get a 9 percent risk-free return and the second one believes she could only get a 7 percent risk-free return. Assume that all rates of return have already accounted for tax effects and transaction costs.
a What mean and variance pair would be selected by using Nash bargaining?
b What mean and variance pair would be selected by an additive utility with equal weights?
c What mean and variance pair would you suggest the club adopt?

CHAPTER **10**

RISK AND TIME

10.1 INTRODUCTION

Decision problems where the key financial results are both risky and spread over time are pervasive in business. Virtually all capital investments, budgeting problems, or planning situations possess time and risk dimensions in their evaluation. One might expect, because it is so common, that evaluation of the time value of money comingled with risk would be handled with noncontroversial procedures by now. Not so; in fact, among the important concerns of financial and quantitative analysis, this issue is one of the least settled and one where theory and practice are most divergent.

This chapter describes the ticklish problems inherent in dealing with risk and time, then presents a range of approaches with their associated pros and cons. The basis for comparisons of alternative methods is a combination of conceptual soundness and ease of use in practice. Two threads of thought and research have been used in comparing approaches: that of decision making under uncertainty and that of finance. The former discipline is more appropriate when evaluating time streams on the basis of your own preferences, for example, choosing among

alternative uncertain streams of personal consumption. The later discipline, of course, is more appropriate for valuing time streams strictly in terms of what the market will pay for them, which would be more appropriate for pricing a stock offering, for example. A problem such as choosing among risky projects presents to the manager some elements of each approach. On the one hand, the manager recognizes that he or she is responsible for the decision and must make it rationally and consistently using appropriate personal judgment. The uncertain performance of any project clearly is risky to the manager, and the manager will wish to deal with this risk. On the other hand, the manager recognizes that the firm belongs to investors and the objective is to maximize their interests, which may be assessed through observance of the financial markets.

The general situation considered in this chapter is one of evaluating uncertain time streams of outcomes. Usually the outcomes are after-tax cash flows minus investment (call it *net cash flow*), but other variables, such as consumption or income, may be treated in the context of the discussion that follows. A stream of uncertain outcomes will be denoted by

$$\mathbf{z}^{\sim} = (z_1^{\sim}, z_2^{\sim}, \ldots, z_T^{\sim})$$

where $z_t^{\sim}$ is the uncertain outcome at time t. The mean of $\mathbf{z}^{\sim}$ will be written $[\mathbf{z}^{\sim}] = ([z_1^{\sim}], [z_2^{\sim}], \ldots, [z_T^{\sim}])$. In general the expression [ ] will denote the operation of taking the expected value or mean of what is inside.

10.2 IN WHAT ORDER AND HOW DOES ONE ADJUST FOR RISK AND TIME?

One of the fundamental concepts of finance is that of discounting a cash flow from a project to adjust it for the time value of money. Thus if cash flow z_t is received at time t, then its present value at time 0 is

$$1 + d_t$$

where d_t is the rate of discount for cash received at time t.

Another fundamental concept is that it is appropriate to adjust an uncertain cash flow, $z^{\sim}$, to account for the undesirability of risk. The value given to $z^{\sim}$ after the adjustment is the amount of cash flow, if it were *certain* to be received, that would be equivalent to the yet *unobserved* amount of cash flow, $z^{\sim}$. Hence it is known as a *certainty equivalent* (CE) for the cash flow $z^{\sim}$.

An individual responsible for making a decision might subjectively estimate a CE directly, or more formal analysis using risk preference concepts (Chapter 7) and utility functions may be used to calculate it. Sometimes it is simplest for the decision maker to consider how much he or she is willing to sacrifice to completely remove the uncertainty in the cash flow—call this the *risk premium* (RP)—then the CE is given by

$$\text{CE} = \text{mean} - \text{RP}$$

The problem in evaluating long-term risky projects is not confusion about how to adjust for time or for risk; the procedures are well developed. But the order in which one makes the adjustments in cash flow $z_t^{\sim}$ does affect the results. Efforts to combine the two adjustments into a single measure of value may be oversimplified.

There are two major needs. One is for better education of managers to understand the available methods and know when and how to use them. The other is for advances in financial theory and decision theory under uncertainty that would provide practical methods of evaluating risk in time streams that are consistent with how financial markets value projects. We can accomplish the former here; we may have to wait for the latter.

The principal alternative methods will now be elaborated and compared, roughly in order of complexity. To avoid ambiguity, the methods will be expressed in equation form as simply as possible; concrete examples will aid the discussion of the concepts behind the methods.

10.3 RISK-ADJUSTED DISCOUNT RATE

The most common method of treating uncertain cash flows is the risk-adjusted discount rate (RADR) approach. The RADR method values a stream of uncertain outcomes according to the quantity

$$\text{RADR}(\mathbf{z}^{\sim}) = \frac{\sum_{t=1}^{T} [z_t^{\sim}]}{\prod_{j=1}^{t} (1 + q_j)} \tag{10.1}$$

where q_j is the RADR for period j and $\sum$ denotes the operation of summing what follows and $\prod$ denotes taking the product of what follows. Note that only the mean of $z_t^{\sim}$ is used; uncertainty is examined only in the process of setting the risk adjustment (greater risk implies larger q_t).

A major reason for using this approach is that much of the theory of financial markets that has been worked out is consistent with it. This means that financial theory may be employed more readily with this approach than with some of the others that follow, not necessarily that (it remains to be seen) financial theory *cannot* be worked out for these other approaches. RADR is itself a special case of the multiple-time-period utility approach to be mentioned later in the chapter.

To use this approach, it is necessary to estimate a RADR for each time period. Each of these RADRs must correspond to the riskiness in that time period of the project being evaluated. It would be the same risk-adjusted rate used to evaluate any other investment opportunity with "comparable" riskiness for that time period. It is no minor task to come up with such risk-adjusted rates. Some approaches are conceptually straightforward—just look until you find projects or securities traded in the markets that have similar patterns of cash-flow uncertainty—then use the rate of return implicit in their market prices. Other approaches are conceptually more formal, involving models of market value, such as the capital asset pricing model. In any case, finding the appropriate risk-adjusted rate is hard to do and invariably involves some qualitative judgments and some art and craft rather than cut-and-dried analysis. It is particularly difficult to find the right discount rates when the time stream is due to a capital project in a firm, such as a new product. There may well be nothing comparable that is traded in the financial markets.

10.3.1 RADR with a Single Discount Rate

The most often used evaluation method in practice is RADR with a simplification. Rather than estimate a different RADR for each time period, the manager uses only one. Thus the following substitution is made in the denominator of Equation (10.1):

$$\prod_{j=1}^{t} (1 + q_j) = (1 + q)^t$$

This simplifies the task, since only one discount rate is needed. It may seem to be only a minor change, but this simplification places very rigid restrictions on the treatment of risk.

The critical assumption inherent in the single-discount-rate RADR is that the cash flows for a risky long-term project should be adjusted for risk by dividing by a factor that grows at a compound rate. Robichek and Myers (1965) argue that there is no justification for this assumption as a *general* rule of financial evaluation.

Using a single rate in RADR may penalize long-term projects relative to short-term projects. Consider the following example of what happens with this approach.

Example 10.1

Suppose you invest a dollar now in either project S (short-term), which has equal chances of producing $.95 or $1.40 at the end of 1 year or project L (long-term), which has equally likely chances of producing $1.65 or $2.35 after 5 years. The success of each investment opportunity hinges on the outcome of a bill before Congress which will be decided in a few months after you make your investment. The higher outcome for each investment applies if the bill is passed. Some relevant facts are that the interest rate on Treasury bills and notes maturing in the 1–5 years happens to be in the vicinity of 12 percent, and that financial research indicates that the rate of return on investments in the class of riskiness of these two investments is 17 percent. Which opportunity (L or S) is more desirable?

The mean cash flow for the short-term opportunity is $1.175 after 1 year and for the long-term opportunity is $2.0. For the short-term opportunity, the single-discount-rate analysis netting out the $1 investment would then be

$$\text{RADR} = 1.175/1.17 - 1 = .0043$$

and for the long-term opportunity

$$\text{RADR} = 2.0/(1.17)^5 - 1 = -.0878$$

Thus the short-term opportunity would be preferred using this approach.

Looking closely at this evaluation, we note that the mean for the 1-year project is adjusted by (1/1.17) and that for the 5-year opportunity is adjusted by ($1/1.17^5$). Since the risk-free discount rate would be approximately 12 percent, the risk adjustment for a 1-year cash flow reduces its value by a small amount to about 96 percent of what it would be with normal 12 percent discounting [1/1.17 = .9573 (1/1.12)]. Yet the risk-adjustment multiplier for a cash flow 5 years out reduces value by a much greater amount to about 80 percent of risk-free discounting ($1.12^5/1.17^5 = .8038$). Why should this be the case when the two projects are subject to the very same risk? There is no reason in this example for the risk adjustment to be so wildly different for the two projects simply because in one case the money comes back sooner!

The problem could be corrected by evaluating Example 10.1 by different discount rates for each period as with Equation (10.1). The adjustment to the discount rates for years 2, 3, and 4 could then reflect the fact that those years do not add substantially to the risk of the project. The riskiness of the project could be accounted for primarily through the discount rate adjustment above the risk-free rate for the first year, since that is when the uncertainty is resolved. Risk adjustments for the other years would be smaller, based on uncertainties in the environment about the performance of competing investment opportunities.

It is not the case that using single-discount rate RADR is never appropriate. For general purposes, however, it is a lot to ask to wrap up the time value of money and the treatment of risk into one number.

10.3.2 Caveats

If you are going to use RADR, then Equation (10.1) is much better than using a single discount rate. Unfortunately, this diminishes one of the principal reasons for the popularity of RADR—simplicity.

There are other reasons why one may look for something more than RADR. If a sophisticated model of a project has been developed containing probability distributions of key variables and

if Monte Carlo simulation provides probability distributions for cash flows, then one would like a more sophisticated method of evaluation that would use in a systematic way more than just the mean of cash flow.

It would also be desirable to capture year-to-year interdependencies in cash-flow risk. Consider for illustration the problem in Example 10.2.

Example 10.2

An investment of $80 will produce after-tax cash flows at the ends of 2 years. There is a choice between investment A and investment B with the following equally likely cash-flow streams:

		Year 1	Year 2
Option A:	.5	$80	$80
	.5	$20	$20
Option B:	.5	$80	$20
	.5	$20	$80

The branches for each option represent the possible outcomes; probabilities of .5 are shown for each branch. Which alternative is best? Assume a risk-free rate of 12 percent and a risk-adjusted rate of 17 percent in the first year and 13 percent in the second year.

Without working out the precise evaluation of the two options in Example 10.2, one can see that they would both have the same RADR value. For each year in each option the mean cash flow is $50 [.5($80) + .5($20)]. Thus for purposes of RADR evaluation the projects are identical (with RADR value = .55 = 50/1.17 + [50/(1.17)(1.13)] − 80). Yet, most managers would not be indifferent between the two projects. Option A exhibits more total risk, since the cash flows are positively correlated; you have either a good year followed by a good year, or a bad year followed by a bad year. On the other hand, option B has one good year and one bad year for either outcome. The RADR approach will miss this interperiod interdependency.

10.4 THE CE OF THE NPV DISTRIBUTION

The RADR approach makes both the risk adjustment and the time-value-of-money adjustment at once in the process of discounting. Alternatively, suppose an adjustment is made first for the time value of money and then a separate adjustment is made for risk.

To be more concrete, the first step of this approach would be to discount each possible net cash flow at the risk-free rate. Since cash flow is uncertain, in general this means discounting the cash-flow stream for each possible outcome, thereby obtaining a probability distribution for net present value (NPV). This step is summarized by the following expression for uncertain NPV:

$$\text{NPV}^{\sim} = \sum_{t=1}^{T} \frac{z_t^{\sim}}{(1+i)^t} \qquad (10.2)$$

where i is the risk-free discount rate.

EXHIBIT 10.1 DISTRIBUTION OF NPV FOR INVESTMENTS L AND S AT A 12 PERCENT DISCOUNT RATE

Outcome of congressional bill	S	L
Bill is defeated	\$−.15	\$−.06
Bill is passed	\$ .25	\$ .33

The second step of this method is to adjust for risk in the probability distribution of NPV. This may be done informally, that is, by subjectively comparing probability distributions visually. Of course, the first thing to do in comparing probability distributions is to check to see if one or more alternatives are stochastically dominated (Chapter 5).

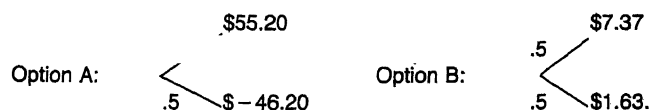
Any of the methods for formally using risk preference discussed in Chapter 7 may also be used for NPV (e.g., assessing a utility function or using a standard model for a utility function such as the negative exponential or logarithmic models). The result of such efforts would be CE(NPV~), the CE of the NPV probability distribution for a project. The use of a utility curve provides consistency of decision making but raises the additional problem in a firm of establishing the degree of corporate aversion to risk.

Example 10.1 may be used to illustrate more clearly how this approach works. If we discount the two possible outcomes for cash flow of the two projects at the risk-free rate (recall that it was 12 percent), we obtain the values given in Exhibit 10.1. The next step of adjusting for risk in the distribution of NPV is simplified by the dominance of the project L over S. Observe that whether or not the congressional bill passes, the long-term investment has a higher discounted value.

The CE of NPV method clearly gives different results from single-discount-rate RADR in Example 10.1. If separate discount rates and risk adjustments for each time period had been used, the CE of NPV method need not give different results from RADR for Example 10.1. But again, the method typically used in practice is single-discount-rate RADR.

The cash flow may be assumed to be a discrete or a continuous quantity. When $z_t^{\sim}$ is a continuous uncertain quantity, Monte Carlo simulation may be the most convenient way to obtain the probability distribution for NPV~. (It is mathematically complicated to otherwise discount each possible outcome separately since there are infinitely many of them.)

The CE of NPV method has advantages and disadvantages relative to RADR. On the positive side, it avoids most of the controversial and perplexing problems of choosing a risk-adjusted "cost of capital." The risk-free rate, usually taken to be the interest rate on federal bills, notes, or bonds, is more easily obtained and less ambiguous. Secondly, in this method, all the adjustments for risk are made in a single quantity, namely present value dollars (which are indistinguishable from other current dollars), rather than dollars accruing at different times. Finally, the CE of NPV approach cuts the time-risk problems into neat separate problems. One always discounts at the same rate and one always adjusts for risk using one set of preferences for money.

This method does account for interdependencies of the type exhibited in Example 10.2. Using the risk-free discount rate the probability distributions of NPV for the two options:

It can easily be verified that the mean of the two options is the same \$4.50. Since the uncertainty in option B is so much lower, it would clearly be favored over A, as expected.

One problem with the CE of NPV approach is that it may oversimplify important information in the time pattern of cash flows. A great many different patterns of cash flow may translate into the same distribution of NPV. More will be said on this point in Section 10.8.

The CE of NPV approach, as we have expressed it, may understate risk in that we presume to know what the risk-free discount rate is over the life of the project. This can be corrected in a Monte Carlo simulation by treating the risk-free discount rate as a random variable. In addition, it may make sense to use a different risk-free rate for each time period.

One criticism that has been made of the CE of NPV approach is that discounting probabilistic cash flows is computationally more difficult than what companies do now. (Just forecasting probabilistic cash flows is more than what most companies do now!) But it is clear that a company cannot treat risk in a project without estimating the uncertainty it faces. Modern computer support makes this approach still quick and easy enough, but one would normally not want to carry it out by hand. The truly significant concern, given current software and hardware availability, is not the computational requirements but rather the need for managers accustomed to traditional forms of analysis to be trained in interpreting and using probability distributions.

A common criticism of this approach is really a criticism of risk-analysis simulation, which is associated with this approach, not the approach itself. Financial theory rightfully considers only the systematic risk (i.e., that risk correlated with the financial market) in valuing a capital investment. It is argued that all other risk (unsystematic or *project-specific* risk) can be diversified away by the investor and thus is irrelevant in valuing the investment. Since projections of cash-flow probability distributions in a risk-analysis simulation model generally include total risk, some claim the CE of NPV approach overstates the risk. This is throwing out the baby with the bath!

The objection is not to the treatment of time and risk, but to the way of modeling risk in the simulation, which is correctable. For example, one may make all of the projections of cash flow conditional on the market. Or even better, make the projections of cash flow conditional on the existing portfolio of investments held by the firm, thereby focusing on the truly relevant risk to the firm. Another possibility is to carry out the analysis of the firm's cash flow with and without the new project. This is easily done in IFPS using the COMBINE command to put together two models or the CONSOLIDATE command to merge data files. Acceptance of the project is then based on whether the total cash flow including the project is superior to that with the project left out.

The user of this approach must keep in mind that while it has its appealing features, there is no theory of financial markets to back it up, like there is with RADR. When in doubt about subtle differences between projects that may be glossed over, do not use it. Or at least compare how other methods may rank alternatives before drawing final conclusions.

10.5 DISCOUNTED CERTAINTY EQUIVALENTS

Another way to treat uncertain cash flows is to adjust for risk first, then for the time value of money. Using the same risk-preference concepts referred to above, CE_t is obtained to replace the uncertain cash flow in each time period t. Then a project is evaluated by the discounted CEs (DCEs):

$$DCE = \sum_{t=1}^{T} \frac{CE_t}{(1+i)^t} \qquad (10.3)$$

This approach will not necessarily rank projects in the same order as the approach in the previous section, even when identical risk preferences for cash flow in any period and for NPV are used. Normally this approach would lead to a lower value than the CE of NPV approach [see Cozzolino (1979)].

Some find this approach conceptually cleaner than the CE of NPV approach, since the risk adjustment is made on projected cash flows rather than NPV. However, the risk adjustment requires more effort, since it must be done separately for each time period. In addition it may be easier to think about risk preference in terms of NPV, which may be viewed similarly to current cash flow. Since the risk adjustment is innately more complex than the adjustment for time, it makes sense to only do it once as in the CE of NPV approach.

A conceptual difficulty with discounting CEs is that generally it is inappropriate to add CEs. For example, the CE of the portfolio of projects consisting of project I and project II is not the sum of the CEs for projects I and II (if it were there would be no value to diversifying).

Example 10.2 provides a more concrete illustration of the fact that discounting CEs misses multiperiod risk interdependencies. Again, option B of that example exhibits a type of multiperiod risk diversification; a good outcome in one year offsets a bad outcome in the other year. Since DCE starts by establishing a CE for each time period, and since the outcomes and probabilities for each period are the same for the two options, the DCE evaluation of A would be identical to that of B. Like RADR, DCE misses this aspect of risk.

10.6 MULTIPERIOD UTILITY

The CE of NPV discounts before adjusting for risk, the DCE adjusts for risk and then discounts, and RADR combines the time-value-of-money adjustment and risk adjustment together in one discount factor. It is possible to combine both adjustments into one calculation in a more general way through the use of a utility function, $u(\mathbf{z})$. This utility function values the entire stream of uncertain outcomes but may have separate components that treat the time value of money and risk. There are many ways to decompose overall utility into one-dimensional utility for cash flow at a single period of time. We will mention only two common forms: additive and multiplicative.

10.6.1 Additive Utility

The additive utility function has the form

$$u(\mathbf{z}) = \sum_{t=1}^{T} k_t u_t(\tilde{z_t})$$

where $u_t(z_t)$ is the utility function expressing risk preference for uncertain cash flow at time t, $\tilde{z_t}$, and k_t is a constant expressing the relative importance of cash flow at time t. Methods of Chapter 7 may be used to assess $u_t(z_t)$, and methods of Chapter 8 may be used to assess the k constants.

If the expression for utility is written into a Monte Carlo simulation of cash flows, then the mean of utility could be used to rank projects. Alternatively, expected utility could be calculated using

$$[u(\mathbf{z})] = \sum_{t=1}^{T} k_t[u_t(\tilde{z_t})]$$

For many applications, it may be appropriate to make some simplifying assumptions:

1 Stationarity of utility: $u_t(.) = u_0(.)$, for all t. In other words, the same one-dimensional utility function is used for cash flow at each period, including time 0.

2 Discounting at the risk-free rate: $k_t = 1/(1 + i)^t$.

The resulting utility expression is then

$$DU = u(\mathbf{z}) = \sum_{t=1}^{T} \frac{u_0(z_t)}{(1+i)^t} \qquad (10.4)$$

While projects may be ranked directly by expected utility using Equation 10.4, it may be desirable in addition to find the CE of NPV. This may be done in the usual way, by finding CE such that

$$u_0(CE) = [u(\mathbf{z}^{\sim})]$$

It has been assumed here that utilities can be discounted at the same risk-free rate that would be used for money. The argument for this is similar to one that would lead to using the same discount rate for CEs as for actual cash flow. If the discount rate reflects the relative importance of receiving money in the future compared to receiving money now, then these relative importances should apply similarly to several different measurements of uncertain money, including CEs and utility. It is, of course, easier to make this argument when utility functions $u_t(.)$ are identical for all t.

Discounting utility is more acceptable theoretically (fewer undesirable assumptions need be made) than discounting CEs. On the other hand discounting CEs may be easier to understand and conceptually more acceptable to some individuals.

Because the utility is *additive*, the interdependencies of Example 10.2 do not affect evaluation; options A and B of that example would have the same value by expected utility. For this reason a multiplicative utility model may be more appropriate.

10.6.2 Multiplicative Utility

The simplest way to account for interactions in a general evaluation model is in the multiplicative form,

$$u(\mathbf{z}) = \frac{\prod_{t=1}^{T} [1 + kk_t u_t(z_t)] - 1}{}$$

where the k_t's have the same interpretation as before and k is a constant expressing the weight on interaction. If k is positive then positive period-to-period correlations are preferred; a negative k implies an aversion to multiperiod risks and negative serial correlation is preferred. Thus it can be verified that if k is negative, option B would be preferred in Example 10.2. See Chapter 8 for a more complete discussion of this form of utility.

10.7 COMPARISON OF METHODS

Each of the four methods described will, in general, place differing values on projects. Exhibit 10.2 shows the resulting NPVs that the methods place on the two projects in Example 10.1.

EXHIBIT 10.2 COMPARATIVE NPV FOR TWO RISKY OPPORTUNITIES USING FOUR METHODS

Single-discount-rate RADR	.0043	−.0878
CE of NPV	.0471	.1329
DCE	.0468	.1313
Discounted utility (DU)	.0401	.0822

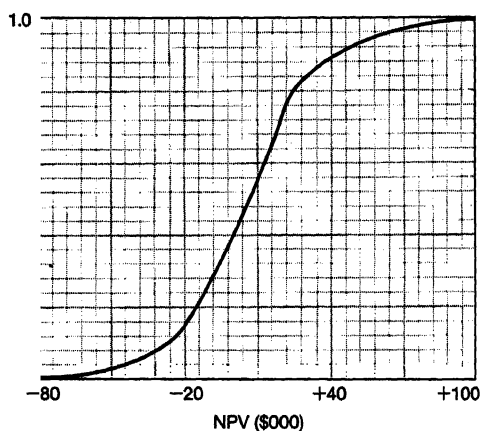
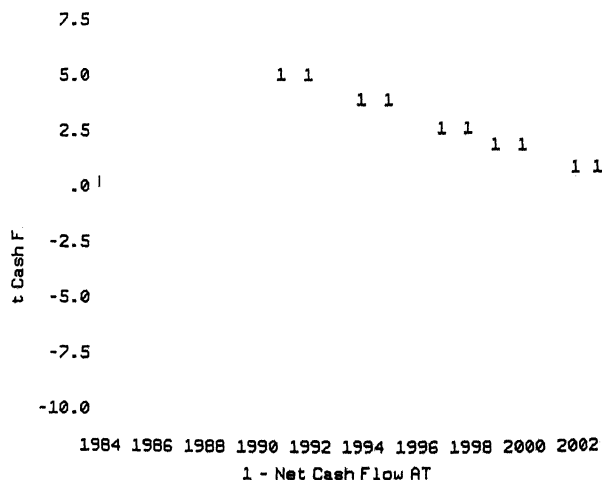
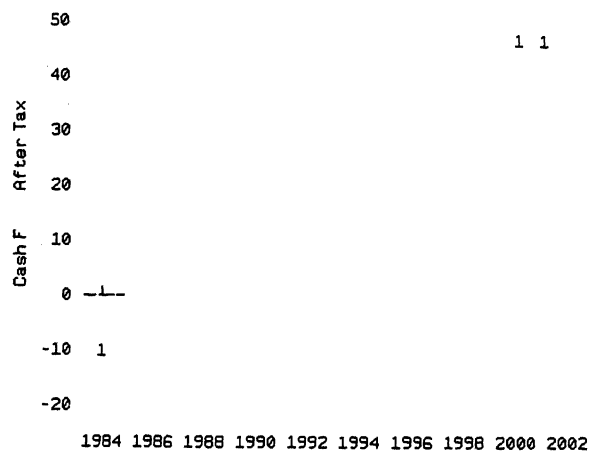
In order to make the calculations for the last three methods, it was necessary to pick an expression for risk preference. For illustrative purposes, a simple constantly risk averse utility function was used. The utility function for dollars was then a negative exponential,

$$u(\$) = 100\ (1 - e^{-.01\$})$$

with risk aversion constant (see Chapter 7) equal to .01. This same utility function was used for dollars received at any time period in the calculations.

Note in Exhibit 10.2 that all of the methods but single-discount-rate RADR favor the long project over the short project. And only the long project evaluated by single-discount-rate RADR has a negative NPV, implying it would be rejected by that method. Again, the bias of single-discount-rate RADR against long-term risky projects is evidenced.

Also note that CE of NPV and DCE methods are close to one another, but that CE of NPV is higher in both cases. In this situation with only two possible outcomes for the two investment opportunities, it is easy to see why this is the case. The range between the two outcomes is

EXHIBIT 10.3 Risk and time graphics for a natural gas development project.

larger in the actual cash flows than in the discounted cash flows. Thus the risk adjustment is larger when made prior to the discounting in DCE than when it is made after discounting.

The DU method gives lower values than either CE of NPV or DCE. It is the most reliable theoretically, although probably the most difficult to employ in practice.

10.8 A SIMPLE MEASURE MAY NOT BE ENOUGH

In truth, none of the measures suggested above or any other single measure that lumps the time and risk characteristics of a project into a single number is perfect. It is possible to construct an example for each measure that shows some anomaly in the way it places value on a project. It may be too much to ask that one number tell us all that needs to be known to compare projects. Indeed, computer models of projects are often built for managers because they want to know more than just the bottom-line evaluation of a project. Of course they also do not want to know all of the detail of the computer model. How can the right risk and time information be selected in order to give insight without confusion?

There are two pictures that capture the risk and time characteristics of a project. A graphical depiction of risk is provided by the cumulative distribution function (CDF) of NPV, an example of which is shown in Exhibit 10.3. On the vertical scale is plotted the probability that NPV is less than or equal to a given level X, which is on the horizontal scale. Sometimes the histogram or frequency distribution, also in Exhibit 10.3, is used.

The time characteristics of a project may be pictured with a profile of cumulative net cash flow (CNCF), that is, the cash flow after tax less investment accumulated up to each time period in the future. For this picture (see bottom example in Exhibit 10.3b), point forecasts of the mean of cash flow are used for each time period. Also shown in Exhibit 10.3 is the net cash flow profile unaccumulated. This shows the stream in and out of actual after-tax dollars associated with the project.

Some may prefer a picture of CNCF that is discounted. This is simply a profile of NPV accumulated for each time period. It would be calculated automatically in an IFPS run.

These pictures tell much more than any of the single measures. Exhibit 10.4 shows the two pictures for a household energy-saving device that has the same single-discount-rate RADR value as the gas well development project of Exhibit 10.3. (Models used to create the graphics of Exhibits 10.3 and 10.4 are given in the Addendum to Chapter 10.) Note that the energy-saving product has both a deeper valley in the CNCF curve and a wider CDF. Thus many would find the energy-saving project less desirable than the gas well investment. Everyone would agree they are quite different.

When comparing two or more projects using the pictures of risk and time, it is useful to plot the projects together as in Exhibit 10.5, which shows projects A, B, and C. Then it is

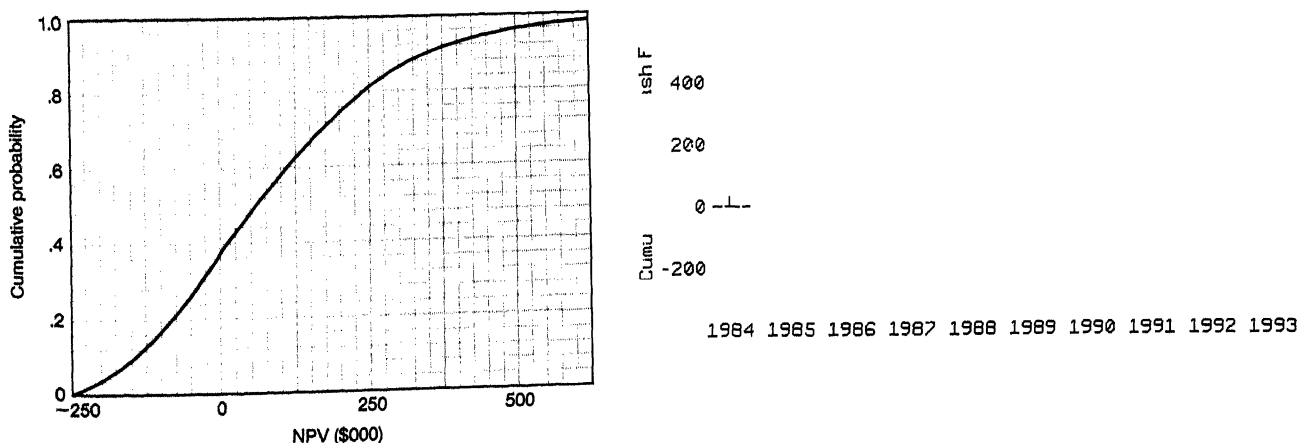



EXHIBIT 10.4 Risk and time graphics for an energy-saving product.

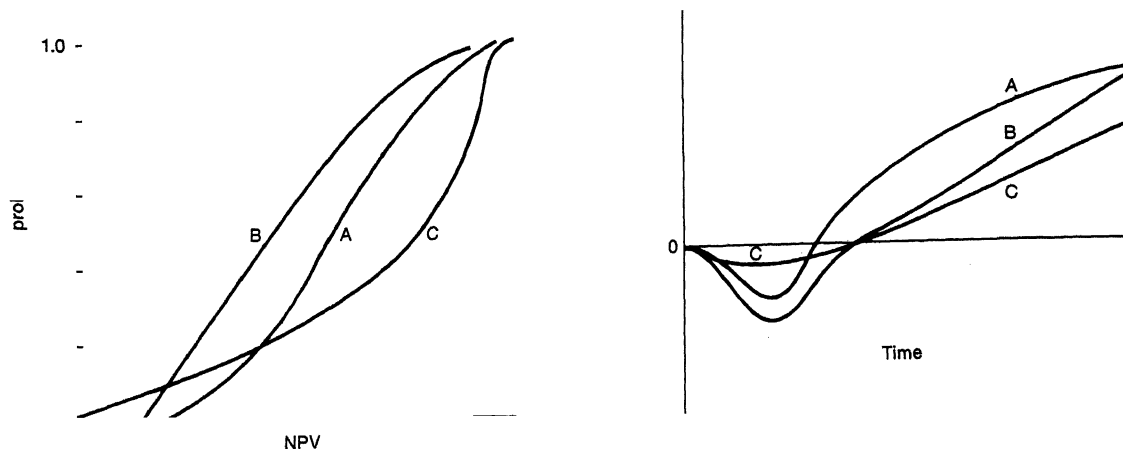



EXHIBIT 10.5 Risk and time cumulatives for hypothetical projects.

easy to check whether a project dominates another. If the project's CDF is strictly under and to the right of another as A is to B, then regardless of the extent of risk aversion of the decision maker, that project is more desirable with regard to risk, by stochastic dominance. Also if the CNCF curve of a project is always above that of another project, as A is to B, then regardless of the time value of money of the decision maker, that project is more desirable. While A is thus indisputably preferred to B in Exhibit 10.5, it is not obvious which of A and C, which have crossing cumulative CDFs and CNCFs, is preferable. Risk preference analysis would resolve the matter.

While the two pictures taken together tell much more than a single measure, additional aspects of the risk may yet be hidden. One aspect is the way in which the risk is resolved over time. For example, the gas wells project pictured in Exhibit 10.3 has a time and risk pattern not unlike that of a capital investment in a geothermal plant, shown in Exhibit 10.6 (the geothermal model is also in the Addendum). Nonetheless the risk is resolved quite differently. With the gas well project a major risk is the amount of reserves that are found, an uncertainty that is resolved after the first year. On the other hand the risk of the geothermal plant lies in the uncertain path of future energy prices. The difference is seen vividly if one examines risk conditional on outcomes in the first year of the project. Exhibit 10.7 shows the CDFs for the two projects, assuming that the uncertainties in the first year are set to their mean level. Note

EXHIBIT 10.6 Risk and time characteristics of a geothermal plant investment.

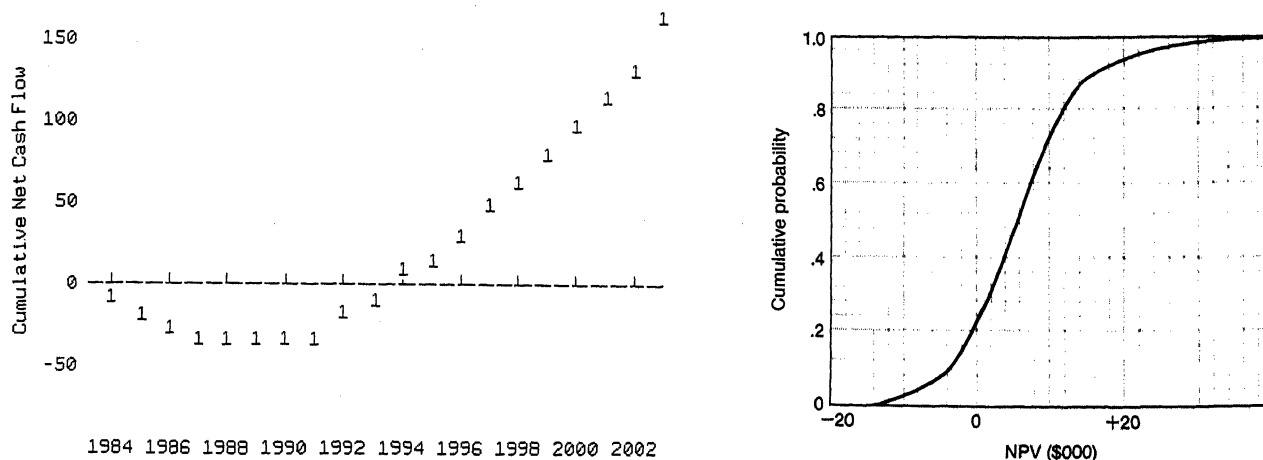
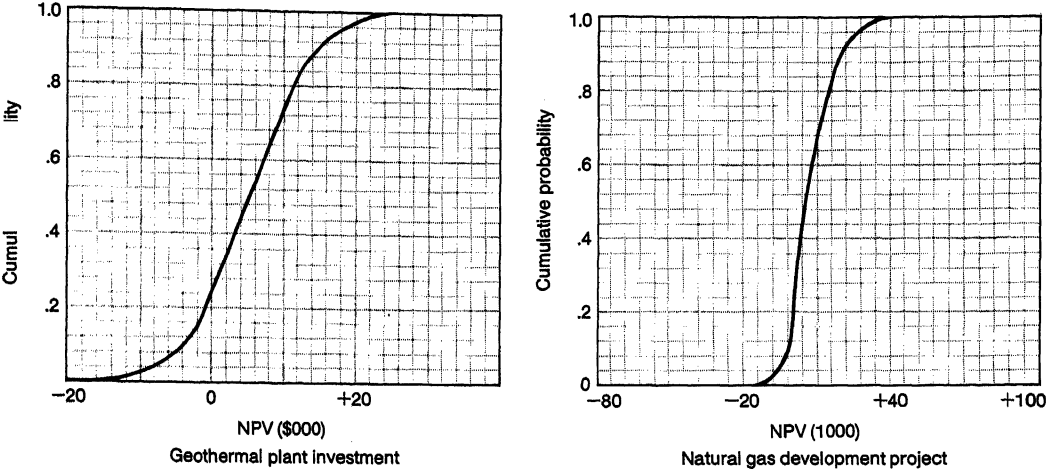
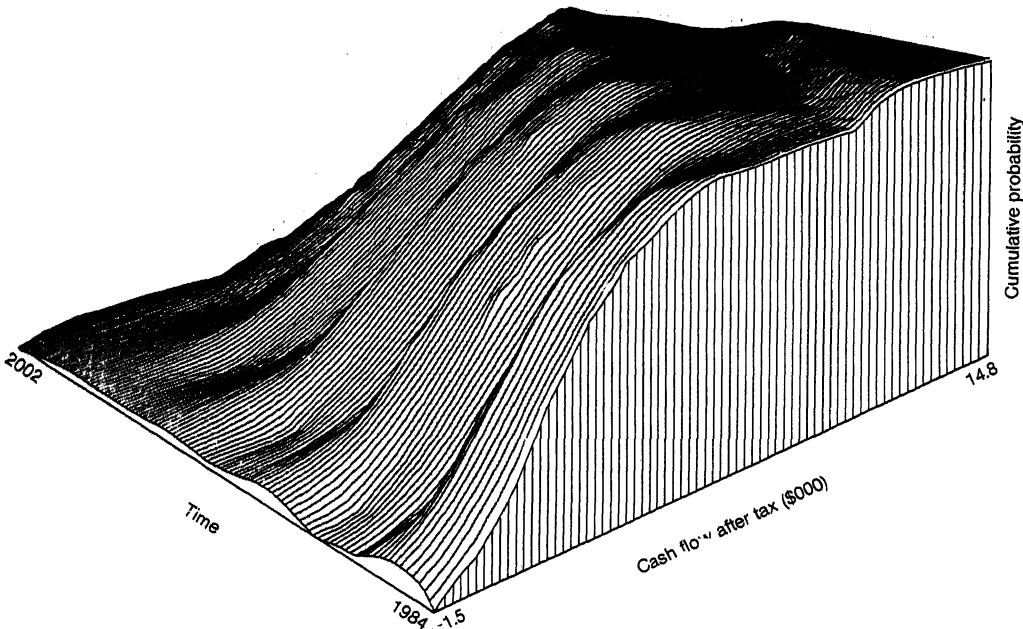



EXHIBIT 10.7 Probability distributions for NPV conditional on first-year uncertainties set to their means.

the great reduction in the width of the CDF for the gas wells project compared to the negligible change in the risk for the geothermal plant. Since it is desirable to know the true value of a project as early as possible, projects having earlier time resolution, like the gas wells project, would be preferred, all other things being equal. Again, the simple measures we have described will not capture this effect.

If more detail of the time and risk characteristics needs to be shown, it may be useful to plot the CDF of cash flow for all time periods. It is most convenient to view this in a three-dimensional plot; an example for the gas wells project is shown in Exhibit 10.8. This picture of cumulative probability on the vertical versus time and cash flow on the horizontal plane shows that the cash flow for this project rises over time and that the risk tends to decrease somewhat over time. Thus time and risk are shown together. But for some decision makers, this may be too much information to readily grasp.

EXHIBIT 10.8 Three-dimensional plot of cash-flow CDFs over time for the natural gas development project.

Interestingly, this three-dimensional CDF plot does not tell all there is to know about the projects, contrary to first impressions. Note in Exhibit 10.8 that there is significant uncertainty in the cash flows out to the end of the project, despite the fact that most of the uncertainty is resolved in the first year of the project. The picture in Exhibit 10.8 is projected from time 0, of course, and does not therefore show the risk reduction perceived after one or more years. This is seen only in pictures of *conditional* risk such as is shown in Exhibit 10.7.

10.9 DISCUSSION

The various approaches to treating time and risk were presented and compared. The right approach for you depends in part on the level of complexity and "correctness" you desire. It will also depend on your purposes—whether you want to develop your own personal or managerial valuation of a project or a valuation by financial markets.

If you are doing a risk-analysis simulation, it is easy to introduce the utility for NPV directly into the model using Equation (10.4) for discounted additive utility. Then expected utility may be calculated as part of the Monte Carlo run and used to rank projects directly. Or without assessing any corporate preferences, the probability distribution of NPV may be compared directly to that of other projects using the CDF or frequency distribution.

Without a simulation analysis and with no computer, RADR will probably be the method of choice. Be aware, however, that using the single-discount-rate RADR approach oversimplifies the problem and introduces an automatic, unjustified assumption about the growth pattern of risk over time.

If one desires to assess a utility function, then it is simplest to do it for NPV and use the probability distribution of NPV. Going to more detail, if you want to use a separate utility function for each time period, it is more acceptable theoretically to use an additive, or even better, multiplicative utility function. A multiplicative utility function captures multiperiod risk characteristics of a project.

Often it is desirable to show graphically how projects differ in time and risk. An effective way to this is with cumulative displays: the CDF of NPV and the cumulative net cash flow over time. In the rare cases where more detail is needed, a three-dimensional plot of CDFs for NPV over time will help to show subtle differences among projects. There may be instances when the time resolution of uncertainty is key to the choice. Plots of risk conditional on results in early time periods will show this.

In this discussion, the focus has been on methods that can be used conveniently, while not ignoring elements of risk and time that need consideration in comparing projects. Recognizably, the subject of risk mixed with time is a deep one, and many issues are still unresolved. I hope the discussion has served to provide you with a better appreciation of available methods, and how and when they may be used. Thereby, practice will be enhanced.

At times decision theory and finance part company on what are appropriate methods of evaluation (although future research may change that). One reason managers are paid as much as they are is to resolve ambiguity at these times.

KEY TERMS

uncertain time streams
risk adjustment
adjustment for time value of money
order of risk and time adjustment
risk-adjusted discount rate (RADR): single or multiple rates
discounted-certainty equivalents (DCEs)
multiperiod utility

cumulative distribution function (CDF) of NPV
cumulative net cash flow (CNCF) profile
CE of NPV distribution
three-dimensional CDF of cash flow over time

EXERCISE

10.1 What approach would you use to evaluate the projects for which each of the three models in the Addendum were written?

CASE ASSIGNMENT

10.1 Stevens and Company

a Act in the role of Steve Condlin, and develop an IFPS model of Blandfield plantation over a 10-year planning horizon. What overall measure will you use in evaluating the investment?

b Make recommendations to Mr. Sandler on whether or not he should buy Blandfield, based on your model and supporting analysis.

ADDENDUM TO CHAPTER 10: Models Used in the Examples

NATURAL GAS DEVELOPMENT PROJECT

```
MODEL WELLS VERSION OF 02/03/83 12:26
1 COLUMNS 1984–2003
2 INVESTMENT = 18.8,12.5,10,0
3 INFLATION = NORRAND(.06,.04) + 1
4 ESCALATION = TRIRAND(.01,.03,.05)
5 SALES PRICE = 2.8,PREVIOUS SALES PRICE * (INFLATION + ESCALATION)
6 RESERVES = NORRAND(50,21.25),PREVIOUS RESERVES - PREVIOUS RESERVES WITHDRAWN
7 RESERVES WITHDRAWN = RESERVES * .13
8 ROYALTY = RESERVES WITHDRAWN * SALES PRICE * .185
9 EXPENSES = .55,1.15,1.7,PREVIOUS EXPENSES * INFLATION
10 NET INCOME = RESERVES WITHDRAWN * SALES PRICE - EXPENSES - ROYALTY
11 CASH FLOW AT = NET INCOME * .664
12 NET PRESENT VALUE = NPVC(CASH FLOW AT,.12,INVESTMENT)
13 NET CASH FLOW AT = CASH FLOW AT - INVESTMENT
14 CUM NET CFAT = NET CASH FLOW AT,NET CASH FLOW AT + PREVIOUS CUM NET CFAT
```

ENERGY SAVING PRODUCT

```
MODEL RSKTIM1 VERSION OF 02/03/83 12:01
2 COLUMNS 1984–1993
3 INVESTMENT = 337.4,0
4 SALES = TRIRAND(175,300,600),PREVIOUS SALES * (1+NORRANDR(.12,.09))
5 COGS = UNIRAND(.60,.65) * SALES
6 SGA = UNIRAND(60,85)
7 AFTER TAX CASH FLOW = .55 * (SALES-COGS-SGA)
8 NET PRESENT VALUE = NPVC(AFTER TAX CASH FLOW,.12,INVESTMENT)
9 NET CASH FLOW = AFTER TAX CASH FLOW - INVESTMENT
10 CUM NET CF = NET CASH FLOW,PREVIOUS CUM NET CF + NET CASH FLOW
```

(continued on next page)

GEOTHERMAL PLANT

```
MODEL GEOTHERM VERSION OF 02/03/83 13:03
1 COLUMNS 1984–2003
2 INVESTMENT = 10 FOR 3,7 FOR 2,5.5 FOR 3,0
3 ESCALATOR = NORRANDR(.07,.07)
4 FUEL PRICE = 30,MINIMUM(PREVIOUS * (1 + ESCALATOR),100)
5 CAPACITY = 0 FOR 3,.05 FOR 4,.1
6 REVENUE = CAPACITY * 10.5 * FUEL PRICE
7 OPERATING COST = .70 * REVENUE
8 STLINE DEPR(INVESTMENT,0,20,DEPRECIATION, BOOK VALUE,CUM DEPR)
9 TAXES = (REVENUE – OPERATING COST – DEPRECIATION) * .40
10 CASH FLOW AT = REVENUE – OPERATING COST – TAXES
11 NET PRESENT VALUE = NPVC(CASH FLOW AT,.12,INVESTMENT)
12 NET CASH FLOW AT = CASH FLOW AT – INVESTMENT
13 CUM NET CASH FLOW AT = NET CASH FLOW AT,PREVIOUS CUM NET CASH FLOW AT + NET
   CASH FLOW AT
```

CORPORATE AND STRATEGIC PLANNING CASES

DATA ELECTRONICS LABORATORIES

Bill Meyers, president of Data Electronics Laboratories (DEL), sat at his desk reviewing the three proposals that had been discussed at the strategic planning committee's September meeting earlier that day. The proposals had been under consideration for some time, and Bill now had to choose among the three alternatives. This was not only a key decision for the company at this time, but Bill felt that the type of decision process used would greatly influence the way a number of decisions would be made in the future.

DEL PRODUCTS

Data Electronics Laboratories, founded 30 years ago in 1949, was a leading manufacturer of computer equipment and related office machines for small- to medium-sized businesses. The company designed and manufactured computers and related peripheral devices. Products were divided into two major product lines. Computer Systems comprised automated information processing systems for the accounting and finance or operations functions of businesses. Office Automation Systems included word processing and office-of-the-future systems designed to increase productivity in various staff and other support positions.

Computer Systems

The company produced two major lines of computers, the D1000 and the D3000 series. The D1000 series, first produced in May 1974, was sold as an "unbundled" system. The customer purchased a central computer(s) and combined this with peripheral devices and application software designed to meet the specific needs of the customer. In 1977 the company introduced the D1200, a central processor designed to process data 8 to 10 times faster than the standard D1000 central processor. From 1977 to 1979 the company had continually updated the D1000 series. Two new additions were the D1000 II, a system at the low end of the product line with limited capabilities, and the D1500, which allowed up to 16 operators simultaneously to share common computer resources such as disk memory and printers.

This case was written by Michael McEneamey, Research Assistant, and Samuel E. Bodily, Associate Professor, as the basis for class discussion. Copyright 1982 by The Colgate Darden Graduate Business School Sponsors of the University of Virginia.

In March 1978, the company introduced its second line of computer systems, the D3000 series. The new system, designed to penetrate the high end of the business computer market, used multiple terminals, multiple languages, and improved programming capabilities. The system provided for up to 128 workstations and over 4.5 billion bytes of on-line disk storage.

Configurations of DEL Computer Systems, depending on the model, peripheral devices, and software chosen by the customer, ranged in price from $12,000 to over $900,000. The company produced 57 peripheral devices, such as printers, terminals, plotters, and disk and tape storage units.

Office Automation Systems

The company entered the word processing and office automation market in early 1972. Since that time DEL made several product innovations that propelled it into a leadership position in the office automation market. The company currently produced 10 basic word processing–office information systems. These ranged from a single-operator stand-alone system to systems that included a small central processor, with 1.2 billion characters of disk storage, capable of supporting up to 48 peripheral devices including work stations, fast-speed printers, graphic printers, low-speed letter-quality printers, and various telecommunications options. The peripheral devices sold with the word processing systems were basically the same units offered with the computer systems sold for business applications. Because of the similarities in the two product lines, an innovation in one line often led to new product introductions in the other.

Although the original office automation systems were designed around the use of an "intelligent" typewriter, the current system used a cathode ray tube (CRT) workstation. The operator sat at the work station, using an electronic keyboard similar to a typewriter, and typed directly onto a CRT display. The information was then edited or corrected electronically, stored on magnetic disks, and then printed when needed. The company's word processing systems sold from under $10,000 for a simple stand-alone unit and up to $250,000 for the top-of-the-line shared-logic system.

DEL's major competitors in the computer systems market included IBM, Burroughs Corp., Digital Equipment Corp., and Data General. In the office automation market, DEL competed with IBM, Xerox, Wang, and Burroughs. These markets were extemely competitive and characterized by rapid technological change. Most of DEL's competitors had substantially greater revenues and assets, larger engineering and research staffs, and larger marketing organizations. DEL maintained that its greatest competitive strengths were its concentration on a specific market segment (small- to medium-sized businesses) and the compatibility of its two product lines.

THE THREE PLANNING PROPOSALS

DEL currently was constructing a 500,000-square-foot manufacturing facility, at a cost of $24.5 million, due to be in operation by September 1980. Final plans for the use of the facility had been discussed by the planning committee during the past month. Because many of DEL's products were similar in construction and because the company was basically an assembler of parts purchased from outside vendors, plans for the use of the new facility could remain flexible up until 10 months before it opened. Ten months would be needed to order and install the specific equipment to be used. It was now 2 months before the company reached this deadline. Bill Meyers knew that he had to reach a decision soon as to which of the three alternative uses of the facility would be best for DEL.

Bill Meyers was aware that if his company did not react to changes in its environment, it would lose its competitive position rapidly. For the past 5 years the company had been growing twice as fast as the industry in general (16 percent growth in the computer industry and 30 percent growth in the office systems industry). Both the computer industry and the word processing industry were expected to maintain their respective growth rates for the next 5 to

10 years. However, even under the most optimistic scenario, Bill did not expect his company to continue to grow at rates much above the industry average. Without consistent improvements in products, DEL could expect to fall quickly behind the industry growth rate. Bill also felt pressure from trends in the cost of goods sold, which had risen steadily over the past 2 years and could reduce profit margins to an unacceptable level if no action were taken.

The finance department had developed simple financial pro forma models to analyze each of the three proposals under consideration by the planning committee. Exhibit 1 shows company performance over the period 1980 to 1985, assuming that the planning committee had taken

EXHIBIT 1 DATA ELECTRONICS LABORATORIES

Base case pro forma model

```
MODEL BASE VERSION OF 06/08/82 10:59
5   COLUMNS 1980–1985
10  *
15  * COMPUTER SYSTEMS
20  *
25  COGS RATE = .45, .46
30  GROWTH RATE = .18,.16,.14,.12,.08
35  *
40  SALES = 167419 * 1.18, PREVIOUS * (GROWTH RATE + 1)
45  COGS = COGS RATE * SALES
50  SGA = .27 * SALES
55  CORPORATE CHARGES = .1 * SALES
60  PBT = SALES – SUM(L45 THRU L55)
65  *
70  WC GROWTH RATE = GROWTH RATE * .95 + 1
75  WC = 91245 * WC GROWTH RATE, PREVIOUS * WC GROWTH RATE
80  *
85  *
90  * AUTOMATED SYSTEMS
95  *
100 GROWTH RATE1 = .45,.35,.30,.25,.25,.20
105 COGS RATE1 = .46,.47,.48
110 *
115 SALES1 = 484507 * 1.45, PREVIOUS * (GROWTH RATE1 + 1)
120 COGS1 = SALES1 * COGS RATE1
125 SGA1 = .32 * SALES1, .31 * SALES1, .30 * SALES1
130 CORPORATE CHARGES1 = .1 * SALES1
135 PBT1 = SALES1 – SUM(L120 THRU L130)
140 *
145 WC GROWTH RATE1 = GROWTH RATE1 * .95 + 1
150 WC1 = 271107 * WC GROWTH RATE1, PREVIOUS * WC GROWTH RATE1
155 *
160 CASH FLOW = PBT + PBT1 + 94968 + 287960 – (WC + WC1),
165               PBT + PBT1 – (WC + WC1) + (PREVIOUS WC + PREVIOUS WC1)
170 NET PRESENT VALUE = NPVC(CASH FLOW,.20,INVESTMENT)
175 *
180 COMPUTER MARKET GROWTH = 1.16
185 COMPUTER MARKET SALES = 2100000 * 1.16, PREVIOUS * L180
190 SOM COMPUTERS = SALES/COMPUTER MARKET SALES
195 *
200 OFFICE SYSTEMS MARKET GROWTH = 1.30
205 OFFICE SYSTEMS MARKET SALES = 3000000 * 1.3, PREVIOUS * L200
210 SOM OFFICE SYSTEMS = SALES1/OFFICE SYSTEMS MARKET SALES
215 *
220 INVESTMENT = 0
END OF MODEL
```

Output of base case model

	1980	1981	1982	1983	1984	1985
COMPUTER SYSTEMS						
COGS RATE	.4500	.4600	.4600	.4600	.4600	.460C
GROWTH RATE	.1800	.1600	.1400	.1200	.0800	.080C
SALES	197554	229163	261246	292595	316003	341283
COGS	88899	105415	120173	134594	145361	156990
SGA	53340	61874	70536	79001	85321	92147
CORPORATE CHARGES	19755	22916	26125	29260	31600	34128
PBT	35560	38958	44412	49741	53721	58018
WC GROWTH RATE	1.171	1.152	1.133	1.114	1.076	1.076
WC	106848	123089	139460	155358	167165	179870
AUTOMATED SYSTEMS						
GROWTH RATE1	.4500	.3500	.3000	.2500	.2500	.2000
COGS RATE1	.4600	.4700	.4800	.4800	.4800	.4800
SALES1	702535	948422	1232949	1541186	1926483	2311780
COGS1	323166	445759	591816	739770	924712	1109654
SGA1	224811	294011	369885	462356	577945	693534
CORPORATE CHARGES1	70254	94842	123295	154119	192648	231178
PBT1	84304	113811	147954	184942	231178	277414
WC GROWTH RATE1	1.428	1.333	1.285	1.238	1.238	1.190
WC1	387005	515684	662655	820035	1014793	1207604
CASH FLOW	8939	7848	29025	61405	78333	129916
NET PRESENT VALUE	7449	12899	29696	59309	90789	134298
COMPUTER MARKET GROWTH	1.160	1.160	1.160	1.160	1.160	1.160
COMPUTER MARKET SALES	2436000	2825760	3277882	3802343	4410717	5116432
SOM COMPUTERS	.0811	.0811	.0797	.0770	.0716	.0667
OFFICE SYSTEMS MARKET GR	1.300	1.300	1.300	1.300	1.300	1.300
OFFICE SYSTEMS MARKET SA	3900000	5070000	6591000	8568300	11138790	14480427
SOM OFFICE SYSTEMS	.1801	.1871	.1871	.1799	.1730	.1596
INVESTMENT	0	0	0	0	0	0

no action and had simply tried to maintain its current position. The cost of the new plant that is under construction is not included in the base case analysis, which serves as a reference point for evaluating the other three alternatives.

The first proposal was to produce in-house the circuit boards and many of the subassemblies used in both the computers and office systems. This was an effort to reduce manufacturing costs and develop a competitive advantage through backward integration. DEL had always assembled products from components and parts purchased from a variety of vendors, with the exception of mechanical portions of certain mass storage units (such as magnetic disk drives) which had been purchased preassembled. Having broadened its in-house design and manufacturing capabilities over the years, DEL was now in a position to manufacture many of the components it had been purchasing.

This option differs from the base case in the amount of investment needed and in the cost of goods sold. Manufacturing equipment costing $50 million would be required in addition to the cost of building the facility. After the facility is opened, the cost-of-goods-sold figure for both product lines would steadily decrease for 3 years and then level off. In addition, corporate charges would increase from 10 to 12 percent under this option (as well as under the other two proposals). This change was to compensate for increases in interest and research and devel-

opment (R & D) (the two components of corporate charges) that each of the three proposals would require. Because this first proposal offered no new products, the growth rates for both product lines would be the same as in the base case model.

The second option under consideration was to introduce a new product, the COMNET System, and to update both product lines to take advantage of this new product. The COMNET System was simply a cable network that allowed communication among "intelligent" machines, computers, word processors, and various peripheral devices such as printers with microprocessors. COMNET would increase productivity in the office by improving the coordination of office equipment. Many of DEL's current products already had telecommunications capabilities. This plan would update those products that did not. The cost of introducing these additions to the product lines would be $24.9 million in new equipment. The basic benefit of this proposal was its ability to keep growth rates in excess of industry growth until the last year of the analysis (1985) where company growth would equal industry growth.

The third option was to introduce a desktop portable computer to enhance the current line of business computers. Several companies, such as Apple, had already introduced similar products and sales were beginning to rise dramatically. Bill Meyers felt that if his company was going to compete in this market, then it had to establish its own product before its major competitors, IBM in particular, came out with comparable products. Bill Meyers considered the desktop computer to be the significant growth area of the future for small businesses and he believed the company could design a model that was particularly attractive to the business consumer given the company's experience in this market. To introduce the computer, the company would have to invest $30.2 million in equipment. The major benefit the company would derive from this strategy was a substantial increase in sales and a reduction in the cost of goods sold for computer systems and a moderate increase, over the base case, in sales in automated office systems.

EXHIBIT 2 SUMMATION OF ASSUMPTIONS USED IN PRO FORMA ANALYSIS

	1980	1981	1982	1983	1984	1985
Option 1: backward integration						
Computer Div. COGS, %	45	44	43	42	42	42
Auto Sys. Div. COGS, %	46	45	44	43	43	43
Corporate charges, %	12	12	12	12	12	12
Investment (millions of $)	74.5	—	—	—	—	—
Working capital (% of sales growth)	90	90	90	90	90	90
Option 2: introduce COMNET						
Computer Div. growth, %	18	20	18	18	17	16
Auto Sys. Div. COGS, %	46	46	47	47	47	47
Auto Sys. Div. growth, %	45	40	40	35	35	30
Corporate charges, %	12	12	12	12	12	12
Investment (millions of $)	49.4	—	—	—	—	—
Working capital (% of sales growth)	85	85	85	80	80	80
Option 3: desktop computer						
Computer Div. COGS, %	45	44	44	44	44	44
Computer Div. growth, %	18	24	24	20	20	20
Auto Sys. Div. growth, %	45	40	35	30	30	30
Corporate charges, %	12	12	12	12	12	12
Investment (millions of $)	54.7	—	—	—	—	—
Working capital (% of sales growth)	85	85	85	80	80	80

Variables not listed are the same as in the base case (see Exhibit 1).

EXHIBIT 3 OUTPUT OF PRO FORMA ANALYSIS

Computer Div. growth,[1] %	11.6	11.6	17.8	21.6
Auto. Sys. Div. growth,[1] %	27.0	27.0	36.0	33.0
Computer Div. SOM,[2] %	6.7	6.7	8.9	10.4
Auto. Sys. Div. SOM,[2] %	16.0	16.0	22.5	20.2
Industry growth,[3] %	26.3	26.3	26.3	26.3
*NPV (millions of $)[4]	134.3	179.8	52.0	74.4
*Adjusted growth,[5] %	25.0	25.0	33.8	31.2
*Adjusted SOM,[6] %	14.8	14.8	20.9	18.7
*Strategic fit	0	45	80	100

*The four attributes to be used in the analysis.
[1]Simple average for 1981 to 1985.
[2]In 1985.
[3]Average growth rate for the two markets in which the company competes adjusted for the level of sales in each market:
Industry Growth = Computer Industry Growth × [Computer Ind. Sales/(Auto. Sys. Ind. Sales + Computer Ind. Sales)] + Auto. Sys. Industry Growth × [Auto. Sys. Ind. Sales/(Computer Ind. Sales + Auto. Sys. Industry Sales)]
[4]For 1980 to 1985 using 20 percent discount rate.
[5]Average growth for the company's two divisions adjusted for each division's percentage of total sales:
Average Growth = Computer Div Growth × (Computer Div Sales/Total Sales) + Auto. Sys. Div Growth × (Auto. Sys. Div. Sales/Tot. Sales)
[6]Average SOM for the company computed the same way as average growth.

Exhibit 2 is a summary of the assumptions used by the finance department to generate the pro formas for each of the three alternatives under consideration. Only those assumptions which were different from the base case are listed in Exhibit 2. Exhibit 3 shows the results of the pro forma analysis for the base case and each of the three alternatives. The financial considerations are summarized by a net present value (NPV) figure for each alternative. Additionally, each option had been evaluated on three other attributes: growth, share of market (SOM), and the strategic fit of the option with the current company strategy.

EVALUATING THE PROPOSALS

Bill Meyers was confident that the models developed by the finance department provided accurate predictions of the benefits to DEL and saw no reason to spend time trying to improve them. But Bill and the other members of the strategic planning committee were puzzled about how to use the projections to make the choice among the proposals. They were satisfied with the near-term forecasts and realized they could not make specific financial predictions in this industry more than 4 to 5 years out. Changes in technology were rapid and many of the products being sold by the company today would be outdated in 5 years. They also realized, however, that the proposals under consideration would take the company along paths they had not explored before and that they had major strategic effects on the company that would extend beyond the 5-year time horizon. For example, a cost control program like backward integration would give them a very different competitive position than the introduction of a new personal computer product line. They did not want the analysis to be biased in favor of projects which might be easier to forecast in the short-term but which would have worse long-term prospects.

Bill did not like the company's current method of financial analysis which jammed all benefits beyond the 5 year horizon into a single number called *residual value*. For some projects (e.g., the personal computer), this number might be more significant than all of the other financial numbers combined. In addition, Bill was concerned that using a measure like NPV put too much emphasis on short-term projects or projects which had significant cash flows in the early years. Bill was concerned that this might undervalue new product introductions which were large cash users in their early years and only produced significant cash flows after the product had been accepted by the market. Instead of using one dollar figure, it made more sense to Bill to consider several measures of an alternative's long-term effect on the performance

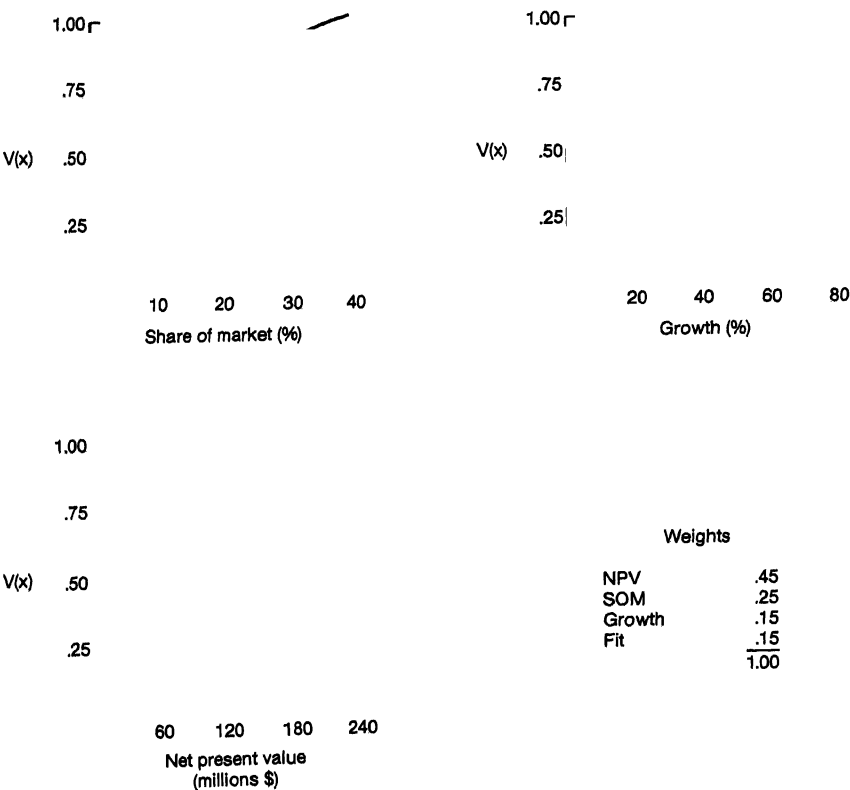
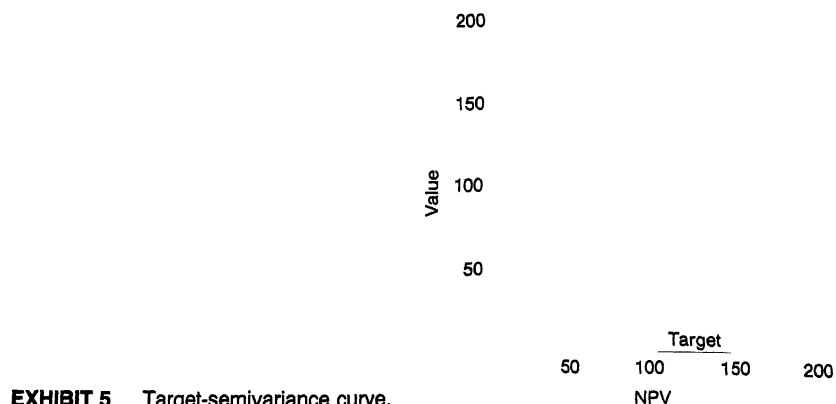
of the company. Share of market and sales growth were measures that Bill thought were good surrogates for the company's long-term competitive position. In order to use these measures, however, it would be necessary to combine them somehow with the NPV measure of benefits accruing within the 5 year period.

In addition, Bill felt that some options, for example a new product introduction, had benefits, such as improving the company image, which could not be accurately measured in financial terms alone. In an effort to evaluate subjective considerations for the three proposals, Bill had rated the alternatives using a scale from 0 to 100, on a subjective score that he labeled *strategic fit* (see Exhibit 3). This attribute encompassed all of the subjective effects an alternative would have on the company's strategic positioning, including its relationship to the definition of markets, company image, competitive strengths, etc. In his rating of strategic fit, however, Bill specifically excluded effects measured by the other attributes: SOM, growth, and 5 year NPV.

Bill now had four attributes which he could use in a model to help him evaluate the three options. Two weeks ago Bill had discussed the decision the company was facing with Margaret Dempsey, V.P. of Finance. Margaret suggested that some of the newer members of her department, recent graduates of various MBA programs, might be able to come up with some new ideas on how to evaluate the new proposals. Earlier in the day Bill had heard presentations by two members of Margaret's staff on ways to merge the various measures in comparing options.

One of the analysts suggested an additive preference model for combining the four attributes. He proposed that the company express a value curve for each attribute and then subjectively assign weights to the attributes; values could be combined in a weighted additive form. Exhibit 4 shows sample value curves that he and others in his group had assessed along with some tentative weights. The strategic fit attribute was by its nature linear and the rating was converted

EXHIBIT 4 Value curves and attribute weights. (Value curve for fit is linear, V(x) = Fit/100.)

EXHIBIT 5 Target-semivariance curve.

to a scale between 0 and 1 by dividing by 100. He had considered recommending a multiplicative combination of attributes, since it seemed to him that the attributes interacted in a complementary fashion. Yet he suggested that the company first become comfortable with an additive form before adding this complication.

The other method that had been suggested attempted to capture some of the interaction between the attributes, but without making things too complex. It was not a major change from the way the company was currently evaluating projects in that it focused primarily on NPV. The analyst proposed that the company set a target for 5 year NPV; performance below this target would be penalized while performance above the target would be rewarded. One way to create a value curve for NPV that could focus on this target would be through a target-semivariance curve, an example of which had been drawn by the analyst in Exhibit 5. The expression for writing such a curve is included in Exhibit 5; the value K is the weight on the penalty for being below the target and the below-target penalty is computed as the square of the distance below the target. The analyst had suggested a K value of .003.

The analyst also suggested that the NPV target should depend on the performance of the other three attributes. He felt that if the SOM were poor, for example, then the target on NPV should be higher. This would reflect an interest in harvesting cash flow if the long-term prospects were not promising. And prospects that had neither short-term nor long-term prospects would be avoided altogether.

Ideally, the NPV target ought to change continuously with the values of the other attributes. But for starters the analyst suggested that the company have two possible targets for NPV. The first target of $40 million would be for an aggressive strategy, i.e., the company's sales-weighted average growth rate over the 5 year period, SOM (in the fifth year), and strategic fit all exceeded some specified level, say 15 percent of SOM, industry growth rate, and a value for strategic fit above 60. The second target level, $125 million, would apply when any two of these measures fell below this specified level. The use of this preference model, he explained, would allow the company to place realistic values on NPV that reflect the important concerns of growth, SOM, and strategic fit. Later on if they wanted to be more sophisticated, they could let the target NPV vary continuously with these other attributes.

Bill Meyers had listened attentively to both presentations. He liked what both analysts had said and felt that either approach could provide insight beyond the company's current method of analysis. It occurred to him that the subjective judgments that would go into creating a preference model for the four attributes were the same kind of judgments that he usually made in coming to a final decision. But he was concerned, however, about being able to explain the model and the judgments it incorporated to his managers, in addition to feeling comfortable about it himself. And there seemed to be so many ways of structuring the model that he needed to think about getting it right. He wanted to try some models of his own for evaluating strategic options. While he was not ready to take a strict quantitative approach to the problem, he felt

that the formal preference model could help him get his thinking straight so that he could make the decision. Maybe the various models would point in the same direction.

Bill now had two problems to solve before the planning committee meeting the following week. He needed to understand better how to combine the four attributes in evaluating strategic options, and he had to decide, based on the resultant structure of the evaluation model, which of the three options to recommend to the planning committee.

UNIVERSAL PRODUCTS

Dan Gomez, president of Universal Products (UP), sat at his desk reviewing his company's performance for the first three quarters of 1981. For the first time in the company's 33-year history it appeared that sales would fall below those of the previous year. Of even greater concern was the strong competition and slow growth in two of the company's three divisions that had significantly reduced profits over the past 2 years and the first three quarters of 1981. Mr. Gomez knew that it was time to consider some serious strategic changes. It might very well be a good time for UP to acquire another company with a better outlook than UP's current divisions, something it had not done since 1966. The company's 5-year budget plan was now being reviewed by the Planning and Budget Committee and they would decide how UP should improve its performance. Besides the possibility of acquiring a new company, there were various other investment opportunities being considered in each division. Mr. Gomez expected the committee to issue the final budget plan by the end of October, just 2 weeks away.

COMPANY BACKGROUND

UP began as one of the first manufacturers of television sets in the United States. As the company grew it expanded its product line, concentrating on consumer electronics, and by 1960 the company had a full line of television, radio, and audio products for both the home and industrial markets. In the early 1960s the company expanded geographically, opening up retail outlets in several key Canadian cities, and grew by acquiring two divisions: a regional manufacturer of kitchen and laundry appliances in 1964, and a national manufacturer of retail cash registers and various sales support machines in 1966. Although the company continued to grow throughout the late 1960s and 1970s, it did not venture into any new businesses, choosing instead to concentrate its efforts on expanding the three divisions which formed its product base in 1966.

In 1981 the company was expecting to have sales of approximately $1.6 billion (see Exhibit 1), down nearly 4 percent from the company's record sales figure of $1.66 billion in 1980. Profit was expected to drop from $92.2 million in 1980 to only $72.6 million in 1981, a drop of 21.3 percent, and nearly 31 percent below the company's record $104.6 million in profit

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EXHIBIT 1 UNIVERSAL PRODUCTS 1981 FINANCIAL RESULTS (IN MILLIONS)

	Consumer electronics	Business products	Major appliances	Total
Sales	$693.8	$515.8	$396.4	$1606.0
COGS				
Fixed	212.6	49.1	70.2	331.9
Variable	301.4	207.4	234.0	742.8
Margin	179.8	259.3	92.2	531.3
Expenses				
Fixed G&A	29.1	38.0	12.6	79.7
Variable G&A	39.5	59.7	13.3	112.5
Selling	44.7	44.9	17.1	106.7
Interest	17.2	13.5	8.7	39.4
Other	14.7	49.0	6.5	70.2
PBT	34.6	53.4	34.0	122.0
Taxes	13.1	22.0	14.3	49.4
PAT	$ 21.5	$ 31.4	$ 19.7	$ 72.6
Current Assets	$362.5	$360.1	$203.7	$ 926.3
Fixed Assets	204.8	119.9	184.8	509.5
Other Assets	51.9	48.9	39.4	140.2
Total	$619.2	$528.9	$427.9	$1576.0
Current Liabilities	$244.7	$191.2	$122.8	$ 558.7
LT Debt	149.4	117.5	75.8	342.7
Equity	225.1	220.2	229.3	674.6
Total	$619.2	$528.9	$427.9	$1576.0

for 1978. Compounding this decline in performance was the outlook for relatively flat sales and declining profits, due to increased competition in the Consumer Electronics and Major Appliance divisions, over the next 5 to 10 years. Only in the third division, the business products division, was the picture reasonably attractive.

Consumer Electronics Division

The Consumer Electronics (CE) Division was formed from the company's original business. It sold television sets, radios, and audio equipment through both company-owned and private retail outlets in the United States and Canada. In 1981 the sales of this division were expected to drop 2.5 percent to $693.8 million from $711.5 million in 1980 (see Exhibit 1), and profits were expected to decline 18 percent to $21.5 million in 1981 versus $26.2 million in 1980. Even with this decline in performance, the division remained the largest with over 43 percent of UP's total sales.

It was extremely hard to make an adequate profit in the consumer electronics industry. Little growth in industry sales for several years had led to intense price competition. Margins were being squeezed as material and labor costs continued to increase. Even major improvements in manufacturing technology, like the increased use of advanced electronics, had not been able to hold the cost of production down.

The weakening of sales can be largely attributed to the nearly 100 percent penetration the industry has achieved. The market has become basically a replacement market since virtually every home in the country already owns a television and radio. The two bright spots in the industry are relatively new products, the large-screen projection televisions and home video recorders. Industry experts also expect a small resurgence in sales over the next several years as the recession ends and the interest in home entertainment combined with new home entertainment alternatives related to the boom in cable TV pushes up demand for various consumer electronic products.

Business Products Division

UP's second largest division, the Business Products (BP) Division, was the one that had undergone the most change over the past 10 years. When UP acquired the division in 1966, the industry appeared relatively mature with little technological change for the prior 20 years. Cash registers, which made up the bulk of the division's sales throughout the 1960s, and other sale support machines (i.e., adding machines for office use) were relatively simple in their production and use. However, the increased use of electronic (instead of mechanical) components and the use of computers in, or attached to, the sale support machines radically altered the industry.

Since the late 1960s, a succession of changes and improved products had resulted in steadily increasing sales. The BP Division now offered electronic terminals ranging from what is basically a large calculator attached to a cash drawer to the top-of-the-line point-of-sales (POS) support system which can include any number of very sophisticated terminals linked directly to a small computer. The larger systems were sold mainly to large national retailers and were designed with automatic scanners which electronically read the product code (the series of bars and lines on most products), increasing both the productivity and accuracy of the checkout clerk while providing data files in the computer for accounting, inventory, and marketing research. The division also sold financial terminal systems used to automate several of the more common banking functions. Some of the company's products sold in this area were drive-in 24-hour automated bank tellers, computerized terminal systems for financial transactions within the bank, and automated loan tracking and delinquent account collection systems.

The BP Division's sales were expected to increase only slightly over the previous year with 1981 sales of $515.8 million compared to $509.8 million in 1980. At the same time the profit for the division was expected to drop from $36.9 million in 1980 to $31.4 million in 1981. The division's management felt the slow sales growth in 1981 was due solely to the recession, since the bulk of sales were to retail stores which had been hit hard by the decline in the economy, and the division was expected to grow at a pretty steady pace (8 to 12 percent per year) for the next 5 years.

The Major Appliance Division

The third and smallest of UP's divisions, the Major Appliance (MA) Division, was in the industry with the poorest outlook over the next 5 years. Market penetration, like that for the CE Division, was relatively high and the bulk of the division's sales were in the replacement market. The industry was also strongly linked to the home construction industry, which was particularly hard hit by the increase in interest rates and the slowdown in the economy. Higher interest rates had also greatly reduced the availability of consumer credit and increased the cost of purchasing major appliances on credit, which was the mode of paying for the majority of replacement purchases.

The major appliance market had also faced a trend of increased consolidation. At the end of World War II there were over 300 manufacturers of major home appliances operating in the United States. In 1980 there were fewer than 20 firms still producing kitchen and laundry appliances and the market was dominated by GE, Whirlpool, White Consolidated, Maytag, Magic Chef, and Sears. Many of the consolidation moves over the past 10 years had been encouraged by the industry's unimpressive growth rates and the benefits of economies of scale in manufacturing. Of the MA Division's major products, only the microwave oven had shown a significant improvement in sales, nearly an 18 percent increase over the previous year, while most of the other products dropped between 5 to 15 percent in sales over the same period. The outlook for the next 5 years, particularly for the smaller producers like UP, was not very good. Sales in most of the product lines were expected to remain relatively flat while increased costs continued to eat away at profits.

Sales for the division were expected to drop about 11 percent in 1981 (see Exhibit 1) to

$396.4 million from nearly $445 million in 1980. Worse yet was an almost 33 percent reduction in profit from nearly $29.1 million in 1980 to an expected level of $19.7 million in 1981.

PLANNING GOALS AND ALTERNATIVES

The management at UP had been concened about these disturbing trends in the company's performance for a long time. As a result a directive had gone out to all divisions at the beginning of the planning cycle, which started in May, to consider any alternative from acquisition to disposition in order to improve the company's outlook for the 5-year budget plan.

The Planning and Budget Committee was composed of the V.P. of Finance, Susan Baker; the V.P. of Operations, Paul Knowles; the V.P. of Marketing, Bruce Jones; and Mr. Gomez. The committee had set several goals for the overall budget at the beginning of the planning cycle. The company had established a limit of spending $85 million or less on new investments in 1982 and set a goal for a positive 5-year net present value (NPV) of cash flow of $50 million above what the company could expect from the base case scenario (the company used a discount rate of 16 percent for its after-tax financial analysis). The management also expected a return on equity (ROE) of 18 percent by 1986 and hoped to achieve an average annual growth in sales in excess of 10 percent over the 5-year period. These goals were for the corporation as a whole and each division was not expected to meet all of the goals as long as the company as a whole did. Mr. Gomez felt that these goals were attainable and although he did not expect the company to turn around overnight, he felt that if the right actions were taken the company could quickly recover from the setbacks of the past couple of years.

Planning Alternatives

The management of each division had identified what it felt were the best two profit improvement alternatives to the base case (i.e., where the company makes no changes from current operations). The Planning Committee would select projects from this combined list of alternatives. The Finance Department had developed pro forma projections for the base case scenario (Exhibit 2) and had indicated what changes would be made to the base case if each of the alternatives under consideration by the Planning Commmittee were undertaken (Exhibit 3).

In the CE Division, the committee was deciding between the introduction of a new product, a home video recorder, or a series of manufacturing improvements designed to control the escalating cost of production. The home video recording market was potentially the biggest new market for the home electronics industry, and although the company was relatively late in entering the market, management felt that it could trade on brand recognition and its reputation for quality in the consumer electronics industry, to successfully penetrate the market.

There were two basic types of video players available on the market. The original and most popular type was a video cassette tape recorder. Existing products in the market offered a wide range of options from fast forward and reverse to slow motion and freeze frame. Although a tape-driven video player allowed both recording and playback options, the producers were having some problems with picture quality, particularly in the slow-motion and freeze-frame modes. The second alternative was a videodisc player which used discs similar to long-playing records. There were also two types of videodiscs available. The first type operated with a tongue-and-groove process, like a phonograph player, and the second type operated with a laser reader which could move to any place on the disc in less than a second. Although the videodisc player could not record, it offered improved picture quality over the tape device, had quicker scan options, particularly with the laser reader, and retailed for about $500 versus the $750 to $1200 for the tape recorders. The company had decided that if it was to produce a videoplayer it would manufacture a cassette tape video recorder because its ability to record *and* playback made it more popular with the consumer.

In the BP Division the committee had to decide between the option of assembling its own computers or opening up a maufacturing plant in Canada. The BP Division currently purchased

EXHIBIT 2 UNIVERSAL PRODUCTS BASE CASE PROJECTIONS BY DIVISION* 1982–1986 (IN MILLIONS)

	1982	1986	Best guess	Range
Consumer Electronics Division				
Sales	$729.6	$892.5	5.5% growth	2–8% growth
Fixed COGS	222.2	264.9	4.5% growth	3.8–5.2% growth
Variable COGS	328.3	419.5	.45,.45,.455, .455,.46 of sales (by year)	±2% of sales
Margin	179.1	208.1	—	—
Fixed G&A	30.4	36.3	4.5% growth	3.8–5.2% growth
Variable G&A	43.8	53.6	6% of sales	5–7% of sales
Selling	43.8	53.6	6% of sales	5–7% of sales
Interest	16.2	12.2	11.5% of debt	—
Other	14.6	17.9	2% of sales	1.5–2.5% sales
PBT	30.4	34.7	—	—
Taxes	11.5	13.2	38% of PBT	—
PAT	18.9	21.5	—	—
Major Appliances Division				
Sales	$402.3	$427.0	1.5% growth	−2 to 5% growth
Fixed COGS	72.7	83.4	3.5% growth	3–4% growth
Variable COGS	241.4	277.6	.6,.61,.62,.635 .65 of sales	±2.5% of sales
Margin	88.3	66.1	—	—
Fixed G&A	13.2	16.1	5% growth	4–6% growth
Variable G&A	17.4	18.5	4% of sales	3–6% of sales
Selling	21.5	22.8	5% of sales	4–7% of sales
Interest	12.8	10.1	11.5% of debt	—
Other	6.3	6.7	1.7% of sales	1–2% of sales
PBT	17.0	−8.1	—	—
Taxes	7.1	−3.4	42% of PBT	—
PAT	9.9	−4.7	—	—
Business Products Division				
Sales	$567.4	$830.7	10% growth	8–12% growth
Fixed COGS	53.5	75.6	9% growth	7.5–10% growth
Variable COGS	229.8	353.0	.41,.415,.42, .425,.43 of sales	±2% of sales
Margin	284.1	402.1	—	—
Fixed G&A	41.2	57.1	8.5% growth	7–10% growth
Variable G&A	63.4	92.8	11.5% of sales	11–13% of sales
Selling	45.4	66.5	8% of sales	7–9% of sales
Interest	12.9	10.6	11.5% of debt	—
Other	50.1	73.4	9.5% of sales	8–10% of sales
PBT	71.1	101.8	—	—
Taxes	29.1	41.7	41% of PBT	—
PAT	42.0	60.1	—	—

Assumptions

1. Approximately 60 percent of depreciation is reinvested in ordinary replacement. This is not included in the investment limit set for the Planning Committee but does influence cash flow.
2. Seventy percent of investment in new projects approved by the committee would be financed with equity.
3. New debt financing would be 20-year notes at 15 percent interest with 20 equal principal repayments at the start of each year (beginning in 1983).
4. Dividends would be about 10 percent of PAT each year.
5. Net working capital and other assets retained the same relationship to sales as in 1981. Working capital changes affected cash flow but not investment restrictions.
6. Fixed assets would increase as required by the committee's decisions. Any new fixed assets were assumed to have a 10-year life and straight-line depreciation would be used.

Depreciation Schedule*

	1981	1982	1983	1984	1985	1986
Consumer Electronics	$20.5	$18.4	$16.7	$14.9	$13.4	$12.1
Major Appliances	15.4	14.1	12.9	11.9	10.8	9.9
Business Products	15.2	13.1	11.5	10.0	8.8	7.7

Debt Repayment Schedule†

	1981	1982	1983	1984	1985	1986
Consumer Electronics	$8.5	$8.5	$8.5	$8.5	$9.0	$9.0
Major Appliances	6.0	6.0	6.0	6.0	6.0	5.5
Business Products	5.1	5.1	5.1	5.1	5.1	5.1

*The depreciation schedule was only for those assets on the books and did not include the reinvested depreciation which the company expected to make each year.

†Debt repayments were made in January of each year and interest was paid on the remaining balance at the end of each year. The average interest rate on the company's long-term debt was 11.5 percent.

EXHIBIT 3 ANALYSIS OF ALTERNATIVES UNDER CONSIDERATION BY THE PLANNING COMMITTEE FOR THE 1982–1986 BUDGET

Consumer Electronics Division

I. Introduce the Video Recorder

The Finance Department felt that the introduction of a tape-driven video recorder would have the following effects on the base case pro forma projections for the division:

1. Sales would most likely grow at a rate of 8.5% per year and would range between 7 to 10%.
2. Fixed cost of goods sold would grow at about 6.0% from the 1981 base (± 1.5%).
3. Variable cost of goods sold would be .5% less than in the base case.
4. Investment would be $18.5 million in 1982 and $5.5 million in each of the years between 1983 to 1986.

II. Manufacturing Improvements

The introduction of new machinery in order to reduce costs would have the following effects on the base case scenario:

1. Sales would most likely grow at 6.5% per year and range between 4 to 8% due to a reduction in unit sales prices.
2. Fixed cost of goods sold would grow at about 5.0% from the 1981 base (± 1%).
3. Variable cost of goods sold would be 42% of sales in 1982 and increase by about one-half of 1% per year.
4. Investment would be $25.5 million in 1982 with no further investment required.

EXHIBIT 3 ANALYSIS OF ALTERNATIVES UNDER CONSIDERATION BY THE PLANNING COMMITTEE FOR THE 1982–1986 BUDGET (*continued*)

Business Products Division

I. Open a New Plant in Canada

The opening of a new manufacturing plant in Canada would have the following effects on the base case scenario:

1. Sales for the division would most likely grow at 15% per year and range between 12 to 20%.
2. Fixed cost of goods sold would grow at about 10.5% per year from the 1981 base (± 2%).
3. Variable cost of goods sold would be about 1% less than in the base case.
4. Selling expense would increase about 1% of sales over the base case.
5. Variable G&A would increase about one-half of 1% over the base case scenario.
6. Investment would be $25.5 million in 1982 and $8.0 million in 1983 to 1986.

II. Assemble Computers in House

The assembly of computers used in the BP Division by NAE would have the following effect on the division base case performance:

1. Growth in sales would most likely be 11% per year and would range between 9.5 to 13%.
2. Fixed cost of goods sold would grow about 10% per year over the 1981 base (± 1.5%).
3. Variable cost of goods sold for the division would be about 39% of sales in 1982, 39.5% in 1983, and 40% for 1984 to 1986.
4. Investment would be $12.5 million in 1982 with a $2.5 million investment in each of the following 4 years.

Major Appliances Division

I. Sell MA Division and Buy New Company

The acquisition of a new company accompanied by the sale of the current MA Division would have the following results:

1. Would require an investment above the income received for selling the company's current MA Division of $57.5 million in 1982 and 12.5 for each year between 1983 and 1986.
2. Would produce the following financial results:

	1982	1983–1986 Best Guess	Range
Sales	$365.8	12% growth	10–14% growth
Fixed COGS	51.2	11% growth	9–13% growth
Variable COGS	212.1	.575,.58,.585	± 2% of sales
Margin	102.4	—	—
Fixed G&A	7.8	10% growth	8.5–12% growth
Variable G&A	23.2	6% of sales	4–9% of sales
Selling	11.6	3% of sales	2–4% of sales
Other	22.0	6% of sales	4–7% of sales
PBT	37.9	—	—
Taxes	17.1	45% of PBT	—
PAT	20.8	—	—
Current Assets	$243.9	Retain same relationship to sales	
Fixed Assets	121.6	60% of depreciation is reinvested	
Other Assets	18.6	Retain same relationship to sales	
Total	$384.1		
Current Liabilities	$128.3	Retain same relationship to sales	
LT Debt	51.2	Retain same relationship to equity	
Equity	204.6	Retain same relationship to debt	
Total	$384.1		

II. Manufacturing Improvements
The introduction of new cost-saving equipment in current plants would have the following effects on the division's base case results:

1. Sales would most likely grow at 4% per year and would range between 1 to 7%.
2. Fixed cost of goods sold would grow at about 4% from the 1981 base (± 1%).
3. Variable cost of goods sold for the division would be about 55.5% of sales in 1982 and increase about 1% per year.
4. Investment would be $11.5 million in 1982 and $3.5 million in each of the next 4 years.

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all of the computers it sold. UP had been approached by a Japanese electronics firm that had offered to sell UP all of the components necessary to assemble its own computers in house. This proposal would require the BP Division to construct a new assembly plant to perform this new operation.

A major alternative for the BP Division was to open a plant in Canada. Although UP had been selling its products in Canada since the early 1960s, it still produced everything in the United States. Management hoped to strengthen its position in the business products industry in Canada by constructing a plant which would produce products specifically for the Canadian market.

In the MA Division, management had narrowed the alternatives down to automation of existing manufacturing plants or the sale of the division and the acquisition of a company which specialized in electronic equipment for the military and other government uses. The first option would mean a major overhaul of the division's manufacturing facilities. If the company introduced state-of-the-art equipment at its plants, then it could reverse the trend of increasing costs in production by holding down labor costs through automation. If UP did not reduce costs at its manufacturing plants then it was felt the only choice was to sell the division. Increasing pressures on prices and rising costs would soon make the industry unattractive.

One of the larger manufacturers of kitchen and laundry appliances had approached UP about the prospect of selling the MA Division to them. They had even gone so far as to mention price and conditions of purchase; outside experts had appraised the price as fair and advised that the purchaser would be able to go ahead quickly with the deal. At the same time UP had been following a company which produced electronic monitoring and sensory equipment for the government. The company's management would not object to a UP takeover and there would be little difficulty completing the purchase. In addition, the increased emphasis on military spending by the government made this industry attractive and the potential for improvement in sales and profits seemed very good. Exhibit 3 shows the net investment required for the sale of the BP Division and the purchase of this new company, including improvements that would be made in the new company.

Although there were two alternatives for each division (in addition to the base case status quo option) there was no requirement that a change be made in each division. In fact the Planning and Budget Committee could opt for both new alternatives or neither in any of the three divisions. The intent was to improve the overall company performance, not to meet specific standards in all three divisions.

DECISION MAKING AT UP

Mr. Gomez was concerned with how the committee should go about making the planning decisions. Like this year, the Planning Committee had always set several goals for the planning process, i.e., NPV, ROE, and growth goals, but then approved projects based on what they felt was "right" for the company. Only after the selection had been made did they examine performance on the goals. Mr. Gomez knew that if the performance varied greatly from the goals, the committee would reconsider its decisions, but the lack of any specific use of the goals in helping the committee to make the planning decisions disturbed Mr. Gomez.

Mr. Gomez wished that it were possible to make a decision based on several attributes (i.e., NPV, ROE, and growth) even if the attributes gave conflicting signals when the performance of alternatives was scored. From his perspective, UP espoused interest in several objectives, while actually making its decisions almost wholly on a single attribute. In all of the planning sessions he had attended while working at the company, the long-term growth of the company and the effect of a decision on investors (who focused on ROE) were always discussed, but the final decision seemed to come down to the ranking of the alternatives on the 5-year NPV figures that the financial analysts provided. He felt that the growth and ROE measures ought to play a bigger role; in his mind they were as important, at least when combined, as the 5-year NPV.

At the last Planning Committee meeting Mr. Gomiez had brought up the prospect of using the three different measures (5-year NPV, ROE at the end of 5-years, and annual growth over 5 years) explicitly in the planning decisions at UP. He proposed that they find a way to maximize overall performance where overall performance merged the three measures.

Reaction to the proposal by other members of the committee was mixed. Susan Baker and Bruce Jones supported the idea, although both felt a little uncomfortable with the concept because they really did not know how it would be done, and Paul Knowles strongly opposed it.

"I think its fine for the committee to establish goals for all of these objectives at the beginning of the planning cycle as we have been doing," Mr. Knowles had responded. "It forces the managers to look at the effect of their proposals on different aspects of our business. At the same time the only really important measure is the bottom line, how our company is going to perform, and the NPV figure we generate from pro forma analysis is as close as we are going to get to an accurate measure of this. If we combine NPV with these other figures, then what do we have, not dollars, but apples and oranges mixed together."

"I don't think there is all that much mystery to using more than one objective in evaluating projects," Susan Baker responded. "I certainly think the idea has some merit and we ought to consider it a little more closely before rejecting it. I agree that putting ROE, growth, and NPV together gives us something that seems a little foreign. But we may come up with something that we could be very comfortable with after working with it for awhile. There is no question that we are making major decisions here and we shouldn't make a hasty decision because the analysis is easy. If we fall back to just NPV, we are ignoring major long-term impacts. In fact, of the three attributes I think that ROE is far and away the most important. After all, it is the stockholder's interests we are supposed to be looking after and ROE is the best measure we have for doing this."

"I agree with Susan," replied Bruce Jones. "The proposal certainly warrants further consideration. If we are going to establish goals for several different objectives, and that is something the company has done in the planning process for as far back as I can remember, then I don't see any reason for not using those different objectives in making our decisions. What you're saying, Paul, is that basically those other objectives don't really mean anything and I disagree. I think that a project's performance on ROE and growth measures can be as important to the company as its NPV performance. The NPV figure we use is strictly short-term and won't tell us how the project is going to affect the performance of the company in the time frame beyond our 5-year planning horizon. I think the different measures tell us different things and, therefore, using all of them in making a decision might help improve our recommendations.

"I think a measure like sales growth gives us some reliable indication of how the project will do in the future. High sales growth tells us that the project is likely to perform well in the future while low sales growth tells us that the project probably does not have any significant added benefits beyond the time frame we have projected. We might also consider some other measures that we haven't used before, for example, share of market or other industry versus company performance measures, as a way of getting an even better picture of the project's performance beyond the planning horizon we currently use."

Mr. Gomez knew that he could not just tell them how to carry out multiobjective planning

in their company. "I think all along we have tried to blend the objectives we established at the beginning of the planning cycle in making our decisions. I would just like to give our process some formal structure. In the end I'll make the final decision on the projects, but I want each of you to sign off on the choice. Therefore we must agree on the process for making the selection. That means we have to understand better what the process is. I'll be careful not to let the formal analysis run away with us; we only want it to give us some help. I would like all of you to at least think about this between now and next Friday and be ready to discuss it."

There were now three problems facing Mr. Gomez. First, he knew that above all else the committee had to produce in 2 weeks a series of recommendations on what projects to approve for the coming 5-year budget plan. This was a crucial time for UP and the recommendations made by the committee would greatly influence the future success of the company.

Second, Mr. Gomez knew that the committee had to decide on how they were going to evaluate projects, this year and in the future. Should the company use a multiobjective optimization or not? Mr. Gomez knew that a switch to the new form of analysis would probably not be very beneficial if everyone on the committee did not agree with it or if the committee had problems understanding the concept. This was too important a decision to force on the committee if the other members were not willing to go along with it. And he knew the decision couldn't be too sensitive to subjective value judgments or it would create problems.

Finally, Mr. Gomez recognized that it was his problem to bring the members of the committee to agreement. But he could see that even if they agreed on a multiobjective analysis, they might not agree on the weighting of attributes. Paul Knowles would undoubtedly put all or almost all of the weight on NPV. He thought the others might be able to agree on a weighting of the three measures. Thus he needed to think about how the choices would be affected by differing weightings on the objectives.

The tasks were not likely to be easy, but if it helped improve the decision making at UP, then Mr. Gomez felt it was worth it.

FINANCE CASES

JOHN DENISON

As he stared into the fireplace at his ski lodge, John Denison suddenly stood up and hurled his brandy snifter at the flames. He had just learned that room, board, tuition, and expenses at an obscure New England liberal arts college for his daughter would cost $9,000 a year, nearly 20 percent of John's income. A quarter-century earlier, when John entered business school after the Korean War, he paid $900 tuition. This reminded John once again how his income was not maintaining the standard of living he had accustomed his daughters to. More than ever, John knew he needed to change his investment policies. Only one thing John would not change: his determination never to work again!

John's new neighbor in the compound walked in, attracted by the sound of shattering glass. "Cabin fever gotcha?" Lee asked, pointing at the snow 2 feet deep and still falling. In response John poured out the tale of his rise and fall. Having read every magazine in the condominium during the storm, Lee encouraged John to keep talking with an occasional "uhuh" or "I hear you."

It was 20 years ago last summer that John, a new MBA with nothing to offer but his labor and $10,000 borrowed from his new bride's family, became operations officer and partner in a phonograph record manufacturer in the Shenandoah Valley. As the music market exploded in the sixties, so did the company's profits. After 7 years of constant expansion and 80-hour work weeks, the stockholders sold out to a go-go conglomerate. John received $700,000 for his share of the business. Promising his family he would spend time with them rather than on a job, John retired to what he thought would be the leisured life of a private investor. Lee groaned enviously as he swept the shards of crystal away from the fireplace.

But the go-go era did not last. John put money into several venture capital investments like the one that had brought him such wealth in the 1960s. He lost $300,000. Gradually John became more conservative, until by 1978 his portfolio was balanced almost equally between "blue-chip" stocks and blue chip bonds. Last year this portfolio yielded 10 percent on a total value of $400,000.

Now, at age 48, John realized he could no longer afford investment losses without severely

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compromising his lifestyle. Since inflation was at 9 percent, John foresaw that his current portfolio would also force him to lower his standard of living eventually. Conservative investment policies were returning him virtually nothing in constant dollars. Yet venture capital investments had made John rich once and had almost made him poor again several times.

John felt his mind turning from past problems to possible solutions. In response to Lee's gentle question, he admitted he had thought about investments that returned more than the securities markets with less risk than equity in new businesses. As he focused on the business problem, John's voice took on the public-speaking style he had learned in presentations to investment bankers. Two factors, he informed Lee, would limit his search. One, he would divert no more than $35,000 at this time to more risky investments. If his current portfolio was not returning him anything in excess of inflation, it was not returning him much less, either. So John did not want to put at risk any larger sum until he felt more comfortable about estimating the risks. Two, John was looking for a return significantly above the rate of inflation. He added 6 percent real return to the 9 percent inflation current in early 1979 and arrived at a target rate of 15 percent. John knew that this figure could be refined, but he recalled his business school training to make up a number and get on with it.

As the conversation moved from complaints to solutions, Lee questioned John more closely. "Well, have you thought about ways to invest the $35,000 you've set aside?" John admitted he had been looking around the last few months. "Did you come up with anything?" Lee pressed. It occurred to John that Lee was awfully businesslike for a psychologist. But replying to the overt question, John said he had been struck by a comment he heard on the TV program *Wall Street Week*, that the most desirable assets were things with a finite supply and increasing demand. Income-producing real estate and fossil fuels, according to this analyst, were better hedges against inflation than collectibles or securities. They could print more stocks and bonds as easily as the Bureau of Printing and Engraving makes more money.

Lee asked if John had researched any real estate or energy investments. "Hey," John protested, "I thought you consulted as a behavioral psychologist. Why are you so interested in portfolio management?"

"For one thing, there's 28 inches of snow on the road now and it's still coming down. I don't think I can even get down the mountain tonight, let alone get back to Charlottesville. For another thing, my undergraduate major was systems engineering. I still get involved occasionally with projects that involve risk assessment and computer analysis. Get me an unbroken glass, and let's work on your problem."

"You see," said Lee accepting a cordial glass, "a lot more can be done with computers to analyze investments now than when you went to Bschool or were in business. I'd like to try modeling your investments with some programs they've got on our time-sharing service. The programs can help you get a handle on the range of risks. I'll need some estimates for the initial cost, residual value, lifetime, and cash flow." Lee was starting to get excited. "What are these investment opportunities you have, anyway?"

"Well," John spread papers over the coffee table, "I don't know a thing about oil and gas, but I really like what I understand about this Elk River partnership to drill for natural gas in West Virginia. Do you realize that the probability of coming up with a dry hole in the Big Injun sands formation is only 3 percent?" The question was only rhetorical, and John rushed on. "The other possibility is a small office building—it's filled now with doctors and dentists. I figure they'll be the last people to miss a rent payment, even in a depression. I've done a little dabbling in real estate, and I feel more comfortable with the building. But I'm not at all sure the building will return more than the gas wells. All the risks are so different, the investments seem nearly incommensurate."

"A lot of this modeling and risk assessment was invented in the petroleum industry. Let's look at that first, since that area gives you more trouble." John's file on the Elk River partnership included the prospectus, some magazine articles, and notes of an interview John had with the developer. They talked as Lee read, and this is what he learned.

THE ELK RIVER PARTNERSHIP

A new partnership was being formed to drill developmental wells in Ritchie County, West Virginia. Although the general partner was relatively new to gas drilling, he had spent many years in West Virginia and had built up numerous contacts with other drillers. Over the previous 2 years he had put together nine partnerships financing one well each. His success was enviable. Exhibit 1 shows the expected results of the nine wells he had drilled and their lifetime earnings based on his own estimates of gas reserves.

Oil and natural gas were first found in West Virginia during the nineteenth century. Early exploration and production took place on an extensive scale. When far larger fields were discovered in Texas, the activity in West Virginia slowed down. The fields in Texas and elsewhere produced gas and oil more cheaply and drove the price down. As a result, wells with only average reserves no longer provided adequate returns for the high risks involved in drilling.

In recent years, as increasing demand for oil and gas pulled the price up, drilling in West Virginia has resumed on an intensive scale. Most of this drilling is termed "developmental,"

EXHIBIT 1 ESTIMATES OF LIFETIME EARNINGS BY ELK RIVER ON NINE CURRENT WELLS–1978 (IN THOUSANDS)*

Year	Million cubic feet	Price†	Gross income	Costs	Net income	Net income discounted at 15%
1	256.2	2.01	516	114	402	349
2	206.2	2.17	448	99	349	264
3	177.0	2.35	415	92	323	212
4	156.2	2.54	396	88	308	176
5	140.1	2.74	384	86	298	148
6	127.0	2.96	375	84	291	125
7	115.9	3.19	370	83	287	108
8	106.3	3.45	366	82	284	92
9	97.8	3.73	364	82	282	80
10	90.2	4.02	362	82	280	69
11	83.3	4.35	361	82	279	60
12	77.0	4.69	361	82	279	52
13	71.2	5.07	361	83	278	45
14	65.9	5.48	360	83	277	39
15	60.9	5.91	360	83	277	34
Total	1831.2		5805	1311	4494	1856 = PV

Well	Initial estimate of reserves, MMCF	
1	180	
2	200	
3	180	
4	230	
5	0	
6	220	
7	200	
8	250	
9	350	Mean = 201
		S.D. = 86

*Numbers may not add due to rounding.
†Before passage of Natural Gas Policy Act of 1978.

because drillers knew gas was in the ground. Extensive activity before 1920 produced a wealth of data on the sizes of reserves, their depths, strata, and geographical locations. Using this information carefully, the driller could avoid many of the uncertainties involved in wildcat, or "exploratory," drilling.

Expenses and Revenues

The promoter proposed a larger partnership than before to engage a contractor to drill 10 wells. Each well had an initial drilling cost of $45,000. After the well had been drilled, core samples and the drilling log (which recorded the rock formation, the pressure, and porosity) would be analyzed to determine where a fracture should be made. An explosive charge detonated at the depth where gas or oil was expected created the fracture. Fracturing and completion costs amounted to $37,500 per well. Thus each well cost $82,500.

The total drilling cost of $825,000 for the package of 10 wells was to be borne by the limited partners, in exchange for 75 percent of the net revenues from the sale of any gas or oil recovered. Before the limited partners received any money, these charges were incurred:

1 Royalty payment of 18.5 percent of gross revenues to the owner of the land.

2 A pumping and service charge of approximately $1800 per well per year payable to the well operator. This would increase at the rate of inflation.

3 Local taxes of approximately 4 percent per year of net income, assessed quarterly.

4 One-fourth of the net revenues to the general partner after royalties and expenses in exchange for procuring the real estate leases, forming and managing the venture, and assuming the legal obligations of a general partner.

The rest of the income would go directly to the limited partners, to be taxed to them as personal income. The promoter offered 25 limited partnership interests in units of $33,000.

Reserves

John had decided to visit West Virginia to see some wells in operation, and he dropped in on the driller while he was there. He learned that local drillers had developed several rules of thumb from a hundred years' drilling experience in this area. Any one gas well could vary widely from these guidelines. Over a number of wells, however, the rules of thumb were reliable. The driller told John that developmental wells in Ritchie County had a 10 percent chance of being dry. A well was dry if it had reserves less than 150 million cubic feet (MMCF), the minimum for commercial exploitation. According to the driller, a reserve of 200 MMCF was the most likely size for a Ritchie County deposit. The chances of hitting reserves less than 150 MMCF or more than 300 MMCF were both about 1 in 10.

Life

Natural gas wells in Ritchie County generally had productive lives averaging 20 years, although this depended on whether the revenues generated covered the fixed pumping charges. Most wells passed the 15-year mark and none outlasted their twenty-fifth year. Wells did not produce evenly. As the reserves depleted, the pressure dropped and caused a slowdown in production. Other difficulties in production were sand blockage, excessive line pressure, and hard winters which interfered with overhauling and servicing the wellhead equipment. John did not believe that these difficulties altered the estimate of recoverable reserves or lifetime, however.

A rule of thumb for Ritchie County wells (sharing similar geographical characteristics) was that 50 percent of the useful reserves were produced in the first 5 years, 75 percent in the first 10, and 82.5 percent by the fifteenth year.

Price

Another uncertainty was the price per thousand cubic feet (MCF) of natural gas in the years to come. The Natural Gas Policy Act, passed by Congress in October 1978, had brought the "stripper" price of natural gas up to $2.80/MCF (before adjusting for Btu content) with provision for annual price increases at the rate of inflation plus 3 percent. John was aware that this price could change suddenly if Congress moved toward deregulation at a different pace. Decontrol of natural gas prices by 1985 was the subject of intense congressional and industrial debate. Political uncertainties were difficult to quantify. Moreover, the shifts by consumers from one source of energy to another created problems in the marketplace, where price is ultimately determined. In addition, the state of the economy in future years could have a major impact on the price of gas since the permissible price rise was tied to inflation.

Overall, John was reasonably certain that the price of gas would not come down because the United States' high energy needs could not be filled by domestic gas production.

Other Considerations

One drawback to purchasing a limited partnership share was the lack of provision in the partnership agreement for resale of the share. In other words, there was no market for selling out if unforeseen events made it necessary for John to raise cash quickly. The most that John could expect would be his initial investment, if the general partner and all other limited partners could agree on what to do with the equity share.

There were several tax advantages to the investment. Under the tax reform act of 1976, John could deduct 50 percent of the drilling costs immediately against other income. For John, this one-time reduction in taxable income effectively reduced the cost of the partnership share by $7425.

Another advantage was the right to deduct 22 percent of the net income to the limited partners as a depletion allowance. Since John was currently in a 45 percent marginal tax bracket he liked these deductions.

Capital gains tax law as of 1979 provided that 60 percent of any capital gain was excluded from taxation, while the remaining 40 percent was taxed as ordinary income at the applicable marginal rate. For John's 45 percent bracket this exclusion was the equivalent of an 18 percent tax on a capital gain.

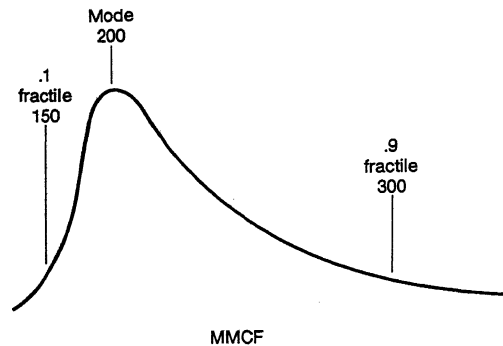
Initial Analysis

"That's a pile of numbers," Lee sighed. "I think we've got most of what we need. I hate to put you off until the next time I get to the Computer Center to run all the nums."

John smiled. "You know, my Christmas present from my wife was an Exidy Sorcerer computer/smart terminal with modem. We celebrated the holidays here on the mountain, and the thing is still in the den. If you remember your account number with that time-sharing company you talked about, you can probably get the cheaper nighttime rates."

"You sly devil, you really are gonna get some work out of me tonight. How about a refill?" Lee started his revision of the figures with the expenses.

Because the service charge was the only annual fixed charge, and because it was likely to increase at a different rate than the price of natural gas, Lee looked for a way of factoring it out of the annual cash flows. This would make his calculations simpler. Since there was no uncertainty about this charge as long as the wells are producing, Lee thought he might discount the annual service charges at his personal target rate of return adjusted for inflation, then add the present value sum to the initial investment of the limited partners. This would group all the payments to the driller into one lump sum.

From his research Lee recognized the probability distribution of reserves as a lognormal curve, as shown.

John had a book on the way oil companies handled such decisions. He read that the distribution on hydrocarbon reserves had a natural tendency to form a lognormal shape, because "nature made more small fields than giant ones."[1] Lee understood that a lognormal distribution was one whose logarithms fitted a normal curve. The observations themselves did not fit a normal curve. Lee determined that the adjusted distribution for one well with all the reserves too small for commercial development set equal to zero had a mean of 207 MMCF and a standard deviation of 87.25 MMCF. Exhibit 2 charts these distributions and their derivation. For 10 wells the probability distribution of recoverable reserves had a mean of 2070 MMCF and a standard deviation approximated by $\sqrt{10} \times 87.25 = 276$ MMCF.

Lee found the parameters of the lognormal curve using the fact that the logs[2] of the known values fitted a normal curve. Since the log of the median must be halfway between the logs of the .1 and .9 fractiles, he calculated the median of the lognormal curve for the distribution of reserves in nature thus:

$$\frac{\ln 150 + \ln 300}{2} = \ln \text{ of median}$$

$$\frac{5.011 + 5.704}{2} = 5.357$$

Taking the antilog, median 212 = exp (5.357)

To find the standard deviation of the logs, Lee used a normal table from an old textbook. He read that a .9 fractile was 1.2815 standard deviations above the mean, so

$$5.357 + (1.2815 \times \text{S.D.}) = 5.704$$

$$\text{S.D.} = \frac{5.704 - 5.357}{1.2815}$$

$$\text{S.D.} = .27 \text{ (in log terms)}$$

Equipped with the mean and standard deviation in log terms and the normal table, Lee could determine the log of any fractile and then convert to the actual value. For example, the log of the .75 fractile is $5.357 + (.67 \times .27) = 5.538$ so the .75 fractile is 254 MMCF.

Next Lee needed a distribution for the commercially available reserves only. But there was no mathematical formula to adjust the lognormal distribution for the worthless reserves smaller

[1] R.E. Megill, *Introduction to Risk Analysis* (Petroleum Publishing Co., Tulsa, 1977) p. 50.

[2] The logarithm of a number is the exponent to which a base must be raised to obtain the original number. Thus the base 10 log of 100 equals 2, because $10^2 = 100$. If no base is specified, logs are calculated using the natural constant e = 2.718 (often written ln). To continue the example, ln 100 = 4.6, because $2.718^{4.6} = 100$. The ln(x) button on many calculators gives the natural log of the number in the display. Logs have many useful properties. Adding the logs of numbers is equivalent to multiplying the numbers, for instance.

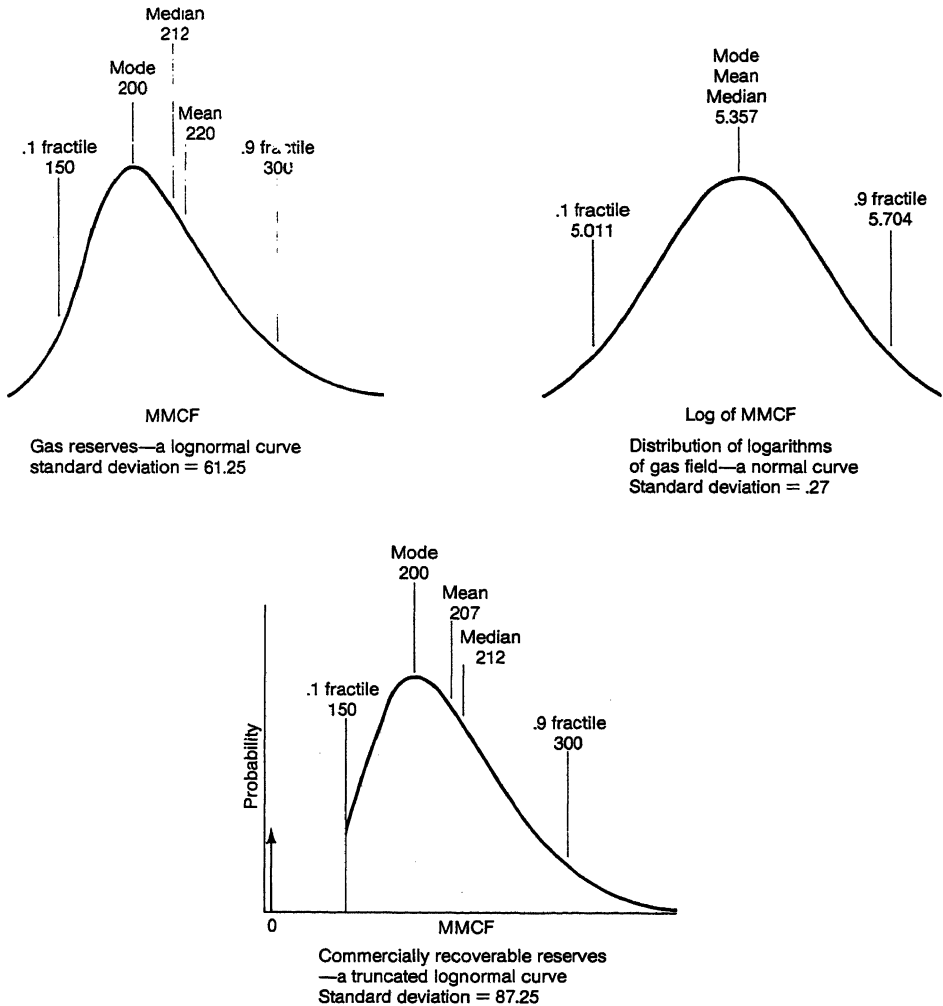



EXHIBIT 2 Probability distribution of gas reserves in Ritchie County, West Virginia.

than 150 MMCF. Therefore, he used the parameters derived above to simulate the distribution. Setting all the randomly generated reserves less than 150 MMCF equal to 0, he found a new mean of 207 MMCF and a new standard deviation of 87.25 for the truncated distribution. Lee was confident of his simulation; the home computer had caught in an endless loop and generated over 3700 trials before Lee pulled its plug.

Lee recognized the decline in production as an exponential decay rate. He could now fit a curve to all five known points of the declining production curve. The equation Percent of reserves remaining = $f(t) = e^{-.1386t} \times 100$ fitted the first 10 years well and understated only the reserves remaining after 15 years. This said, in effect, that 12.9 percent of the remaining reserves were pumped out each year. Lee had figured out how to use the trend line analysis on the home computer with logarithms. From there it was a simple step to see that the percent of the original reserves withdrawn in any one year t was $f(t - 1) - f(t)$ as shown in Exhibit 3.

The price trend also reminded Lee of an exponential time series. This one, however, was increasing rather than declining. The congressionally allowed 3 percent real price increase seemed like a reasonable estimate, so Lee combined it with several different rates of inflation and the depletion rate. Exhibit 4 charts these life-cycle profiles for gross sales revenue of the partnership using the expected reserves.

Converting those total revenue figures to the share of the income John would receive after

EXHIBIT 3 RESERVES WITHDRAWN IN A YEAR*

Year	Original reserves withdrawn in year, %	Original reserves remaining at end of year, %
1	12.9	87.1
2	11.3	75.8
3	9.8	66.0
4	8.5	57.4
5	7.4	50.0
6	6.5	43.5
7	5.6	37.9
8	4.9	33.0
9	4.3	28.7
10	3.7	25.0
11	3.2	21.8
12	2.8	18.9
13	2.5	16.5
14	2.1	14.4
15	1.9	12.5
16	1.6	10.9
17	1.4	9.5
18	1.2	8.2
19	1.1	7.2
20	.9	6.3
21	.8	5.4
22	.7	4.7
23	.6	4.1
24	.5	3.6
25	.5	3.1

*Using the formula remaining reserves = $e^{-.1386t}$ where t is the year.

taxes and discounting each year should provide a present value that John could compare with the cost of the gas wells. Before Lee called the time-sharing service, however, he asked John about the other choice, so he could put both sets of figures into the computer simulation model.

819 CHERRY HILL DRIVE

"It's in a small group of commercial buildings, but not right in the downtown area. That's why I have some difficulty comparing it to other real estate. There aren't many properties similar to 819 Cherry Hill Drive," John explained.

Lee broke in to say that the contrast with a fungible good like gas, where a lot of data on identical commodities had been collected, was what made the comparison interesting to him. He realized each piece of land is unique, but Lee hoped enough information existed to construct at least judgmental probability distributions on the real estate.

The Property

The building itself was in a medium to poor state of repair but was located on a main road through the city. The current annual income of $10,440 came from leasing all six office units. The turnover among tenants was very low because the rent was below average. John felt that an annual increase in excess of 3 percent might cause some of the tenants to think of leaving, unless he poured a lot of money into renovations.

To maintain the building in its present condition cost about $2000 annually, including utilities. This sum had been rising at a rate of 6 percent a year and showed no signs of slowing down. The local property taxes had been stable at $1215 annually, and depreciation would come close to $3125 a year.

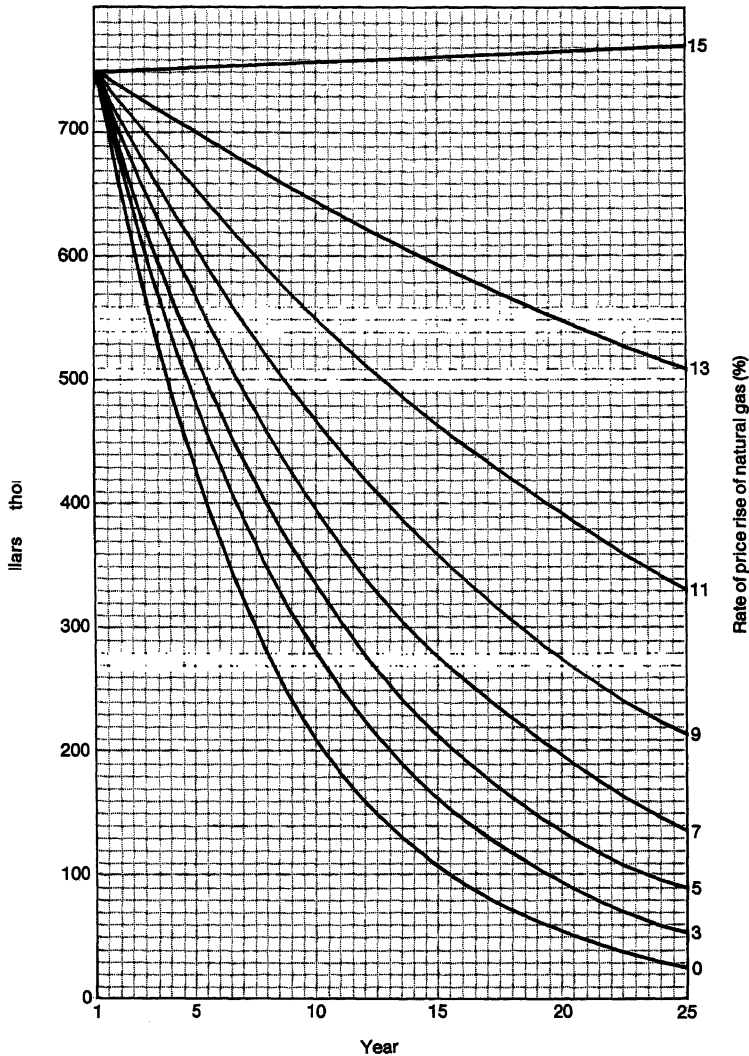



EXHIBIT 4 Annual sales revenue for natural gas reservoirs of 2070 MMCF (assuming annual depletion rate of 12.9 percent).

An appraiser whom John had hired indicated that the property had a market value around $72,000. Although the asking price was about $80,000, John heard from a friend that the seller had not received any offers and would undoubtedly be interested in an offer close to $72,000. The figure $72,500 was mentioned.

John had asked both his real estate broker and the appraiser for their estimates of the appreciation of commercial property compared to inflation in Charlottesville. Neither would answer until he pressed them and promised them it was "off the record." They guessed about 3 percent above the general trend of prices but warned there was no guarantee on any particular property.

John handed over Exhibit 5, the sheet of notes he had taken in the Deed Room of the City Court. "The problem is that the neighboring commercial properties are an electrician's supply house, a convenience store, and a drive-in photo chain store—nothing like a medical building," John explained. "So I'm dubious about the reliability of these numbers. Still, it gives you a rough idea how the value of the properties in the area varies from the long-term trend."

John's banker had agreed to finance up to 90 percent of the purchase price at a rate of 9.5 percent over 10 years. The banker explained to John that the state of repair, while adequate for the current use of the building, prevented him from offering a longer-term loan. John was

EXHIBIT 5 PROPERTY TRANSACTIONS

		412 Berkshire	426 Berkshire	827 Cherry Hill Dr.	819 Cherry Hill Dr.*
Recorded purchase prices:	1961	18.9			
	1966			85	
	1969		48		
	1970				55
	1973		105		56
	1976			127	
	1978			130	
Total sq. feet (000)		10.5	20	10.2	11.5

Growth rates for property values

Property	Years	Rate	Years	Rate	Years	Rate
827 Cherry Hill Dr.	10	4.1	2	1.2	12	3.6
819 Cherry Hill Dr.	3	.6	5	5.3	8	3.5
426 Berkshire	4	21.7				

*Mr. Denison was interested in this property, appraised at $72,000 in 1979 with likely sales price $72,500.

EXHIBIT 6 AMORTIZATION SCHEDULE BASED ON A PURCHASE PRICE OF $72,500, AN INTEREST RATE OF 9.5%, AND A 10-YEAR TERM

	90% debt, $7250 down payment			70% debt, $21,750 down payment			50% debt, $36,250 down payment		
Year	Interest	Principal	Remaining balance	Interest	Principal	Remaining balance	Interest	Principal	Remaining balance
1	6199	4193	61,057	4821	3262	47,488	3444	2330	33,920
2	5800	4592	56,465	4511	3571	43,917	3222	2551	31,369
3	5364	5028	51,437	4172	3911	40,006	2980	2793	28,576
4	4887	5506	45,931	3801	4282	35,724	2715	3059	25,517
5	4363	6029	39,902	3394	4689	31,035	2424	3349	22,168
6	3791	6601	33,301	2948	5134	25,901	2106	3667	18,501
7	3164	7228	26,073	2461	5622	20,279	1758	4016	14,485
8	2477	7915	18,158	1926	6156	14,123	1376	4397	10,088
9	1725	8667	9,491	1342	6741	7,382	958	4815	5,273
10	902	9491	0	701	7382	0	501	5273	0

EXHIBIT 7
PRO FORMA INCOME STATEMENTS AND CASH FLOWS FROM 819 CHERRY HILL DR

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
90% debt financing										
Income statement										
Revenues	10440	10753	11076	11408	11750	12103	12466	12840	13225	13622
Interest	6199	5800	5364	4887	4363	3791	3164	2477	1725	902
Maintenance & utilities*	2000	2120	2247	2382	2525	2676	2837	3007	3188	3379
Real estate taxes	1215	1215	1215	1215	1215	1215	1215	1215	1215	1215
Income before depreciation	1026	1618	2250	2924	3647	4421	5250	6141	7097	8126
Depreciation	3125	3125	3125	3125	3125	3125	3125	3125	3125	3125
Profit before tax	(2099)	(1507)	(875)	(201)	522	1296	2125	3016	3972	5001
Taxes at 45% or refund	945	678	394	90	(235)	(583)	(956)	(1357)	(1787)	(2250)
Profit after tax	(1154)	(829)	(481)	(111)	287	713	1169	1659	2185	2751
Cash flow										
Cash after tax before deprec.	1971	2296	2644	3014	3412	3838	4294	4784	5310	5876
Less principal repayment	4193	4592	5028	5506	6029	6601	7228	7915	8667	9491
Net cash flow	(2222)	(2296)	(2384)	(2492)	(2617)	(2763)	(2934)	(3131)	(3357)	(3615)
70% debt financing										
Income statement										
Change in interest from 90%	1378	1289	1192	1086	969	843	703	551	383	201
PBT	(721)	(218)	317	885	1491	2139	2828	3567	4355	5202
Tax at 45% or refund	324	98	(143)	(398)	(671)	(963)	(1273)	(1605)	(1960)	(2341)
PAT	(397)	(120)	174	487	820	1176	1555	1962	2395	2861
Cash flow										
Cash after tax before deprec.	2728	3005	3299	3612	3945	4301	4680	5087	5520	5986
Less principal repayment	3262	3571	3911	4282	4689	5134	5622	6156	6741	7382
Net cash flow	(534)	(566)	(612)	(670)	(744)	(833)	(942)	(1069)	(1221)	(1396)
50% debt financing										
Income statement										
Change in interest from 90%	2755	2578	2384	2172	1939	1685	1406	1101	767	401
PBT	656	1071	1509	1971	2461	2981	3531	4117	4739	5402
Tax at 45%	(295)	(482)	(679)	(887)	(1107)	(1341)	(1589)	(1853)	(2132)	(2431)
PAT	361	589	830	1084	1354	1640	1942	2264	2607	2971
Cash flow										
Cash after tax before deprec.	3486	3714	3955	4209	4479	4765	5067	5389	5732	6096
Less principal repayment	2330	2551	2793	3059	3349	3667	4016	4397	4815	5273
Net cash flow	1156	1163	1162	1150	1130	1098	1051	992	917	823

*Increase at 6% a year.

EXHIBIT 8 PROJECTED RESIDUAL VALUES OF 819 CHERRY HILL DR.*

Rate of property appreciation, %	Debt %	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
	90	$ 29,786	$ 35,842	$ 42,490	$ 49,842	$ 57,947	$ 66,876
0	70	38,653	43,224	48,284	53,877	60,056	66,875
	50	47,520	50,624	54,078	57,912	62,165	66,875
	90	39,254	47,360	56,156	65,701	76,065	87,321
3	70	48,121	54,760	61,950	69,736	78,174	87,321
	50	56,988	62,160	67,744	73,771	80,283	87,321
	90	46,210	56,043	66,692	78,227	90,723	104,263
5	70	55,077	63,443	72,486	82,262	92,832	104,263
	50	63,944	70,843	78,820	86,297	94,941	104,263
	90	53,717	65,592	78,503	92,538	107,793	124,372
7	70	62,584	72,992	84,297	96,573	109,902	124,372
	50	71,451	80,392	90,091	100,608	112,011	124,372
	90	61,807	76,078	91,716	108,850	127,616	148,165
9	70	70,674	83,478	97,510	112,885	129,725	148,165
	50	79,541	90,878	103,304	116,920	131,834	148,165
	90	70,512	87,570	106,467	127,397	150,572	176,229
11	70	79,379	94,970	112,261	131,432	152,681	176,229
	50	88,246	102,370	118,055	135,467	154,790	176,229
	90	79,868	100,146	122,902	148,437	177,087	209,232
13	70	88,735	107,546	128,696	152,472	179,196	209,232
	50	97,602	114,946	134,490	156,507	181,305	209,232
	90	89,911	113,885	141,178	172,251	207,634	247,933
15	70	98,778	121,285	146,972	176,286	209,743	247,933
	50	107,645	128,685	152,766	180,321	211,852	247,933

*Residual values (RV) are derived from the formula RV = SP − .18CG − LB
where SP = selling price = $72{,}500(1 + A)^t$
A = rate of appreciation of resale price = inflation plus 3%
t = year of resale
CG = capital gain = SP − BV
BV = book value = 72,500 − 3125t
LB = outstanding balance on loan from Exhibit 6

not particularly worried by this as he was only considering owning the building for around 7 years. His real estate broker told John that the market for commercial real estate was not highly liquid in Charlottesville. If he wanted to sell the property for a high price, he must be prepared to hold out for some time. On the other hand, John did not see much point in holding onto this property after the tax-shielding effects of the interest payments expired.

"I developed some pro formas somewhere here," John was shuffling through the piles of papers on the coffee table. "Using loan levels of 90, 70, and 50 percent, I derived these amortization tables," handing over Exhibit 6, "which went into these cash flows," he passed to Lee Exhibit 7, "and these estimates of residual value for several possible rates of appreciation on the property." He flourished Exhibit 8.

"These numbers cover most of the inputs we need for the real estate, if you'll just tell me which of these are the likely ones, and the extremes possible. But first, why do these cash flow figures show losses with the 70 and 90 percent debt?" Lee asked.

"I'm using the rental income to pay off the mortgage, but the rent doesn't cover all the mortgage payments. So I have to add more of my own money after the down payment."

As Lee gathered up his notes and began dialing the computer number, he glanced over at the case writer who had fallen asleep by the fire. "Too bad he will miss the fun part," he muttered under his breath.

UNICRON (A)

Humanity's fascination with creating a robot goes at least as far back as the thirteenth century when St. Albertus Magnus, a German philosopher, was said to have spent 30 years constructing a servant of deceptively human appearance out of metal, wood, wax, and leather. The creature allegedly would open the door and greet visitors, engaging them in polite conversation. Frankenstein, Hal the computer from *2001,* and *Star Wars'* R2D2 are more recent cinematic versions of this age-old fantasy.

To Greg Willet, robots were not a fantasy. His future and that of his company, Unicron, depended upon the success of the robot industry. Greg, a financial analyst with the Robotics Division of Unicron, had worked on the Model CN2 project (the division's first industrial robot with vision) for the past month after transferring from the Energy Conversion Division. He had seen the CN2 perform complex tasks and was amazed. But, could the project be a financial success? It was Mr. Willet's responsibility to present, in three days, the CN2 project's 5-year financial plan to the New Product Review Committee (NPRC). He had just received from pricing and forecasting the information needed to prepare his reports. This was his first exposure to senior management so Greg wanted to make as good an impression as his robot friend had. He knew that for the NPRC he would need to have ready, at the drop of a hat, a response for any question about the CN2's financial prospects.

ROBOTS

Webster's defines robot as "a machine of human form that performs the mechanical functions of a human being but lacks emotions and sensitivity." The term *robot* comes from the Czech word for forced labor and was invented by Karel Capek, a 20th-century writer. Robots currently working in an industrial setting suggest a redefinition as they resemble lobsters more often than humans.

Industrial robots (IRs) are reprogrammable manipulators of basically two types: those which can vary the speed of the tool control point (TCP) and stop anywhere in its range of motion (servo-controlled) and those whose TCP moves with uniform speed and stops only at the end points of each axis (non-servo-controlled). There were in 1981 3000 IRs working in American

Case prepared by William Long, Research Assistant, and Samuel E. Bodily, Associate Professor, as a basis for class discussion rather than to demonstrate effective or ineffective administrative practice. Copyright 1981 by The Colgate Darden Graduate Business School Sponsors of the University of Virginia.

factories. Because they perform hazardous and mundane tasks more reliably and less expensively than their human counterparts with higher product quality, robots receive rave reviews from their employers. Industry analysts expect an annual growth rate of 40 percent for the next several years. Increased robot usage has been launched by improvements in technology and wage inflation. Microprocessors (minute computers that serve as the robot's brain) have decreased in cost, keeping the hourly cost of operating a robot under $5. The cost of a factory worker, including fringe benefits, was more than $15 per hour in 1981.

The issue of international competition has added force to the sweeping wave of automation. Many economists tie Japan's emerging competitive advantage in the steel, auto, and electronics industries to their productivity gains and high product quality, both of which can be increased through automation techniques such as robots. Japan has three times as many robots as the United States.

While it is clear the robotics industry is poised for significant growth, several issues cloud the horizon. The industry structure would change drastically if large computer manufacturers such as Texas Instruments and Digital Equipment Corp. entered the market. If so, it is likely that prices would be significantly reduced, boosting demand to, perhaps, 200,000 units per year by 1990. Robots with sensory capabilities (touch, sight, etc.) are expected to be the next generation of robots. The entry of sensory robots that see and touch would considerably augment industry demand but exacerbate labor relation problems. To date robots have taken over dangerous and monotonous jobs and, in general, displaced workers have been retrained for other jobs. Organized labor has complained little. Rapid expansion of the robot industry will likely put thousands of workers out of jobs. For example, Westinghouse has embarked on an automation program that could replace half of its 37,000 factory workers with robots. According to James S. Albus, head of the robotics research lab at the National Bureau of Standards, "The human race is now poised on the brink of a new industrial revolution that will at least equal, if not far exceed, the first Industrial Revolution in its impact on mankind."

THE CN2 ROBOT

Unicron's model CN2 represented a significant breakthrough in robot technology as it could perform rudimentary analysis of what it "saw" and make necessary mechanical adjustments. The robot was thought to have a large market for material-handling applications because it could pick parts from a disarrayed bin. Prior to this time, parts had to be delivered to robots in a precise manner. This was often expensive and time consuming. It was felt that CN2 technology could, in the future, be used in part inspection and monitoring.

As financial analyst, Greg Willet was responsible for the financial evaluation of new products (see Exhibit 1 for the positioning of his job and those with whom he worked in the corporate structure). He worked closely with the project administrator whose job it was to nurse the CN2 project through a series of stages that culminated in market introduction. The project administrator kept marketing and engineering in touch to ensure that designers could and would design a product that met the customer's needs.

EXHIBIT 1 Corporate structure.

EXHIBIT 2 VOLUME FORECAST*

Selling price, $	Sales, units				
	1982	1983	1984	1985	1986
70,000	150	218	315	457	663
80,000	120	174	252	366	530
90,000	100	145	210	305	442

*Volumes for years 1983 to 1986 were computed from initial year (1982) forecasts assuming growth of 45 percent annually.

Consequently product modifications were routine. Once the product was well defined, the administrator made initial selling price assumptions and worked with corporate forecasting to develop volume forecasts for each assumed price. The official company forecast (Exhibit 2) was the result of lengthy negotiations between the project's administrator and the forecaster.

Product cost figures had been developed for each forecasted volume. Product costs were of two varieties, direct and indirect. Direct product cost was projected by a division cost estimator and included material, labor, overhead, and development. Exhibit 3 shows projections of these costs. Corporate practice was to treat the manufacturing costs as variable with the quantity manufactured.

Indirect costs greatly concerned financial analysts in general, and Greg Willet in particular. The CN2 project would be charged cost allocations and operating expenses according to a schedule (Exhibit 4) prepared by corporate finance. Corporate policy stated that current products should fund the research and development of future products in addition to providing for the company's growth and profitability. These allocations were based on three characteristics of a project: revenue ($.27 on the dollar), manufacturing costs (an additional 20 percent above actual costs) and development costs (20 percent extra tacked on). As a result, allocated costs often accounted for over 50 percent of a project's total costs (see Exhibit 4). Greg Willet had accepted the cost allocation schedule as a fact of life at Unicron although he frequently voiced the opinion

EXHIBIT 3 COST ESTIMATES, 1981. (MFG. QUANTITY = 100)*

Manufacturing costs:	
Material	$1,900,000
Labor	200,000
Overhead	120,000
Material handling	80,000
Supplies & equipment	30,000
Insurance	25,000
Fringe benefits	110,000
Depreciation	90,000
Building & grounds	145,000
Total mfg. costs	2,700,000*
Total mfg. costs per unit	27,000
Annual development costs:	
1/5 of development costs to date	$ 200,000†
Sustaining engineering	200,000†
Market research	80,000†
Product development engineering	120,000†
Total development expense	600,000†

*Mfg. costs based on a production quantity of 100 units.
†This is an annual cost for each of the 5 years 1982–1986.

EXHIBIT 4 INDIRECT COSTS* CN2—1981

Allocated costs:	
Revenue allocations (27% of sales)	2,430,000
Mfg. allocations (20% mfg. costs)	540,000
Development allocations (20% development costs)	120,000
Total allocated costs	3,090,000
Operating expenses:	
Selling	60,000
Marketing	90,000
Administrative	150,000
Total operating expenses	300,000

*Allocated cost percentages are constant throughout the project's life. Operating expenses will increase over time at the same rate as the growth in the market. A sales price of $90,000 and manufacturing quantity of 100 was assumed.

that a project should be evaluated on an incremental rather than a full-cost basis. He argued that by including only those revenues and expenses that would be avoided if the product were not introduced, a better evaluation of a project's true effect on the corporation could be made. As it was, he barely had time to complete the evaluation using current company methods and would leave the problem of developing better methods to some other time.

The CN2 project's final step before market introduction was a review by the NPRC. In preparation Greg was to construct a 5-year pro forma income statement, primarily to see net income after taxes (current tax rate was 46 percent) and evaluate the return on sales (ROS) using the aforementioned forecasts. With that information, he was to recommend a selling price. The typical ROS in the firm was about 14 percent.

One major uncertainty nagged at Mr. Willet as he laid out his spread sheet. Sales volume was forecast to grow at the industry growth rate. Analyst's predictions of that rate varied widely, between 30 and 60 percent, with an average of 45 percent annually. The NPRC was known for grilling financial analysts about alternative scenarios. For this reason and, in order to make a more well-informed selling price recommendation, Greg intended to prepare an analysis for each growth rate and each of the three price/volume estimates.

GENERAL CASES

CALIFORNIA OIL COMPANY

Carl Shimer, Research and Development Director for California Oil Company (COC), had been studying the proposed construction of a supertanker port and pipeline. The new facility would supply COC's Richmond refinery in the San Francisco Bay area. The port would consist of a single-point mooring 2 or 3 miles from shore to unload supertankers. Submarine pipelines would take the oil into a shore-based pumping station where it would enter the pipeline to the Richmond refinery. After a preliminary screening, four sites had been selected for more detailed evaluation: Moss Landing, Estero Bay, Port Hueneme, and Oso Flaco Dunes. Shimer had to recommend a site to COC's Research and Development Committee.

The major considerations in evaluating the sites were economic, political, and environmental. These were then refined and expanded into ten criteria (see Table 1). The importance of each criterion was discussed at length at a committee meeting, after which Shimer received the following memo from his assistant:

TABLE 1 CALIFORNIA OIL COMPANY TEN CRITERIA SELECTED FOR PROPOSED SUPERTANKER PORT EVALUATION

Economic	Facilities
	Port characteristics
	Location
	Initial cost
	Annual cost
	Possibilities for future development
Political	Attitude of local populace
	Attitude of local politicians
Environmental	Environmental impact from operation
	Environmental impact from placement of facilities

Adapted from Elwood S. Buffa and James S. Dyer, *Management Science Operations Research* (New York: John Wiley & Sons, Inc., 1977). Source: Unpublished report by G. Hill, A. Kokin, and S. Nukes. Reproduced with permission.

TABLE 2
CALIFORNIA OIL COMPANY ASSESSMENT OF PORT SITES

	Alternatives			
Criteria	**Moss Landing**	**Estero Bay**	**Port Hueneme**	**Oso Flaco Dunes**
Attitude of local politicians	Possibly opposed	Possibly favorable	Favorable	Possibly favorable
Initial cost	$40 million less than Estero Bay	The cost-base location	$60 million more than Estero Bay	$5 million more than Estero Bay (estimate)
Annual cost	$2 million per year less than Estero Bay	Base location	$5 million more than Estero Bay	Near cost of base location
Location	Close to Richmond, farther from Elk Hills than base location	Base location	90 miles farther from Richmond than base location	Central location
Possibilities for future development	Area already populated	Rolling terrain will hamper large expansion	Navy interference	Area available, subject to local politicians
Port characteristics	Fair	Good	Excellent	Good
Attitude of local populace	Possible opposition	Vocal opposition	Little effect on population	Little effect on population
Environmental impact from operation/accident	High impact: area is sandy to marshy, possibly difficult to clean up; possible long-term effects	High impact: tourism and fishing industry will be seriously affected; marshy area and rocky coastline extremely difficult to clean up; possible long-term damage to bird sanctuary and oyster beds	Minimal impact: area sandy; easy cleanup; area already industrialized	Minimal impact: area sandy; easy cleanup
Facilities	No	Some	No	No
Environmental impact from placement of facilities	Tank farm highly visible	Tank farm hidden; major restructure of existing creek	Tank farm visible (no nearby population)	Tank farm visible (no nearby population)

TO: Mr. C. Shimer, Research and Development Director
FROM: Mr. D. Klopp, Assistant Director of Research and Development

SUBJECT: Tanker Port Selection

At Friday's Meeting (April 2), it was decided that ten criteria be used for evaluating promising sites. The ten criteria were ranked in order of importance. The list developed on Friday places the "attitude of local politicians" as the *most* important factor and "environmental impact from placement of facilities" as the *least* important factor. I have enclosed the ranked criteria with comparative site descriptions for your information.

DATE: April 5, 1978 SIGNED: D. Klopp

With this information, Shimer pulled out his notes from the committee meeting which held evaluations of each of the sites broken down by the ten criteria. He placed the criteria in the order suggested by the committee (see Table 2). Shimer examined each criterion and decided the "best" and "worst" situation possible under the circumstances. For example, the "best" outcome for the criterion "attitude of local politicians" would be a favorable vote assured, and the "worst" outcome would be an unlikely favorable vote (see Table 3).

For each criterion, Shimer assigned the "worst" situation a value of 0 and the "best" situation a value of 1. He then hoped to assign values between 0 and 1 for each site characteristic, weighing them by the relative importance of each of the ten criteria to achieve an overall evaluation of each site (see Figure 1). He anticipated this method would help differentiate

TABLE 3 CALIFORNIA OIL COMPANY REFERENCE PORT DESCRIPTIONS

Criteria	"Worst" value for each criterion	"Best" value for each criterion
Attitude of local politicians	Favorable vote unlikely	Favorable vote assured
Initial cost (Estro Bay cost as base)	$60 million above base	$60 million below base
Annual cost (Estero Bay cost as base)	$5 million above base	$5 million below base
Location (Estero Bay as base)	Near Los Angeles with poor access to the San Joaquin Valley and Richmond	Between the Elk Hills oil field and San Francisco, but closer to Elk Hills with easy pipeline access to San Joaquin Valley
Possibilities for future development	No future development or expansion possible after initial part is completed	No limit on future growth or expansion of facilities
Port characteristics	Very rough seas and more than 4 miles from shore	Calm seas and 1 mile from shore
Attitude of local populace	Large, strong, vocal, and effective opposition	Small, weak, and ineffective opposition
Environmental impact from operation/accident	Oil spill would seriously disrupt the community and harm wildlife; extreme danger due to proximity to military operations or other industry	Oil spill could be cleaned up relatively swiftly with no serious effect
Facilities	No facilities to support supertanker operations	All facilities completed for supertanker port operations
Environmental impact from placement of facilities	Extreme blight on the area and interference with the natural environment	No major adverse effects from placement of facilities

FIGURE 1 CALIFORNIA OIL COMPANY WORK SHEET

Criterion	Alternatives			
	Moss Landing	**Estero Bay**	**Port Hueneme**	**Oso Flaco Dunes**
Attitude of Local Politicians				
Initial Cost				
Annual Cost				
Location				
Possibilities for Future Development				
Port Characteristics				
Attitude of Local Populace				
Environmental Impact of Operations and Accidents				
Facilities				
Environmental Impact from Placement of Facilities				

between sites rated favorably on high-priority criteria from those with good ratings on low-priority items. At the moment, he favored Moss Landing because it was superior for the criteria ranked second, third, and fourth. Specifically, Moss Landing was:

1 The cheapest to build.
2 The cheapest to operate.
3 The closest to Richmond.

WHIRLPOOL RESEARCH AND ENGINEERING DIVISION (A)

In the summer of 1975 Dr. Gale Cutler, Director of Corporate Research for Whirlpool Corporation, was beginning to prepare the annual research and funds allocation plan for the coming year. The question in Gale's mind was whether the methods that had been used to allocate funds could be improved.

WHIRLPOOL AND THE RESEARCH AND ENGINEERING DIVISION

Whirlpool manufactured home appliances such as clothes washers, refrigerators, air conditioners, and vacuum cleaners. Annual sales were expected to reach $1.5 billion in 1975. Over 50 percent of these sales were made to Sears and Roebuck, which sold Whirlpool appliances under Sears' own brand names.

The company was organized into two major product groups and one corporate group, as shown in Exhibit 1. Each product group was responsible for its own research, product design, and manufacturing engineering. To supplement these activities, the Research and Engineering (R&E) Division, a part of the corporate group, provided basic research and development (R&D) and engineering support for the company as a whole.

The division had six clearly defined "mission areas," so-termed by the R&E division "charter."

1 *Current product R&D* Improving the designs of current products with respect to features, materials, safety, etc. (a complement to product engineering work done elsewhere in the corporation).

2 *Manufacturing process R&D* Both developing new manufacturing processes and improving existing ones in order to increase quality, reduce cost, reduce energy consumption, etc.

3 *New business opportunities* Developing new products and services for the corporation.

4 *Technological research* Monitoring and developing new technologies to further the corporation's competitive position.

5 *Continuing support* Providing the corporation with centralized services of a technical nature.

This case was prepared by Jeffry L. McNair, Research Assistant, and Samuel E. Bodily, Assistant Professor, as a basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative decision.
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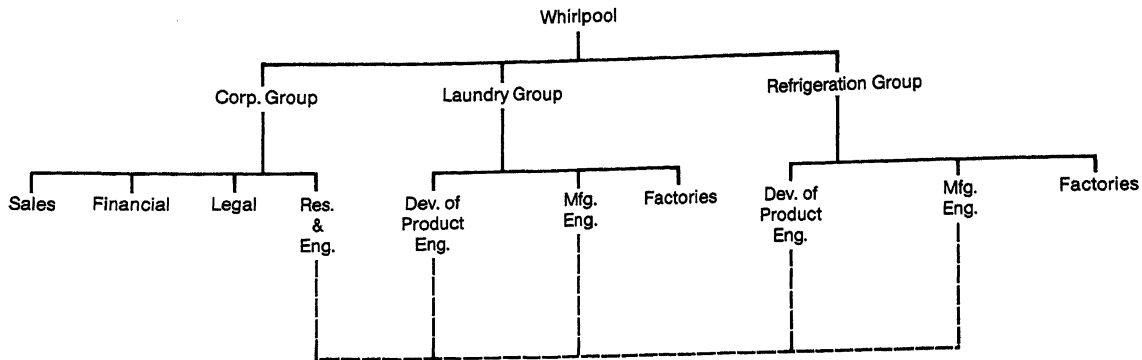


EXHIBIT 1 General organization of corporation. (Source: corporate records)

6 *Other* Achieving objectives with respect to continuing organizational development, satisfaction of people needs, and good corporate citizenship.

Although these missions remained constant, the relative emphasis that the division placed on each mission area fluctuated from year to year. (See Exhibit 2 for the division's organization chart.)

As Exhibit 1 shows, the R&E Division had no direct control over the R&D and engineering activities of the two product groups. Total corporate expenditures on R&D approached $20 million annually, but only about $7 million was controlled by the R&E Division. In theory, there was much interplay between the corporate budgeting process which determined the size

EXHIBIT 2 Organization of corporate Research and Engineering Division. (Source: corporate records)

Corp. Vice President of Research & Engineering E. A. Baillif

Corp. Engineering
- Corp. Adv. Dev.
- Home Econ.
- Product Eng. Mfg. Eng.
  - Refrig. Group: Evansville, Ft. Smith, Danville
  - Laundry Group: St. Joseph, Marion, Clyde, Findlay
  - St. Paul
  - Heil Quaker, Thomas Int'l., Inglis

Director of Corp. Research G. Cutler
- Science Res. Dept.
- Elec. Sys. Res. Dept.
- Mech. Sys. Res. Dept.
- Info. Center Dept.

Corp. Mfg. Res.

Spec. Projects

Controller

Personnel

Env. Control

Prod. Safety

Director of Tech. Planning G. Stoll
- Tech. Forecasting Dept.
- Indust. Des. Dept.
- Product Res. Dept.

of the R&E budget pie and the R&E Division's allocation process which sliced the pie. In recent years, however, the total allotment to R&E had remained about the same, when measured in constant dollars.

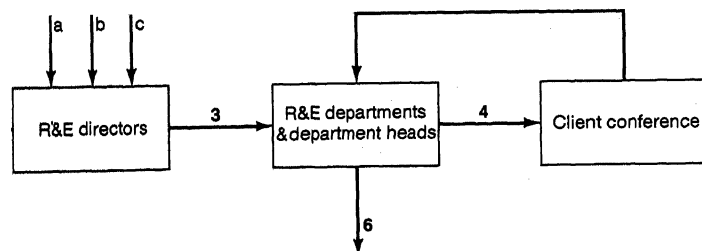
R&E PLANNING

Gale Cutler's immediate problem was to decide how to divide the total 1976 R&E budget among the six mission areas, although this decision would be only an intermediate stage in the R&E planning process. These allocations of funds to mission areas would then be further divided among research programs, many of which continued from year to year. Finally, specific projects within each program would be funded.

Annual R&E planning began in a committee of senior corporate executives that formulatd corporate objectives for the coming year. (See Exhibit 3 for a diagram of the planning process.) From these objectives and from their own knowledge of previous funding levels, R&E directors could determine the likely size of the R&E budget. (Whirlpool's president did set an upper limit on the R&E budget.) The R&E directors would then forecast the support needs of the marketing groups at Sears and Whirlpool, and of the design and production elements of the refrigeration and laundry groups at Whirlpool. Finally, the technology forecasting department submitted its assessments of recent technological and ecological developments and of their likely effects upon Whirlpool's business.

Using their understanding of the enduring missions of the R&E division and of the recent changes in corporate objectives, client needs, and external factors, the R&E directors would begin to prepare the annual research plan. At this time Dr. Gale Cutler and others, such as Gerhard Stoll, Director of Technical Planning, had to agree on the relative importance of conflicting missions and corporate demands. They then prepared a tentative plan allocating research funds to the six mission areas.

The tentative plan then passed down to the heads of the research departments. Each department head prepared a list of specific proposed research programs and projects. Typically, the total cost of the projects proposed by all departments exceeded the total division budget for the year. Because the mission areas to which funds were allocated were broadly defined, each department normally had projects in more than one mission area. Department heads prepared reports showing how their projects fit the various mission categories, as well as how this distribution of projects fit the emphasis given to each mission by the R&E directors.

EXHIBIT 3 The Research and Engineering Division annual research planning process. (Source: article on Whirlpool's R&D planning by G. Cutler.)

1. The 6 stated missions of the R&E Division remain constant from year to year.
2. Annual/continuous inputs from various sources:
   (a) corp. planning group ("corp objective")
   (b) client functional areas—marketing, engineering, etc. ("client needs")
   (c) technical forecasting dept. report ("outside factors")
3. A general (rough) allocation of emphasis/funds to 6 mission areas for the year in question.
4. Framework from #3 (above) filled in with specific research project proposals.
5. Detailed evaluation of each of #4's projects by prospective clients.
6. Operating research plans for the year at research department level.

After reviewing these reports, the R&E directors held a client conference with senior representatives of client engineering, marketing, customer service, and quality control groups. Clients were invited to evaluate the projects being considered in terms of their urgency, importance to specific functional areas, and importance to the company as a whole. The R&E directors used these evaluations to determine final fund allocations to each project. These plans were returned to the R&E department heads, who prepared the final departmental research plans, which often included lists of alternate projects to be undertaken if additional money became available.

The corporate vice president for R&E then approved the year's budget and its distribution to missions and programs. Individual projects and changes were approved by members of his staff. As the final step in the planning cycle, the vice president reviewed the annual plan—its projects, operating expenses, and labor requirements—with his own superior, the president of Whirlpool.

THE DIRECTORS' ALLOCATION PROBLEM

Each year Gale grappled with two key problems in allocating funds to mission areas. The first problem was deciding how to orchestrate the division's competing aims, as articulated by the six mission areas of the R&E Division. Gale naturally wanted the best possible performance in each area, but funds were limited. The problem was particularly cumbersome because the annual budget could be divided among the six areas in an infinite number of ways. Gale envied managers who simply had to select one of a handful of alternatives for a single activity rather than to choose one of an infinite range of potential levels for several activities.

His second problem was that R&D performance could not be easily measured or predicted. The division's work was far removed from final sales, so its performance could not be measured based on changes in the bottom line. Further, it was not known in advance how much time and money would be needed to obtain a desired result if that result could be obtained at all. On the other hand, research programs sometimes brought entirely unexpected benefits, often to mission areas other than that which the program officially served. And because they had no quantitative means of measuring performance, the directors were often unable to communicate the reasons for their allocation preferences. Each director relied on intuition and experience, but with no precise means of communication such intangibles could be easily mistaken for prejudice and favoritism.

A few years earlier Gale had tried one method of quantitative measurement. Department heads and clients had evaluated proposed projects on several performance scales. The R&E directors had then used the total scores to select projects, thus eliminating the intermediate step of allocating funds to the separate mission areas. However, R&E department heads had objected to this approach because they had believed it to be their professional prerogative to select individual projects. They had felt that the R&E directors were less familiar with the projects and thus not as qualified to make final decisions about their worth. The R&E directors themselves had soon realized that this scoring system was inadequate, since both department heads and clients naturally tended to rate their own projects highly. Further, the scoring method had not guaranteed that the research would fulfill the company's true needs in the long run. Thus, the current system was adopted. Individual projects were selected by R&E department heads, while the R&E directors provided guidance by allocating funds to the mission areas.

Gale saw no answer to the question of how to approach the allocation problem more systematically than in the past. It appeared that the 1976 allocation would again be based on the unaided intuition and experience of Gale and his associates.

WHIRLPOOL RESEARCH AND ENGINEERING DIVISION (B)

In July of 1975 Dr. Gale Cutler and the other directors of the Research and Engineering (R&E) Division of Whirlpool Corporation were preparing a tentative plan to allocate the division's 1976 research funds among the division's six mission areas. When this plan was complete, it would pass down to the division's department heads, who would prepare lists of specific research projects that would draw upon these allocated funds. After the division's clients evaluated both the allocations and the proposed projects, the R&E directors would juggle funds among the various projects and mission areas. They looked for a final allocation plan that would maximize the benefits that the R&E division delivered to the company.

Gale was concerned that these allocation plans relied exclusively on the directors' vague, intuitive judgments. At lunch one day he mentioned this worry to a member of his staff, Dr. Gerry Eisenbrandt. Gerry informed him that a doctoral candidate in industrial engineering from one of the state schools was working with the product engineering department on a similar problem.

"Don Keefer has some ideas about methods of quantifying some of the soft issues in resource allocation decisions. We've found that his methods have helped us deal with uncertainties and conflicting objectives. When he finishes his project, he will be making a presentation to us anyway, since R&E has supported his work. You could ask him then if he would be interested in testing his methods further on your problem. Or, if you like, I'll give you his phone number now."

After obtaining the phone number, Gale returned to his office and called Don Keefer. Don sounded interested and promised to send some notes describing the multiattribute decision analysis that he was doing for the product engineering department. Don and Gale agreed to meet during September, after Don gave his presentation on his current project.

Gale realized that it would be injudicious to jettison the present allocation process for a system not yet developed, and in any case the final allocation plan was due in the hands of the corporate vice president for R&E by late November. Thus, Gale decided to continue the normal planning procedure while Don prepared an allocation proposal based on multiattribute decision analysis. When both plans were complete, Gale and his colleagues could compare the two procedures and their results. Gale hoped that the approach Don developed would be useful in future years.

This case was prepared by Jeffry L. McNair, Research Assistant, and Samuel E. Bodily, Assistant Professor, as a basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative decision.
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MEETING #1

In the last week of September, a few days after his presentation on the product engineering study, Don arrived at Gale's office as scheduled. Gale had also invited Gerhard Stoll, director of Technical Planning and the person responsible for the division's planning for longer-range (5- to 20-year) technological advances—a significant segment of annual R&E research planning. Gerry Eisenbrandt, also present, was on the R&E Division's staff and was serving on Don's doctoral committee. Gerry's role was to give Don confidential advice as his work progressed.

Cutler: I noticed in your report of the work with the product engineering department that you used management preferences in all of your analysis. The department managers must have spent a great deal of time with you.

Keefer: Not really. Our total interaction time was less than 20 hours, and much of that time was used to structure the problem, identify their objectives, and determine ways to measure their group's performance. I hope that we can do that part of the work for your budgeting today. To get started, let me ask about the nature of your annual planning and your objectives in making the plan.

Cutler: The task is to divide the division's budget pie among the ongoing research activities. Our objective is to get the package of activities that best meets the needs of the company.

Keefer: Do you actually determine which projects to undertake?

Cutler: No. We tried that. But the department managers didn't buy the idea that we could make those decisions as well as they can. I believe they're right. Individual project selection is best done by someone closer to the projects; it makes sense to keep those decisions decentralized. We now have our activities divided into six mission areas. We allocate the budget to these missions; department managers select the specific projects.

Keefer: Then how do you know what allocation "meets the needs of the company," as you put it?

Cutler: Well, the company would like us to develop new products or to improve manufacturing efficiency on existing product lines.

Stoll: Just as important, I believe, is the company's interest in having its own base of research support and in finding new technologies that may be of use in the future but are still far from use in a specific product.

Keefer: Let's try to work out a complete list of the objectives of the R&E Division.

A lively discussion about the various missions of the R&E Division followed.

Cutler: After going around and around on this, I think that our breakdown of the division's purpose into the six mission areas pretty well defines our objectives. I don't think we can improve on it.

Keefer: Then our task is to decide how performance can be measured in each of those areas.

Cutler: The problem of measuring our effectiveness in meeting objectives is one of the main problems that has prevented our using a more formal allocation procedure. Frankly, I'm a little skeptical about what we'll be able to do now. Our division's work is so far removed from the bottom line that we have no clear measures of the benefits that the company gets from our efforts. We might try to count the number of new products or pieces of new technology developed, but we could only guess at such numbers.

Keefer: That's a place to start. Of course, regardless of the scales we use to measure performance, the numbers must be forecast. We may as well admit to uncertainty and incorporate probabilities into the analysis. What we need now, though, are numerical scales associated with levels of achievement of each of the objectives.

During the next hour, the group discussed relevant scales for each of the objectives. The scales finally chosen are shown in Exhibit 1.

Keefer: It's important that these scales have meaning to you—that you be able to assign numerical values to varying levels of achievement on each attribute (or mission) scale. You

EXHIBIT 1 PERFORMANCE SCALES FOR R&E DIVISION (BY MISSION AREA)

Mission	Performance level	Description
1 Current Product R&D	0	Progress toward at least one feature is made
	1	One new feature developed or progress made on at least two
	2	One new feature developed and progress made on at least five
	3	Two or more new features developed and progress made on others
	4	Five or more new features developed to a point of definite operating division interest, and progress made on others
2 Manufacturing process R&D		
	0	No significant new development of interest to the operating divisions
	1	Progress made toward at least one improved process of substantial interest
	2	One new process developed and improvement made on one other process
	3	One new process developed and improvement made on several others
3 New business opportunities		
	0	No significant progress made on a new product or new business
	1	Several concepts well developed, but none at a major effort stage
	2	Two major efforts underway and one concept well developed
	3	One new product (or business) developed, one major effort underway, and one concept well developed
4 Technological research		
	0	No new applied technology found or being pursued
	1	One new technology found and under development for application
	2	At least two new technologies found and under development for application
	3	At least one new technology applied and another under development for application
5 Continuing support		
	0	Corporate needs for centralized technical support not met
	1	Corporate needs partially (1/3) met
	2	Most (2/3) of corporate needs met
	3	All corporate needs met
6 Other		
	0	No improvement in R&E organization, related corporate objectives not met
	1	Some improvement in R&E organization, related objectives partially met
	2	Significant improvements in R&E organization; related corporate objectives met

EXHIBIT 2 LIMITS ON BUDGET ALLOCATIONS BY MISSION AREA*

Mission # (i)	Smallest fraction acceptable (a_i)	Largest fraction acceptable (b_i)
	.15	.30
	.20	.30
	.15	.35
	.10	.25
	.05	.20
	.01	.05

*If x_i is the fraction of the budget allocated to mission i, then $a_i \le x_i \le b_i$, $i = 1, 2, \ldots, 6$.

will also need to be able to evaluate tradeoffs among the mission areas and among their various achievement levels. Finally, you will need to describe the probabilities of the possible outcomes of each proposed allocation strategy. Do you think you will be able to do this? In other words, are these scales written in terms in which you could think while evaluating R&E programs?

Cutler: Well, while the scales are somewhat arbitrary, I think we could give preferences at a rather coarse level. Actually, though, I think that question will just have to wait until we try some of the evaluations you are referring to.

Keefer: Which raises another question. Would a value of, say, 2.5 on one of the scales have meaning to you?

Cutler: Again, since the scales do not represent hard numerical results anyway, I suppose noninteger values would make as much sense to me as integers only. But I would certainly have a difficult time assigning values to small differences in results on these attribute scales.

Keefer: O.K., for the present, let's assume that these scales which you have agreed upon are useful and that they are continuous. As we progress through future sessions, we'll find ways to check the validity of these assumptions. Before our next meeting, please review what we have discussed today and write down any questions you have or changes you wish to make. Also, be certain that the two of you are really in agreement—Gale has done most of the talking so far. Finally, please develop for me a list of the maximum and minimum levels of funds that you feel you could assign to each mission area.

Stoll: I'm not sure what you are asking for, Don.

Keefer: Well, we have developed a measure of the results of your allocation decision in the form of the six performance scales. We have assumed that the variable you control is the portion of the budget going to each mission area. Does this sound reasonable so far?

Stoll: Yes.

Keefer: The total assignment to all six areas must be equal to one—that is, to the whole of the budget. But I feel certain that you could not suddenly assign all of the budget to mission 1. After all, constraints are imposed on you by existing facilities and labor and by the demands made by other parts of the corporation. Consequently, it seems reasonable to think in terms of upper and lower limits on the proportion of the budget allocated to each area.

Stoll: I see that may not be easy, but we'll work at it. (Exhibit 2 shows the upper and lower limits that Gale and Gerhard eventually chose.)

MEETING #2

As soon as the meeting opened, Gerhard began to express his and Gale's reservations about the evaluation technique developed at the previous meeting.

Stoll: Some of the measures of success seem rather artificial. We aren't certain that you can actually count new technologies or that you can always determine when progress has been made toward a new manufacturing process.

Keefer: Well, as you will begin to see, we will not go out and actually count anything. Our purpose is to forecast the outcomes of various allocations, while explicitly accounting for uncertainties. Since these forecasts are subjective, the important thing is that the scales are convenient for the two of you to think about and use in discussing preferences and possible outcomes.

Stoll: We have found that we can communicate better using these scales, so let's continue. I've tried to think of measures with less ambiguity and haven't come up with anything.

Keefer: Today, I'd like to tie the input—your spending level in a mission—to the output, as measured on our new attribute scales. To do this, we must do some conditional forecasting. We need to quantify your judgments concerning the results of a given level of allocation. Now, there are an infinite number of budget fractions that could possibly be allocated to any given mission. I have an interpolation technique that makes it necessary to consider only three allocation levels. We will use the upper and lower bounds on allocation levels that you have chosen (Exhibit 2) plus the point midway between these limits. Are you with me so far?

Stoll: Yes, that's straightforward.

Keefer: O.K. For each allocation level, I want you to give me the median of the corresponding probability distribution of performance. By median I mean the point along the attribute scale that divides it into equally likely intervals. It is just as likely that the actual outcome will fall above the point as it is that it will fall below the point. Then we will pick two extreme outcomes for the same attribute, one that you feel has a 1 in 100 chance of being exceeded and one that has a 99 in 100 chance of being exceeded.

Cutler: You are asking for—let's see—three estimates for each of three allocation levels. That's nine values for each mission?

Keefer: That's correct. Let's get started. If you spend the smallest possible amount, 15 percent of your budget, on current product R&D, what would be the median outcome on the attribute scales for that mission? What point divides the attribute scale into equally likely intervals?

Cutler: I guess I would say . . . well, we might get one new feature out of that and we might not. We certainly would make some progress on new features. I'll stick my neck out and say 1.5.

Stoll (after some thought): I'll go along with that for now.

Keefer: O.K. Now, for the same attribute and allocation level, what about the pessimistic outcome level—the level which has a 99 in 100 chance of being exceeded?

Stoll [staring at a copy of the performance scales (Exhibit 1)]: My choice would be a 0.5.

Cutler: Good. I agree.

Keefer: Now, the optimistic value?

There was some debate but eventually Gale and Gerhard settled on a value of 2.0. Don then proceeded to name other allocation levels. Without a great deal of difficulty, he elicited all of the responses shown in Exhibit 3.

Keefer: In the forecasting we have just done we have assumed what's called *probabilistic independence*. My question is this: Do you feel that the level of outcome in a mission depends only on the funds allocated to that mission and not on how the remaining funds are allocated among the remaining missions?

Stoll: Well, of course, we've been assuming that each mission's achievements would be independent. The only problem I see is that some projects cross mission lines in the benefits that they provide.

Cutler: Sure, Gerhard . . . but they often cross mission lines in funding as well. Both allocations and benefits will have to be prorated between missions. But the assumption that the progress in one mission resulting from a given project is independent of that project's success in another mission is still a good one.

Stoll: I guess the independence assumption works partly because we unconsciously defined the missions so that it would.

EXHIBIT 3 ASSESSED PROBABILITY FRACTILES

Mission	Budget fraction	Fractiles .01	.50	.99
1	.15	.50	1.5	2.0
1	.225	.80	2.25	3.0
1	.30	2.0	3.0	3.75
2	.20	0	.50	1.2
2	.25	0	1.2	1.8
2	.30	.50	1.75	2.5
3	.15	0	.30	1.0
3	.25	0	1.3	2.0
3	.35	1.0	1.8	2.75
4	.10	0	.50	1.3
4	.175	.40	1.3	2.2
4	.25	.90	1.8	2.7
5	.05	.20	.70	1.2
5	.125	1.0	1.5	2.0
5	.20	2.0	2.5	3.0
6	.01	0	.25	.50
6	.03	.50	1.0	1.5
6	.05	1.0	1.4	1.8

Keefer: Fine. That takes care of our "conditional forecasts." Now I want to review what we have done so far. After we defined your missions, we developed attribute scales which allow us to measure the performance of the missions. We have said that the actual outcome in a mission will depend upon the portion of your budget you spend in that area, and we have predicted outcomes with various probabilities, given various spending levels. Next time, we'll begin with some progressively more difficult material. I will send you some information—maybe 10 pages or so—that will introduce you to the concepts and techniques involved.[1] It may have occurred to you that we have not yet talked about the relative value you place on various levels of outcome, nor do we have any way of getting one combined value for a set of outcomes in the six missions. These are things we'll work on next time.

MEETING #3

Don started off by going over the concepts and techniques described in the notes he had given Gale and Gerhard. When they felt comfortable with these ideas, he continued.

Keefer: Today, I am going to get you to make some judgments for me. All that really matters is that you give me your best answers to my questions. By the time we finish, I believe you will understand why I am doing this, since you are already familiar with the concepts involved. We will be developing one-dimensional utility functions. In each situation, I will ask you to make judgments about only one attribute—that is, judgments about performance with respect to one mission. I'm looking for your risk preferences concerning various achievement levels in that one mission area. Let's start here. (Don pulled out a form like that shown in Exhibit 4.) You are given the choice between a lottery and some fixed value of outcome. The first set of prizes for current product R&D are results of values 0 and 4. You have a ticket to a lottery in which you have a 50 percent chance of winning an outcome of 4—that is, five or more new

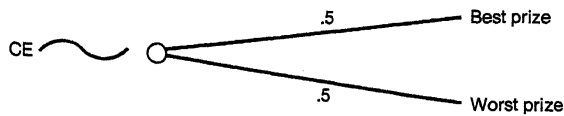
[1]The notes Don sent are not included in this case. They provided a brief introduction to each of the following concepts and techniques: lotteries, certainty equivalents, risk preference, utility and the assessment of one-dimensional utility functions, and utility and preferential independence.

Progress toward one or more features	One new feature or progress toward two or more	One new feature and progress toward five or more	Two or more new features and progress toward others	Five or more new features and progress toward others

EXHIBIT 4 Development of one-dimensional utility functions. (Example for mission 1—current product R&D.)

features and progess on others, and a 50 percent chance of winning a 0—which would mean only progress toward one or more new features. I am offering you a guaranteed outcome of a 1 on the scale, if you will give up the lottery ticket. Will you accept?

Stoll: I don't think so.

Cutler: Definitely not.

Keefer: What if I assured you of an outcome of 3? Would you take that?

Cutler (after a moment's thought): Yes.

Stoll: Surely.

Keefer: O.K., I gather from your responses that an outcome of 3 is more than enough to buy the lottery from you. That means that somewhere between 1 and 3 is an outcome that is roughly as desirable to you as holding the 50-50 lottery between 0 and 4. What is the minimum certain outcome that you would just accept in exchange for the lottery? What I am looking for is the point of indifference—the level of certain outcome for which you would be indifferent between having the certain income and having the 50-50 lottery.

Some discussion followed, but Gerhard and Gale finally agreed on a value of 2.3. Don brought out a table like the one in Figure 1. He wrote 2.3 as the row 1 certainty equivalent. He also wrote 2.3 down as the value of the worst outcome in row 2 and as the value of the best outcome in row 3.

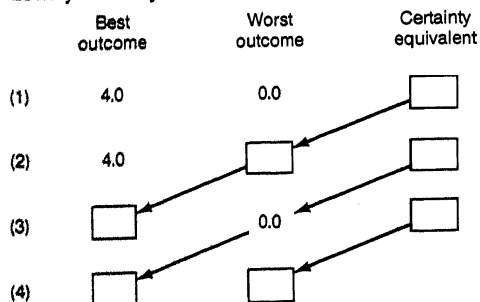
Keefer: O.K. Let's consider another equally likely lottery between the 2.3 you just gave me and 4. What amount makes you indifferent between accepting my offer and retaining the lottery ticket (he pointed at the second row of the figure as he spoke).

Stoll: I'd have to say 3.2., or maybe even . . .

Cutler: Gerhard, your long-term gambling instinct is really showing up here. When we get up above two new features, a bird in the hand is worth two in the bush. I wouldn't take as much of a chance—I would sell out at around 3.0.

Some additional discussion followed, with Gerhard finally agreeing with Gale on a value of 3.0. Don wrote 3.0 as the certainty entry in row 2 and also as the best outcome entry in

FIGURE 1 Lottery/certainty values for Mission 1.

	Best outcome	Worst outcome	Certainty equivalent
(1)	4.0	0.0	
(2)	4.0		
(3)		0.0	
(4)			

row 4. Then he asked for a certainty equivalent to associate with a lottery between 0 and 2.3. This time Gerhard's outlook won out. The two men agreed on a value of 1.5. Don placed the 1.5 in the certainty entry on row 3 and in the worst outcome entry in row 4.

Keefer: Now, we have a lottery between 1.5 and 3.0. What certain value would you just accept for this lottery?

Cutler: Uh . . . 2.1 maybe, no, say 2.2.

Stoll: I think I would probably have started at 2.3 and come down to 2.2. But shouldn't our answer here be the same as our answer to the first question?

Keefer: Well, yes. This is a consistency check—the results of questions 1 and 4 should be about the same. Although a .1 difference on a scale of 4 is not too severe, why don't you see if you can resolve the conflict?

In the discussion which followed, Gale and Gerhard were finally able to decide that they wanted their answer to question 1 to be 2.2 instead of 2.3. They also discovered that it was easier to determine certainty equivalents when the interval between lottery outcomes was smaller. Don made the necessary changes on the chart, then went through the second, third, and fourth questions again to confirm that these results would not change. No further inconsistencies arose.

Don pursued a further breakdown of the directors' preferences within mission 1 by requiring the directors again to halve the segments obtained so far. This additional round of questions (which obtained certainty equivalents for 0 vs. 1.5, 1.5 vs. 2.2, 2.2 vs. 3.0, and 3.0 vs. 4.0 lotteries) seemed to have divided the scale as finely as was practical. Don recorded the results. Using similar techniques, Don obtained preference data on missions 2 through 6.

Keefer: We have completed data collection for the one-dimensional utility functions. I am interested in the fact that we have done so without any questions arising about the attributes not in question in a given case. I would like to check this formally. First, let's say I set the levels of attributes 2 through 6 at .5 each and asked you for your certainty equivalents for various lotteries on mission 1. Then suppose I set the value of attributes 2 through 6 at levels of 2 and again asked for certainty equivalents on mission 1. Would your answers in the second case be the same as in the first case?

Cutler (after moments of thought): Yes—and I guess that assumption of independence was implicit in our development of those functions in the first place, wasn't it?

Keefer: Yes, it was. Gerhard, what do you think?

Stoll: I think my assessments would remain the same.

Keefer: Good. Can I generalize and say that the same would be true regardless of the combination of levels I picked for nonsubject attributes, and regardless of which attribute is the subject attribute?

Stoll (glancing at Gale for confirmation): Yes, I would say so.

Keefer: Fine. As we discussed at the beginning of today's session, the property that we have just explored is known as utility independence. And as Gale observed, we really could not have developed one-dimensional utility functions as we did if utility independence did not hold. That does it for this session. I'll have a summary of these results for you next meeting.

MEETING #4

Don opened this meeting by presenting graphs of the data collected at the previous meeting (see Exhibit 5).

Keefer: Your values for the possible outcomes of a given mission are ranked on a scale from 0 to 1 on the vertical axis. We frequently use the word "utility" instead of "value" when referring to scales like these—as you saw in the notes I sent you. Let's take the first case that we did last session. You set 2.2 on the scale for mission 1 as the value of a 50-50 lottery between 0 and 4 on that same scale. Now, as we defined the scale earlier, 0 is the lowest possible value

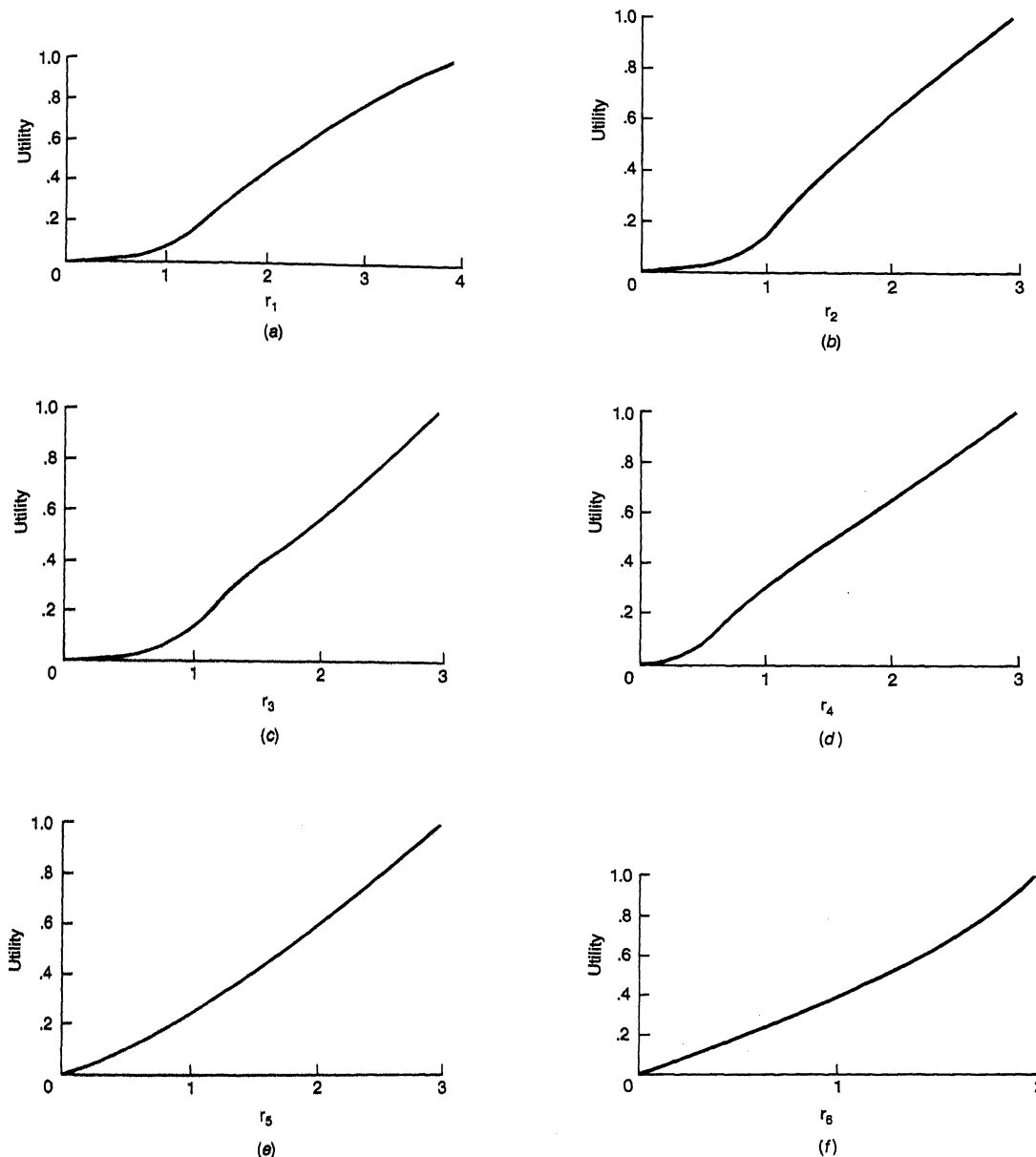



EXHIBIT 5 One-dimensional utility curves: (*a*) current product R&D; (*b*) manufacturing process R&D; (*c*) new business opportunities; (*d*) technological research; (*e*) continuing support; (*f*) other.

and 4 the highest. We set the utility of an outcome of 0 at 0 on the vertical scale, and the utility of an outcome of 4 at 1 on the vertical scale. We then say that a 50-50 lottery between a utility of 0 and a utility of 1 must have a utility of .5, so we put a point at the 2.2 you gave, opposite .5 on the vertical scale. Similarly, we put your 3.0 at a utility of .75 and your 1.5 at a utility of .25. After plotting all available data, we put a curve through the points. Curves for all six missions were prepared in this way. Any questions on what these curves represent?

Cutler (after a moment's hesitation): No, I don't think so.

Stoll: Makes sense to me.

Keefer: O.K., then I want to ask you some questions. Can you explain why some of the curves are convex in the lower portion of their ranges?

A good deal of discussion followed, Don guiding but giving no answers. When Gale and Gerhard felt that they had an answer to the question, Don asked them to restate the answer in terms of risk.

Cutler: O.K., Don, I'll give it a try. Because we value the lower level outcomes so little, we are willing to take a proportionately large risk to produce potentially better results. For example, in the current product R&D there is little difference to me between the 0 and 1.5 levels of outcome. Consequently, I am willing to give up a certainty of 1.5 for a chance at a more acceptable level of outcome (2.2). That wouldn't be true, though, if the worst possible outcome (0) brought some really dire consequence. In a way, we are seeking risk.

Keefer: Then what about the upper regions of the curves, where they are either flat or concave?

Cutler: There we get back into what I call the "bird-in-the-hand" syndrome. As the certain results move up to more acceptable levels, we become less willing to risk a lower outcome in order to possibly achieve a better one. We begin to try to avoid risk.

Keefer: O.K., that sounds good—we normally say you are risk-seeking in the first case and risk-averse in the second. Now, the question is, do you believe these curves? That is, do these statements or risk preferences describe your feelings?

After some discussion, Gerhard and Gale concurred.

Keefer: Good. Now, today's topic is tradeoffs among outcomes in different missions. Suppose that you were facing a very bad year, in which the performance measures for the six missions would all be at 0. And suppose further that you could raise one and only one mission's performance to its highest possible level. What mission would you wish to have at its highest level?

Stoll: In others words, which mission is most important?

Keefer: What I want you to do is maximize total utility, as you see it, by raising one attribute's value to its highest level.

Cutler: I'd say, raise the results in mission 1, current product R&D.

Stoll: Well, I agree performance in mission 1 is critical, but I'm tempted to say that new business opportunities—mission 3—is a more important attribute to the company. It's a difficult choice.

During the discussion which followed, it became apparent that both directors found it a close decision between missions 1 and 3. Further, the two directors observed that Gale's short-term orientation led him to favor mission 1 slightly, while Gerhard's longer-term orientation led him to favor mission 3 initially. The two finally agreed in favor of mission 1.

Keefer: O.K. Now suppose that the situation is unchanged, except that it is not possible to raise the value of mission 1. Now which one would you choose to raise?

Cutler: Now I would go with mission 3.

Stoll: So would I.

Don continued to ask similar questions, each time excluding all previous choices from consideration, until only mission 6 had not been chosen. The missions were chosen in this order: 1, 3, 2, 4, and 5.

Keefer: That was just a warm-up for what is to come. I will now ask you a set of only six questions. They may be difficult questions, however, because they present combinations of outcomes under certain conditions that may be difficult for you to imagine. These are very important questions, so please take your time in answering them. (Don took out a sheet like the one shown in Exhibit 6.) The situation is this: The certain outcome in current product R&D (mission 1) is at its highest level—five or more new features will be developed, and progress made on others. But with this option missions 2 through 6 will be at level 0. Your second option in this case is a lottery in which all attributes are at their highest levels in the good

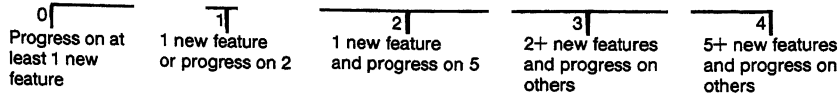



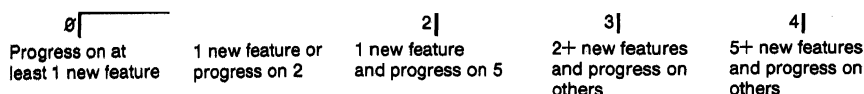
EXHIBIT 6 Evaluation of objective function constants. (Sample for Mission 1.)

outcome or at their lowest values in the bad outcome. Now, what must the probability of a good outcome be, for you to be indifferent between having the certainty and having a ticket to the lottery?

Cutler: Don, you've got to be kidding! You want us to consider a gamble between the worst possible year and the best possible year? Oh boy. . . .

After a heated discussion, Gale and Gerhard settled on a probability of .70. Don continued to pose similar cases for each attribute. In each case they chose the probability to make the lottery equally desirable to the certain option where the subject attribute was at its highest level and all other attributes at their lowest levels. By the end of an hour, Gale and Gerhard had agreed on the following probabilities: .40, .65, .30, .25, and .10 for missions 2 through 6.

Keefer: Good. Notice that a ranking of these probabilities by size corresponds to the order in which you picked the missions in our first set of questions today. Now, evaluating attributes or missions in this way requires that we assume that tradeoffs between attributes are independent of the remaining attributes. I would now like to confirm this independence assumption by asking you to make some more comparisons (he pointed to a sheet like Exhibit 7). Let me summarize

EXHIBIT 7 Confirmation of preferential independence. (Sample for Missions 1 and 2.)

for you. You have two situations—A and B. The difference between situations A and B is that the levels of missions 3 through 6 are low in situation A and high in situation B. Your task is to set the level of the current product R&D outcome so that you are indifferent between the two sides of each situation.

In the discussion that followed, Gale and Gerhard had as much difficulty settling on a specific number as they had had in the previous six cases. They did agree that the number they chose would be the same for the two situations. At that point, Don broke in to say that he did not need an actual number as long as he knew that the numbers for the two situations were the same.

Don posed additional questions, using different missions as the subject missions. He varied the levels of the four missions that were not being directly considered, but these four did always have the same levels in the left and right situations in each case. He concluded that in all cases their tradeoffs between the two attributes were unaffected by the levels of the remaining attributes. Don explained that this property was the pairwise preferential independence which he had hoped to confirm. He asked if the directors had any questions about the concept of independence.

Cutler: As a matter of fact, I do. I think all this independence talk oversimplifies the problem. To the choices you gave us last time and again today we could only give the most honest responses we knew—but there are still some interdependences that you are missing, I think. For example, the total benefit that the corporation receives from our efforts is not simply the sum of six individual levels of benefit.

Keefer: Your concern is a valid one, and you have intuitively glimpsed something that we will develop in more detail later this session. If your answers expressed your true feelings, then our independence conclusions are supported. Let me point out, though, that we have not ruled out all interrelationships among attributes. For example, we are not ruling out the possibility you mention, that the total value of a particular combination of levels of outcomes will differ from the simple sum of the individual benefits.

Cutler (after a pause): Well, Gerhard, what do you think?

Stoll: Don has addressed my immediate concern, but I'm not sure how what he just said squares with the independence conditions we have developed.

Keefer: Well, I can clarify this by explaining my purpose in obtaining the last set of probabilities. If the six probabilities you just gave me (.70, .40, .65, .30, .25, .10) had summed to 1, I would have concluded that the total utility to the company of any year's outcome was simply the weighted sum of the six separate one-dimensional utilities for that outcome. However, the probabilities sum to 2.4, so the expression for total utility must be a multiplicative form, rather than the simpler additive one. This multiplicative form means, for example, that if one attribute has a high outcome, it is more than half as desirable as both attributes having a high outcome; the attributes substitute one for another to a certain extent. That is, when results in one area are good they partially make up for poor performance in other areas. To use an analogy from economics, they are like substitutable goods. Do these ideas make sense to you, and do you feel that they are true in your case?

Cutler (after some thought): Well, I agree with the comments Gerhard made earlier. The total value of the result of a year's expenditures in R&E is not going to be the sum of the six simple utility values. I'm not sure about the substitutable nature of these things, though. . . .

Stoll: I've been thinking . . . Don, what you are really saying is that having one attribute at a high level is more important if that attribute is the only high one. Adding a second high-valued attribute increases the total value less than the first does, the third high one less than the second, and so on. And that seems right to me.

Some discussion followed, and Gale eventually agreed with Gerhard that the mission results were indeed substitutable.

Keefer: Gentlemen, we have reached a milestone. I have all the data I need. Before we meet again, I will set up the problem and run an initial solution to it.

MEETING #5

Cutler: O.K., Don, after all these hours of effort, we expect to see great things today.

Keefer: Well, Gale, before I tell you what the optimal solution appears to be, let me show you how I arrived at it. The general approach is to combine your preferences (represented by the utility functions I have assessed) with your beliefs about the performance of a given budgeting level (represented by the probabilities I have elicited) by computing "expected" utility. Utility values are multiplied by their probabilities and summed over all possible outcomes to obtain the expected utility, a measure of the relative desirability of a particular budget allocation. The optimization computer program will find the budget allocation that provides maximum expected utility.

Stoll: This is heavy stuff for nine o'clock in the morning, Don. I'm a bit confused about how you evaluate the budget for all mission areas, since the probabilities and preferences we have discussed have only involved a single mission area at a time.

Keefer: I can explain that, but it's easiest to do it using some symbols. Here (Exhibit 8) is a complete statement of the problem. Our intent is to choose $x_1, x_2, \ldots, x_6$, the fraction of

EXHIBIT 8 WHIRLPOOL R&E DIVISION (B) MATHEMATICAL STATEMENT OF BUDGETING PROBLEM

$$\underset{x_1, x_2, \ldots, x_6}{\text{Maximize}} \left\{ \prod_{i=1}^{6} (1 + kk_i[u_i \cdot (x_i)]) - 1 \right\} / k \qquad (1)$$

subject to

$$a_i \leq x_i \leq b_i \quad i = 1, 2, \ldots, 6 \qquad (2)$$

$$\sum^{6} x_i = 1 \qquad (3)$$

where $[u\ (x_i)]$ combines the probability distribution

$F_i(r|x_i)$

probability

with the utility function

u_i

$$[u_i\ (x_i)] = \int_{R_i} u_i\ (r_i)\ \ dF_i\ (r_i \mid x_i) \qquad (4)$$

where r_i = results in mission area i

$F_i\ (r_i \mid x_i)$ = cumulative probability distribution* for results in mission area i if budget level is x_i

k_i = preference weights on mission area i

k = interaction parameter found from $1 + k = \prod_{i=1}^{6} (1 + kk_i)$

*Probability that results are no greater than r_i.

EXHIBIT 9 APPROXIMATION OF EXPECTED UTILITY

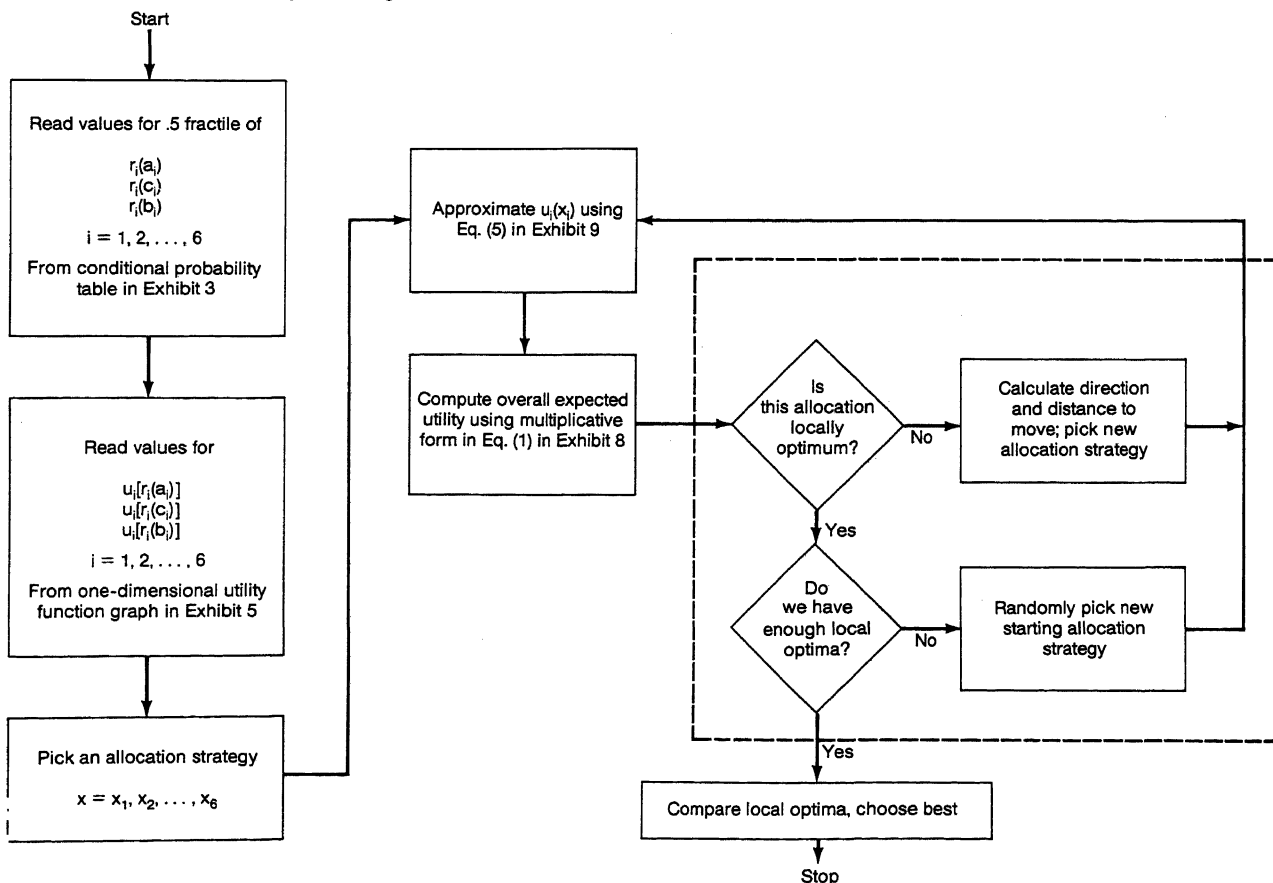
$$[u_i(x_i)] \approx (1 - 3h + 2h^2)\, u_i(r_i^{.5}(a_i)) + (4h - 4h^2)\, u_i(r_i^{.5}(c_i)) + (-h + 2h^2)\, u_i(r_i^{.5}(b_i)) \tag{5}$$

where $c_i = \dfrac{b_i - a_i}{2}$

$h(x_i) = \dfrac{x_i - a_i}{b_i - a_i} \quad i = 1, 2, \ldots, 6.$

and $r_i^{.5}(a_i)$ is the median (.5 fractile) for the result of mission area i when the fraction of total budget allocated to i is a_i.

the total budget assigned to the six mission areas so as to maximize expected utility for the overall performance in the entire division. Your responses to my preference questions (utility independence combined with pairwise preferential independence) imply that your utility for performance in the entire division is found by combining your utilities for performance in mission areas multiplicatively. Thus expected utility for the complete budget is given by expression (1), where $[u_i(x_i)]$ is the expected utility for the performance of mission i if x_i is budgeted to it. Again, the expected utility combines the probability distribution with the utility function. Since there are an infinite number of outcomes and associated probabilities (we have taken r to be continuous), the summation of probabilities times utilities requires an integral ($\int$) as in expression (4).

EXHIBIT 10 System diagram: Keefer's final optimization model.

Stoll: I think I see what's going on, but what are the k's?

Keefer: The k_i's are weighting constants that you helped me find by responding to my tradeoff questions. The k is a constant that accounts for interactions between the mission areas. I can calculate it from the k_i's. The maximization is subject to two constraints. Expression (2) is the constraint that the budgeting level for each mission area must fall within the upper and lower limits you set. Expression (3) ensures that the budgets for the mission areas add up to the whole budget (remember the x_i's are defined as fractions of total budget).

Cutler: Don, I don't see how you are going to do those expected utility computations. I only remember your asking a few probability questions of us. Where did you get the probability curves?

Keefer: It's true that I only have three points of the cumulative probability curve (the .01, .5, and .99 fractiles) and not the whole curve. And I only have these for three budgeting levels: the upper and lower limits you set and halfway between. It was too much to ask for the entire probability distribution for every budgeting level. I will approximate expected utility using the median of the probability distribution in an approximation that quadratically interpolates expected utility for all values of x_i [see expression (5) of Exhibit 9]. I took the .01 and .99 fractiles to test whether the approximation is close enough. I have already looked at the closeness of the approximation and believe that it is accurate enough for our purposes. Don't worry about expression (5); how to calculate the expected utilities is a technical mathematical issue not important to this discussion.

Cutler: I think we can settle for a layperson's understanding of the math; I'm more interested in discussing your results.

Keefer: I used a nonlinear programming computer package at the university to find the budget allocation that maximized overall expected utility. The way the whole system works is shown in Exhibit 10. The part in dotted lines is what the computer does to find the optimal solution. The rest of the diagram shows how the numbers for the computer are calculated. My optimal solution is this (he pointed to the first line of Exhibit 11). Gale, since you gave me your proposed allocation strategy when we were on the phone last week, I calculated its expected utility. The other two lines show other "local" optima. If you think of the region of possible solutions as geographical terrain, then there are three peaks in the area. The highest is the "global" optimum and the other two are "local" optima.

Cutler: Well, now. The .716 is quite a change from the .588. I had begun to feel that the strategy which we arrived at in our normal discussions here at Whirlpool might not be nearly optimal. But I am a little surprised at the amount by which it is off, and I wonder how significant the difference really is.

Keefer: I anticipated this question. Suppose I give you two sets of outcome levels: one that has the same total utility as the expected utility for my optimal allocation, and another set that has the same utility as the expected utility for your allocation. These sets of results will differ in the value of only one attribute—let's say, in the manufacturing process R&D. The set of outcomes, 1.90, <u>2.93</u>, 1.10, .73, 2.08, 1.40 (he wrote them on scrap paper as he went), has a .716 utility—the same as the optimal solution. The set of results, 1.90, <u>1.07</u>, 1.10, .73, 2.08, and 1.40, has a utility of .558—the expected outcome for your allocation. Since my solution yields 2.93 in manufacturing process R&D and yours yields 1.07, I would get one

EXHIBIT 11 ALLOCATION STRATEGIES

Strategy	x_1	x_2	x_3	x_4	x_5	x_6	Expected utility
Global optimum	.30	.28	.26	.10	.05	.01	.716
Local optimum	.30	.20	.31	.12	.05	.02	.708
Local optimum	.30	.30	.15	.17	.05	.03	.699
Planned 1976 policy	.19	.24	.23	.12	.17	.05	.558

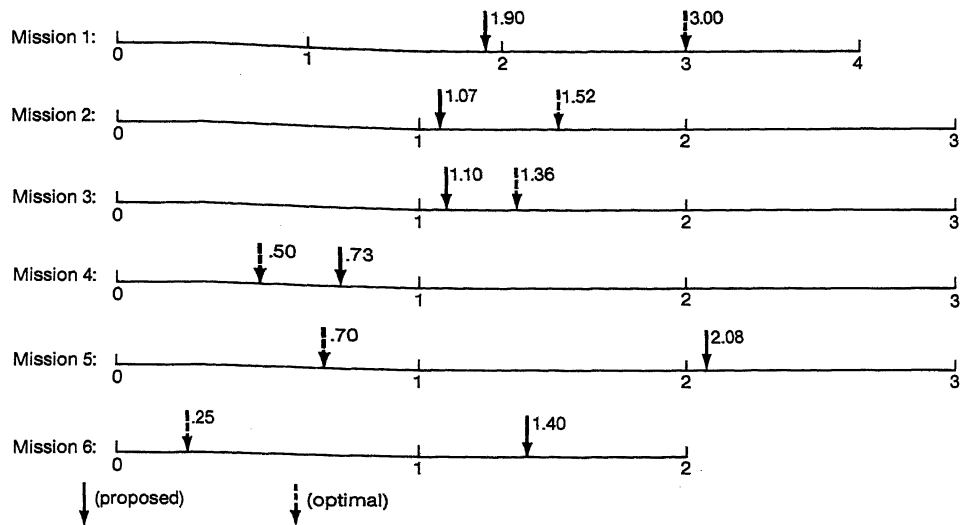



EXHIBIT 12 Comparison of certain equivalents corresponding to proposed and optimal budget allocations.

EXHIBIT 13 SENSITIVITY OF (NOMINAL) OPTIMUM POLICY TO SCALING CONSTANTS

Run no.	Scaling constants						Optimal solution					
	k_1	k_2	k_3	k_4	k_5	k_6	x_1	x_2	x_3	x_4	x_5	x_6
1	.70	.40	.65	.30	.25	.10	.30	.28	.26	.10	.05	.01
2	.60	—	—	—	—	—	—	—	—	—	—	—
3	.50	—	—	—	—	—	—	—	—	—	—	—
4	.40	—	—	—	—	—	.25	.30	.29	—	—	—
5	—	.35	—	—	—	—	—	.27	.27	—	—	—
6	—	.30	—	—	—	—	—	.25	.29	—	—	—
7	—	.25	—	—	—	—	—	.25	.29	—	—	—
8	—	—	.75	—	—	—	—	.26	.28	—	—	—
9	—	—	.85	—	—	—	—	.25	.29	—	—	—
10	—	—	.60	—	—	—	—	—	—	—	—	—
11	—	—	.55	—	—	—	—	.29	.25	—	—	—
12	—	—	.45	—	—	—	—	.29	.25	—	—	—
13	—	—	—	.35	—	—	—	—	—	—	—	—
14	—	—	—	.40	—	—	—	—	—	—	—	—
15	—	—	—	.45	—	—	—	.27	.25	.12	—	—
16	—	—	—	—	.30	—	—	—	—	—	—	—
17	—	—	—	—	.35	—	—	—	—	—	—	—
18	—	—	—	—	.40	—	—	—	—	—	—	—
19	—	—	—	—	—	.15	—	.27	—	—	—	.02
20	—	—	—	—	—	.20	—	.26	.25	—	—	.04

Key: — Denotes same value as in base case (run 1).

EXHIBIT 14
REQUESTED SUPPLEMENTAL SENSITIVITY RUNS

Run no.	Change from base case	Nominal optimum*							Global optimum						
		x_1	x_2	x_3	x_4	x_5	x_6	Utility	x_1	x_2	x_3	x_4	x_5	x_6	Utility
1	Base case	.30	.28	.26	.10	.05	.01	.716	—	—	—	—	—	—	—
2	$x_5 \geqslant .10$	—	.24	.25	—	.10	—	.691	—	.20	.29	—	.10	—	.697
3	R.P. 1	—	—	—	—	—	—	.658	—	—	—	—	—	—	—
4	$x_6 = .05$	—	.25	.25	—	—	.05	.703	—	.20	,30	—	—	.05	.704
5	R.P. 1, $x_6 = .05$	—	.25	.25	—	—	.05	.643	—	.20	.30	—	—	.05	.644
6	R.P. 1, $x_5 \geqslant .10$	.29	.25	.25	—	.10	—	.631	—	.20	.29	—	.10	—	.636
7	$x_5 > .10, x_6 = .05$	—	.20	.25	—	.10	.05	.687	—	.30	.15	—	.10	.05	.689
8	R.P. 1, $x_5 \geqslant .10$, $x_6 = .05$	—	.20	.25	—	.10	.05	.624	—	.30	.15	—	.10	.05	.627

Key: R.P. 1 denotes reduced productivity for x_1; — denotes same value as in nominal base case solution.
*The results of adjusting the original optimum to meet the new constraints without searching for the new optimum.

new process and several improved ones, while you would get progress on one new process. That difference should give you an idea of the significance of changing from your intended allocation plan to the optimal policy.

Stoll: In other words, all other things being equal, the difference in expected outcomes between our allocations would be one new process and at least two improved ones. (Don nodded.) But what you've just described is not what you would predict would happen—the increased benefit would actually be split among missions. Surely your expected outcomes are not higher in all six missions?

Keefer: You are correct—again. Assuming that increased benefits would show up in only one mission area simply helps us to compare the predicted outcomes. I have made up a chart here (Exhibit 12) that shows the expected levels of each attribute, using your solution and using mine. Let me tell you here, too, that I have run some sensitivity analyses on my solution, and it turns out that the principle of the solution changes very little even if the data (scaling constants, probability data, or utility data) are varied by as much as 30 percent. These results mean that a minor change or inaccuracy in your feelings on some of these items would not change my recommendations. (Exhibit 13 shows an example of the results of the sensitivity analysis that Don ran.) However, such a change or inaccuracy can cause just enough change in the expected utility value that one of the local optima becomes the global optimum.

Cutler: Well, Don, I think Gerhard and I are somewhat surprised at your results and certainly impressed by your efforts. I also think we will want you to do some added sensitivity analysis. There could be a good deal of resistance within Whirlpool to any major alteration in our strategy of allocation. It might be that we could soften that resistance by moving in a direction you recommend, but perhaps not as far or not in all missions. This will take some additional thought on our part. We'll let you know what additional sensitivity runs we would like.

During the following week, Gale received the results of the additional sensitivity analysis that he had requested from Don (Exhibit 14). With these results in hand, Gale and Gerhard faced the question of what significance the study had for them and for Whirlpool's R&E Division.

WHIRLPOOL RESEARCH AND ENGINEERING DIVISION (C)

"There is no question that Don Keefer's study was useful to us. Don's model captured several elements of the problem which we had never been able to fully explore previously." Dr. Gale Cutler was responding to questions posed by the writers of the Whirlpool (A) and (B) cases, during a follow-up visit in July, 1979. The writers continued by asking whether Don's methods were still in use.

"I wish I could say they were. Don caused us to think hard about our allocation procedures in his interviews and then to use a new technique—multiattribute decision analysis—to find a budget allocation. However, when he completed his doctoral work, he had no reason to leave any of his system with us. While we all understood in principle what he was doing, only Gerry Eisenbrandt had a good handle on the detail of his methods. I had more pressing things for Gerry to do, so we have not worked with this any further."

Gale reported that there had not been sufficient time for the directors to alter their 1976 allocation of funds after Don had returned the results of his study. Though the policy adopted in 1977 was not Don's optimal strategy, it had been strongly affected by the results of Don's study.

Gale went on to say that some things had changed since the time of the (B) case. Gerhard Stoll had retired in 1977 and had been replaced with Sam Pearson, an engineer who had worked with the operating groups of the company for 20 years. Also the "other" mission area had been eliminated and the remaining mission areas restated as follows:

1 Current product R&D Extend and complement division engineering—including evaluation, design, features, components, material, aesthetics, safety, and quality.

2 Manufacturing systems R&D Develop and apply new and improved technology to current manufacturing systems for quality improvement, cost reduction, environmental protection, minimized energy consumption, and increased productivity.

3 Long-range product research Investigate and demonstrate the feasibility of new long-range concepts for products of current interest to the corporation through consideration and application of changes in technology and resources.

This case was prepared by Jeffry L. McNair, Research Assistant, and Samuel E. Bodily, Assistant Professor, as a basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation. Copyright 1979 by the Colgate Darden Graduate Business School Sponsors of the University of Virginia.

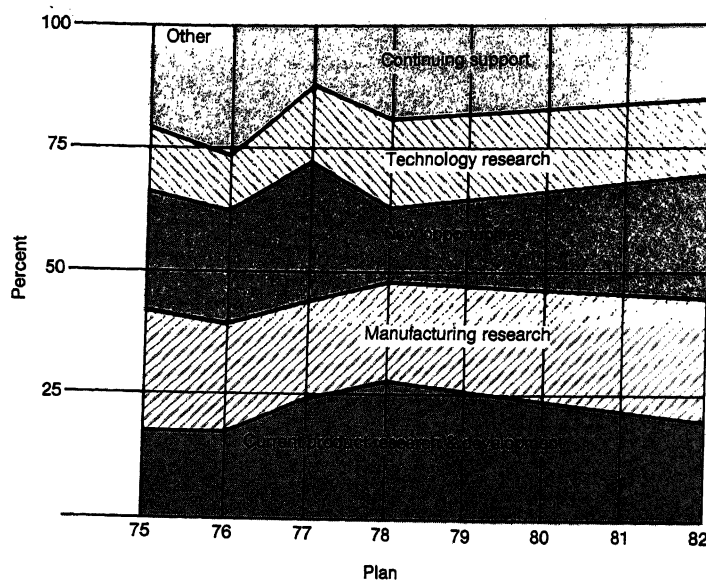



EXHIBIT 1 Mission distribution.

4 Technological research Seek, identify, and develop technologies to improve Whirlpool's competitive position.

5 Continuing support and administration Meet the corporation's needs for research and engineering support and service.

Gale showed the budget allocations for the years since the Keefer study (Exhibit 1) and pointed out that the most significant change was in 1977.

"I would like to be able to repeat Don's study in, say, two days every year or two. But as it is we don't have the nonlinear programming package to use and some of Don's interviews would be hard for us to do on our own. We just don't have much time to devote to it. I wish there were a simpler version of the analysis that we could do."

MARKETING CASE

DISCOMALL, INC.

David Jackson was well aware that his upcoming presentation to the financial committees of Compton's Department Stores, Venture Capital Associates (VentCap), and First Manhattan Guaranty (FMG) would decide the fate of DiscoMall, Inc. Committee members were certain to solicit a workable structure of prices and financial arrangements and investigate the prospects for success of the new business concept, its risks, and its sensitivity to major assumptions before committing resources to the further study of the joint venture.

The DiscoMall concept allows customers to shop for merchandise by operating an intelligent computer terminal using a catalog with a large amount of pictorial, audio, and written information on videodiscs. Items would be ordered and billed through a central computer system and shipped to the customer within 24 hours.

DiscoMall, Inc., is a company envisioned by three men: James Harrison, a computer design and telecommunications expert with a Ph.D. from MIT; Frank McKinley, who worked for a large bank in their information systems and credit card departments; and David Jackson, a research specialist for a large Boston consulting firm. The three men hope to convince Compton's, VentCap, and FMG to participate in a joint venture, DiscoMall, Inc., with themselves as operations vice president, financial vice president, and president, respectively. As initially conceived, Compton's role would be as a large installer of retail marketing terminals, VentCap would provide equity capital, and the bank would loan part of the initial investment and provide a line of credit for future working capital needs. The next step was to determine the willingness of the parties to participate and the specifics of the financing arrangements.

VIDEO MARKETING INDUSTRY

For many products, a principal cost element is marketing and distribution expense. While manufacturing costs can be reduced by achieving high volumes, marketing costs resist similar economies of scale. The past century has brought improved marketing efficiency through discount, catalog, mail order, and other forms of direct marketing. It was estimated that total sales of all forms of direct marketing exceeded $100 billion in 1981. This $100 billion comprises mail order of goods and services, vending machines, door-to-door and in-home selling, and

This case was prepared as a basis for class discussion by Richard P. Ten Dyke, IBM Corporation, Samuel E. Bodily, Associate Professor, and William D. Long, Research Assistant. Copyright 1981 by The Colgate Darden Graduate Business School Sponsors of the University of Virginia.

EXHIBIT 1

July 14, 1981

New Channels for Sales

With Video Shopping Services, Goods You See On the Screen Can Be Delivered to Your Door

By JOHN E. COONEY
Staff Reporter of THE WALL STREET JOURNAL

The popular new television show purports to be nothing more than what it is: a slickly produced, extended commercial.

An attractive host and hostess glide through sets designed to simulate a home where the furnishings change faster than a soap opera's subplots. Moving from room to room, they dangle such baubles as cordless telephones, portable television sets and fine china before the audience. Viewers dial a toll-free number to order any of the above–and much, much more.

Welcome to the "Shopping Channel," a nearly continuous cable-television program that is the latest and most ambitious experiment to date in "videoshopping," or selling directly to the consumer using cable TV, teletext and even videodiscs. This amalgam of retailing and the various new communications technologies ultimately could change the buying habits of the nation.

"This is the wave of the future," predicts Bernard Shusman, executive producer of the video division of Newsweek Inc., a unit of Washington Post Co. "It will be a billion-dollar business."

Direct-Marketing Tool

Videoshopping has much appeal for the nation's merchants. Saddled with rising costs and depressed sales, they see direct marketing as one of the few bright spots in their business. Indeed, sales from catalogs, coupons, direct mail, and radio and TV offers are rising 10% to 15% annually, or double the rate of increase for retail sales.

Using television as a direct-marketing tool isn't new, of course. Commercials soliciting orders for everything from "golden oldie" records to storm windows are often aired by local TV stations. And years ago, individual cable-TV systems tried–with little success–to peddle real estate and general merchandise on a variety of shopping programs.

Today, however, the potential audience is much larger: Nearly 25% of U.S. homes receive cable, and more are being wired all the time. What's more, recent technological advances in two-way, or interactive, cable make shop-at-home services–whether merchandise demonstrations or "video catalogs"–more feasible. Though retailers have yet to overcome videoshopping's major drawback–the high cost per purchase–they "are looking at the new technologies as a way to expand their profit centers," says a spokesman for the Direct Mail Marketing Association.

Books, Cookies and Washers

A number of manufacturers, for example, display their products on the successful "Home Shopping Show," which is transmitted to some 35 million households by Modern Satellite Network, a cable-TV program service based in New York. (Modern Satellite recently acquired the show from its innovator, Washburn Associates, a Chicago cable-program producer.)

The Monday-through-Friday program charges manufacturers some $6,000 for a 10-minute product demonstration that is repeated five times in 35 days. Maytag washing machines, the Encyclopaedia Britannica and Walt Disney books are among the products that have been featured on the show. "Business is great," says Michal Laidlaw, the show's executive producer. Noting that there is no dearth of companies wishing to advertise products on the show, she adds: "We have more work than I can handle."

Procter & Gamble Co. experimented with the "Home Shopping Show" and apparently was pleased with the results. P&G presented a Pillsbury cookie recipe and afterward got 10,000 viewer responses to its coupon offer for cookies. "It's certainly an indication that people are watching the show," says John Levine, a P&G marketer.

The New York based production company Great Image had less happy results with its "American Teleshopping Network," a cable program that was transmitted to 112,000 homes in affluent Nassau County on Long Island. The show, which ran for several weeks last fall, presented 60 items from the Neiman-Marcus Co. catalog, at prices ranging up to $500 for china and silver candlesticks.

"The response, from a sales point of view, wasn't good," says Ronald Fopper, a Neiman-Marcus senior vice president. He adds, however, that the department store was "just testing the waters, and we will continue to investigate a video cable catalog."

A "Big-League" Venture

By far the most attention has been riveted on the well-financed and well-produced Shopping Channel, a joint venture of Times Mirror Cable, a unit of Times Mirror Co., and Comp-U-Card of America Inc., a private shopping service based in Stamford, Conn. Paul Kagan, a publisher of cable-TV newsletters, terms Shopping Channel "the first big-league attempt at the videoshopping business."

Shopping Channel is aimed at a total of 150,000 cable-TV subscribers in six markets served by Times Mirror cable systems: Hartford, Conn.; Louisville, Ky.; Springfield, Ill.; Midland, Texas; Newark, Ohio, and Orange County, Calif. The show runs seven days a week, 16 hours a day, and each half-hour segment is repeated five times a month. Introduced last April, it may become available to additional cable systems in September.

The show's concept is simple. It is a video extension of Comp-U-Card's main business: operating a buying service for consumers who, after paying $18 in annual dues, are entitled to discounts of as much as 40% on merchandise offerings.

The notion of televising the service came after Federated Department Stores Inc. acquired 10% of Comp-U-Card last year. Federated owns such well-known retail outlets as Bloomingdale's and Abraham & Straus in New York and I. Magnin in San Francisco. With Federated's access to merchandise and Comp-U-Card's existing billing and distribution system, a videoshopping service seemed a natural. For a variety of reasons, I. Magnin was selected as the source of goods for the program.

30,000 Items to Order

Shopping Channel viewers, after paying Comp-U-Card dues, can order displayed merchandise by calling a toll-free number. In addition, they can–and do–order any of the over 30,000 items I. Magnin stocks in its stores. "The number of products sold to members we gained through the Shopping Channel greatly exceeds the number sold to our regular membership." says Walter A. Forbes, Comp-U-Card's chief executive officer. "Things couldn't be better."

The spread of two-way cable should hasten the growth of videoshopping, cable-industry people predict. Instead of telephoning, viewers will be able to order goods with a hand-held, calculator-like device linked to a central data base by cable or telephone wires. Interactive systems are part of virtually all cable franchises currently being awarded, and companies like Warner Communications Inc. and Dow Jones & Co., which publishes this newspaper, are investigating applications for them, including home shopping services.

"The ultimate shopping will be the ability to summon up on the TV screen what you want to buy," says Gus Houser, the president of Warner Amex Cable, the cable TV joint venture of Warner Communications and American Express Co.

Indeed, a number of companies are experimenting with a version of videoshopping that is more akin to the traditional merchandise catalog. One is Viewtron, a joint venture of Sears, Roebuck & Co., Knight-Ridder Newspapers Inc. and American Telephone & Telegraph Co. Viewtron enables viewers to summon up a catalog's text and pictures on a specially adapted TV set. Cox Cable Communications Inc., a Cox Broadcasting Corp. unit, and Viewmart, an American Can Co. unit, are testing a similar system known as Interactive Data Exchange, or Indax. With both systems, viewers make purchases using an order form that appears on the screen.

Merchandise offered on Viewtron comes from such big retailers as Sears and J.C. Penny Co., as well as from some 15 local merchants in the Coral Gables, Fla., test-market area. Indax draws merchandise from about 30 retailers, ranging from big department stores to a lumberyard. "We are after any reputable retailers for the system." says Stephen Rowley, Viewmart's vice president.

In a separate experiment, Sears is putting portions of its famed catalog on videodiscs in an effort to spur sales.

The major drawback of all videoshopping services is their high cost per sale. Everything from the purchase of satellite time for cable TV to establishing a merchandise data base is costly, and sales thus far haven't justified the selling costs.

Another potentially sticky problem is thrashing out who takes responsibility for incorrectly processed orders: the cable system operator, the retailers or the program producer. "People are starting to think of such things," says Dick Smith, executive vice president of Tulsa-based Satellite Syndicated Systems, which is developing its own cable shopping show. But, he adds, "there may be no answers until a lawsuit resolves the issue."

other nonstore retailing such as TV "not available in any store." Some analysts predict that by 1990, one-third of all retail sales will be nonstore generated.[1] In-home selling, including "party plan," has declined in popularity in recent years. Mail order, however, has been experiencing growth rates significantly higher than conventional retailers.

Expectations were that 15 percent of all consumer purchases would be made by mail (including in-store catalog desks as well as telephone and interactive TV) by 1985. In 1982 sales by mail would surpass $35 billion, over 11 percent of general merchandise sales.

There are two factors that are encouraging these developments. First is a number of demographic and life-style shifts that are resulting in consumers being more concerned about the time and effort required for such activities as shopping; present-day pressures are mounting from a shortage of "discretionary time" rather than discretionary income. The other factor is the wide variety of communications technologies which are being experimented with that will enable the consumer to shop efficiently and satisfactorily without leaving the home.

Several experiments in telemarketing have already been conducted:

- The shopping channel, a joint venture between Comp-U-Card and others, advertises merchandise to cable TV watchers at substantial savings (See Exhibit 1).
- Both Sears, Roebuck & Co. and J. C. Penney are testing the QUBE system from Warner Amex Cable, the only operational two-way interactive cable system, in order to increase nonstore revenue.

According to The *Wall Street Journal,* July 14, 1981 (Exhibit 1), the problems of telemarketing are not conceptual, but economic. People are willing to buy through such a channel, just as they are willing to buy through direct mail and catalog sales, but the cost to bring the message to the consumer per sales response is high.

It is not difficult to discern why the experimental methods are uneconomical. Shopping is a very individual activity. For sales messages to have immediate impact, a potential buyer has to control the order in which they are presented. If a consumer is interested in digital watches, he or she wants to see sales presentations concerning digital watches and not about lawn mowers, no matter how effective a sales presentation it might be. (Regular TV commercials do not usually require immediate viewer response to be effective: their impact is delayed and cumulative.) So it is a matter of low statistical probability that a viewer watching a merchandising channel will be viewing what is important to him or her at any particular moment. Clearly what is needed is an approach where the shopper is in control.

To offer such a system with cable or satellite channels is prohibitively expensive. A viewer would need to "own" the channel for a short period of time, while his or her unique needs were being met. Simple arithmetic shows that it is infeasible.

The DiscoMall Inc. solution is to make use of inexpensive video disks. Sales and product information can be delivered inexpensively by this means, and interactive communication channels need be used only to permit the customer to place an order.

The DiscoMall system thus provides the best of both worlds: marketing efficiencies of mail order or cable TV shopping channels and the customer control of conventional shopping.

[1]Standard and Poor's 1980 Industry Survey.

THE DISCOMALL SYSTEM

A DiscoMall user will browse through the system's library of product information much as he or she would through a catalog, or a library of catalogs. Under computer control, the customer is provided digital, pictorial, and voice descriptions (much like a slide show), short moving video segments, specifications, prices, and information concerning availability of products. When the viewer decides to buy, he or she places an order for merchandise through the terminal keyboard.

The DiscoMall terminal is the principle interface between the system and the customer. Named ACORN, for *a*utomatic *c*atalog *o*rdering *n*etwork station, it should be visualized as an attractive desklike arrangement with three viewing screens (Exhibit 2). Two of the screens will provide pictorial information in the manner of a television screen. The third will provide written product information. In front of the screens sits a keyboard for customer response and, to the right, a 24-position printer for hard copies of orders and product information.

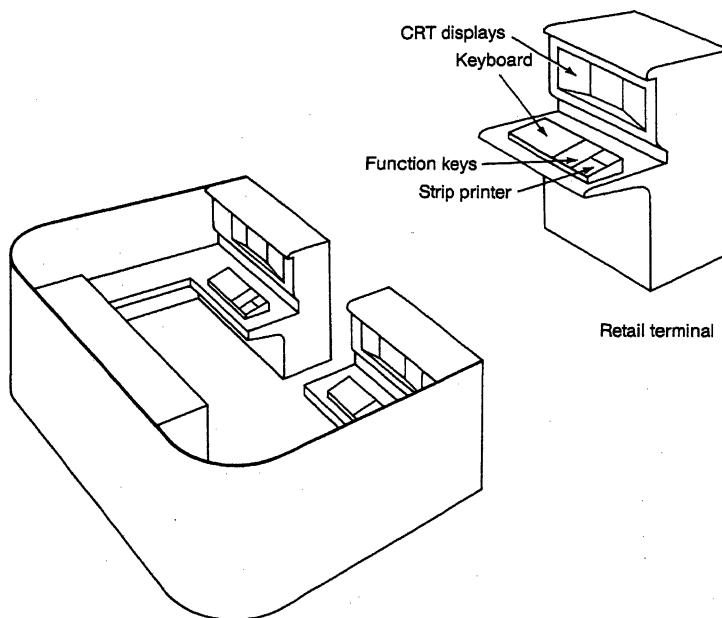
Material will be displayed on all three screens simultaneously. For example, it will be possible to have a motion picture sequence on the left screen, still pictures on the right screen, and written information appearing on the screen in the center. The presentation of material will be coordinated, so that the "speaker" can refer to all three screens.

Instructions for use of the terminal will appear on the screens as well. The instructions will be written and verbal. The user will be given choices and will respond to the system by pressing keys. For example, the digital screen may show a menu of options, with each option associated with a key on the keyboard. The customer will respond to the choice by depressing one of the keys.

The terminal will also permit the introduction of words and phrases typed in from the alphabetic keyboard. For example, the customer could type the word "camera" and the system would be able to respond with information about cameras.

All browsing and product description operations will be conducted from data stored within the terminal. When the customer wishes to place an order, he or she will depress a key and only then will the terminal connect to the central data base over a telephone link.

The ACORN terminal is designed to use available data storage and communications tech-

EXHIBIT 2 Acorn terminal.

nology. The key is the ability to store and access vast amounts of information from the terminal location without the need for an expensive telephone or cable link to the central system. This is accomplished by having two video disk players which access up to 30 video disks within the ACORN terminal. A typical Sears Roebuck catalog offers about 40,000 items, while it is estimated that the 30 video disks will be able to describe over 400,000 items. A video disk management unit facilitates playing any of the 60 disk sides on either of the video disk players under computer control. The video disks, capable of storing digital as well as pictorial information, are also able to store programs and data to be used by the computer.

The ACORN terminal also contains the equipment necessary to make the telephone connection to the central processor at the appropriate time in the ordering process.

The possibility of designing a separate communications network for the system was discarded, due to the complexity and risk associated with the approach. Instead, DiscoMall, Inc. plans to use the services of Telenet, a commercial packet-switching network. Each selling terminal will be connected to the network by means of a dial-up telephone connection. Telenet has offices in more than 200 cities in the United States. The installer of the selling terminal will pay local telephone charges only.

When the customer at the terminal decides to place an order for merchandise, he or she will communicate with a central data base that contains product availability and price information as well as individual account credit information. When the customer has verified that the order is correct, he or she receives a written confirmation of the order by means of the printer attached to the system. The orders are received by the central system and forwarded to the supplier by telephone link. If for some reason the supplier is unable to fill orders beyond a certain level of sales, it is the supplier's responsibility to notify the central system of that fact.

The central data processing center will serve as a recordkeeping and clearing house facility for all transactions. For example, it will forward orders to suppliers, maintain a data base of product information, and update commission and customer credit records. The product file will include instructions for obtaining the appropriate information from the customer when taking an order. The center will use programs that are modified versions of those designed for the airline industry. DiscoMall itself will not bill customers but will pass along all billing data to the appropriate credit agency (e.g., MasterCard). As a basis of comparison, a typical airline reservation system has been studied as a starting point for determining the size and capacity of the central processing center for this application.

MARKETING

DiscoMall can be divided into industrial and retail segments. In an industrial setting, the terminal would probably be installed in the purchasing department. For retail marketing, a store owner will install a terminal in a shopping mall, retail store, or other public location. Eventually, perhaps, the home will be the setting for a personal computer to connect to the DiscoMall system.

DiscoMall is conceived as a highly efficient marketing channel, combining the economics of mail order with the availability of product information at the user's demand, and with several times more in the way of selection than even the largest catalog. From the supplier's viewpoint, it reduces marketing costs because:

- Personnel costs in the selling situation are minimized.
- Inventory requirements are significantly reduced.
- Shrinkage due to loss or theft is reduced.
- Merchandise which has a thin demand may be handled; for example, merchandise designed for left-handed people or specialized industrial components.

Advantages of DiscoMall from the customer's standpoint include:

- A wide selection of merchandise options—more than otherwise would be available in a retail store

- The convenience of shopping from a seated position
- The ability to place an order immediately and to use any one of a number of credit options
- The opportunity to comparison shop for brands, features, and price
- Lower prices for some merchandise offerings
- The ability to receive instant response regarding availability of merchandise and credit authorization
- For the industrial user, lower costs, better selection, and quicker delivery.

David Jackson felt that DiscoMall was a natural merchandising device for industrial purchasing agents, since their normal shopping behavior was to browse through a library of catalogs. Only if additional information was needed would they call a salesperson. The DiscoMall system could reproduce exactly what purchasing agents were now doing. In addition, its speed, convenience, wide selection, and lower prices were all very important factors to the industrial customer.

In the retail segment, however, it was less clear how the customer would react. The customer could neither touch nor feel the merchandise. Also, the buyer could not take a purchase home immediately and might find it harder to return an item or have it repaired.

The buying public has been able to adapt to mail order and David Jackson believed they could adapt to DiscoMall. Still, DiscoMall, at least in its retail (out-of-the-home) form, did not provide some things that customers had grown to expect from mail order: to stay at home, to be able to shop after normal hours, and to avoid dealing with salespeople. While he was concerned at the moment with whether DiscoMall could be financially viable, David Jackson knew that at some time the marketing questions surrounding DiscoMall would need to be addressed.

DISCOMALL COSTS

Six major cost elements have been identified. They are catalog operations, terminals, Telenet, sales force, central facility, and administration. Each will be discussed below.

Catalog Operations

Catalog operations involve the preparation of materials to be included on the video disks which are installed in the terminals. The supplier provides copy and price information regarding the products to be offered for sale. Additional formatting and checking is required to install the data on the disks and to install the correct information in the central data files. This cost is not dependent upon the number of terminals or orders processed by the system, but it is dependent upon the number of disk refreshes expected over the course of a year.

Ten new disks would be issued per month, five in the retail segment and five in the industrial segment. About .6 labor-years of effort is required to prepare a master disk for production. Thus a department of 72 people will be required for catalog operations. The system will start out with an initial entry of 12 disks each for retail and industrial, which are estimated to require 1 labor-year of preparation for each side, a total of 48 labor-years.

In addition to these costs, the manufacturing of the disks themselves involves a $6000 charge for creating the master disk (both sides) and after that, production costs are $12.00 per disk pressed (both sides).

Four of the five disks issued will be regular disks divided into 180 "books" each containing the following amounts of information arranged as desired by the buyer:

100 still color pictures
10 minutes of sound
100 pages of text

The fifth disk will contain 120 books consisting of moving video sequences of 30 seconds each.

EXHIBIT 3 ACORN TERMINAL SYSTEM SUMMARY COST ESTIMATES

Disk management unit	$1522
Video displays (2)	560
Digital display	1270
Videodisk players (2)	1438
Control computer	986
Keyboard	365
Printer	310
Assembly and test	515
Cabinet	980
Storage, shipping and misc.	540
Contingency	849
Total	$9335
Service cost estimate (per unit installed)	
Labor-hours per month	.9
Cost per labor-hour	$62.00
Refurbishment cost	$1200.00

The supplier will buy one or more regular or motion books, which will be installed on the disks. In total there will be 120 disks issued a year consisting of 20,160 books. The disks would be inserted in the terminals for a 6-month period, when they will be removed.

Terminals

DiscoMall has developed sources for all of the necessary components of the ACORN terminal and will be directly involved only in the final assembly and testing of the terminal package. With the exception of the disk-handling unit and the cabinet, the components are slightly modified (hardened) versions of existing equipment. The disk-handling unit required special design but is based on the same technology used in "juke box" designs of the thirties and forties. The cabinet is to be made partly out of wood and partly out of sheet metal, and is not expected to present design or manufacturing problems. Cost estimates for the ACORN terminal are attached as Exhibit 3.

EXHIBIT 4 TELENET CHARGES

Charge per kilopacket* = $1.00
Charge per hour In-WATS† connect time = $22.00
Charge per hour of local access connect time‡ = $4.50

1 Kilopackets used
Packet = 128 characters
DiscoMall message: Terminal to central computer = 15 characters
Central computer to terminal = 300 characters
Total characters per message 315

Retail order = 12.8 messages
Industrial order = 28.0 messages

2 Average connect time§
Industrial order = 240 seconds
Retail order = 105 seconds

*Kilopacket = 1000 packets.
†Calls placed into an 800 number.
‡Available in 200 cities nationwide including most major retail and industrial areas.
§Minimum charge of 1 minute per each connection.

EXHIBIT 5 TELENET DISCOUNTS FOR HIGH-VOLUME USERS

Total monthly charges,	Monthly discount rate, %
0 to 14,999	0
15,000 to 29,999	1.5
30,000 to 49,999	3.0
50,000 to 69,999	6.0
70,000 to 99,999	9.0
100,000 to 249,999	11.0
250,000 +	13.0

To be conservative, DiscoMall estimates that the typical ACORN terminal will be installed at the same location for 2 years. After that time it will be reinstalled at another location directly, at a cost of about $400, or will be returned to the factory to be refurbished before being reinstalled. (The physical life of a terminal is expected to be 6 years with one refurbishing).

Telenet

As mentioned earlier, DiscoMall terminals will be connected to the central processing facility via the Telenet system. Exhibit 4 shows the Telenet charges to DiscoMall, Inc. Discounts for large monthly purchases are shown in Exhibit 5.

Sales Force

The rental of terminals is considered to be key to the success of the program. It will be necessary to find responsible installers who are willing to pay the monthly rental in exchange for commissions from sales made through the system.

DiscoMall plans to use a direct sales force. The number of salespersons required by DiscoMall would depend on the level of terminal sales and the total number of terminals in operation. It was estimated that each salesperson could sell around 40 terminals a year or service about 100 existing terminals. Each salesperson's time would be divided between these two functions.

Sales force estimates did not include marketing management, administration support, and training requirements which would add 50 percent to labor requirements. It was also felt that, due to early inefficiencies and startup problems, a minimal level of 50 salespersons was required. Each new salesperson hired would require initial training at a cost of $15,000. Turnover is expected to be 40 percent per year.

Central Facility

Based on estimates for construction and operating costs across the United States, the principals have developed plans to open the operating facility several miles from the center of a medium-sized city, but not near the eastern corridor, Chicago, Los Angeles, or San Francisco. There is no cost advantage to locating in these areas since communication costs are independent of location if the facility is within reasonable distance of a major telephone switching center. This decision was also made for security reasons. Omaha, Nebraska is one area being considered.

DiscoMall expects to construct or lease a facility in the 50,000 to 100,000 square foot range. Building costs are expected to run from $100 to $125 per square foot for office or terminal assembly and storage. Building costs for the computer facility are expected to be 150 percent of the costs for office use. Lease costs are estimated at $9 per square foot per year. Operating costs, including taxes, normal power (excluding computer operations), insurance, heat, water, cleaning, etc. are estimated at $6.50 per square foot per year. Operating costs will not include capital costs (interest) which are 19 to 21 percent at the time of the estimates.

The building size is to be determined by current and expansion needs. Office space is planned at 225 square feet per person. Assembly facilities for terminals are estimated at 24 square feet per terminal shipped per year. Computer system facilities are estimated at 5000 square feet.

The central site capacity is measured in terms of its message handling capability. For example, a system consisting of an IBM 4331 processor will handle up to 20 messages per second (mps), and an IBM 4341 system has been designed to handle up to 60 mps.

Based on the following assumptions, an IBM 4331 can support 1902 retail *or* 1285 industrial terminals:

- The average retail and industrial order will contain 1.6 and 4.1 transactions, respectively. A transaction is the order of one line item by one customer.
- The average retail and industrial transaction requires 7.875 and 6.830 messages, respectively.
- The system can achieve a throughput of 6.5 times the peak hour demand.
- The system will be utilized 50 percent of the time, 75 hours per week. (Note, the system day is longer than that of any terminal because it services three time zones.)
- The peak hourly order rate is six for retail and four for industrial terminals.

An IBM 4341 can support three times as many terminals as the IBM 4331.

Computer Operations The Computer Operations Department operates the central system facility. As an on-line system, it will require constant attention but not manipulation except in times of emergency. A staff of 24 will be sufficient to manage the 60 mps, IBM 4341–size facility, and only slightly more will be required to handle facilities of a larger size. An additional six people are required to deal with Telenet and AT&T with respect to communications facilities.

The cost of operating the computer itself would depend on the capacity required to support terminal operations. The IBM 4331, 20-mps capacity, would cost $1,377,100 to purchase with an additional $6750 monthly maintenance fee. An alternative would be to rent the 4331 for $55,200 a month with no additional maintenance fee. An IBM 4341, 60-mps capacity, would cost $3,505,800 to purchase with a $15,855 monthly maintenance charge or $151,100 a month to rent.

Just as important as the computer system is the backup provision in case of mechanical or power failure. (The operating system has been proved many times in actual operations, having been tuned and improved in reliability since its introduction in the late sixties.) Without backup, power or mechanical failures are the most likely causes of major system failure. Therefore, complete backup power and air conditioning systems will be provided at a cost of $450,000 for the 4341 system. Such backup costs will be roughly the same for systems of other capacities.

Programming Approximately 40 labor-years of programming will be required to tailor the application and management systems to DiscoMall specifications. The estimate also includes programs for catalog operations required to translate catalog information into digital code for the video disks.

Following the initial programming load, DiscoMall expects to maintain a staff of approximately 26 programmers for system maintenance and improvements. These figures include direct management but do not include secretarial or support services which will add an additional 20 percent to the totals.

Administration

In addition to the above, a number of expense items are included for which no specific estimates have been developed. These include

Administration This includes management, legal, personnel, accounting, and other staff

departments. In the long run this is expected to average about 6 percent of DiscoMall revenue (terminal rentals, book sales, and commissions less commissions paid to installers), but will be greater as a percentage of DiscoMall revenue in the start-up years.

Employee Benefits Included are the company's share of social security, medical, educational, and retirement provisions. This has been estimated at $9000 per person per year.

Research and Development (R&D) This is not considered a key factor in the early years, but there is to be a provision for R&D in a few years to support and service growth for the business. No estimate has been completed.

Interest VentCap and FMG will provide the capital required to start DiscoMall (facility construction and initial software development). Any working capital requirements would be financed by the bank, through a line of credit. In the summer of 1981, the expected interest expense was around 20 percent, but the interest rate may fluctuate over the life of the project. It is not anticipated that any dividends will be paid on common stock in DiscoMall, Inc. for at least 10 years.

Taxes Local and state property taxes (not income taxes) are dependent upon locations picked for facilities and other factors and are estimated to run between 2 and 4 percent of DiscoMall revenue. The higher number will be incurred if terminals remain the property of DiscoMall, Inc. State and federal income taxes are expected to be 46 percent of net before taxes, minus investment tax credits (ITC) on purchased equipment. ITC is estimated at 10 percent of acquired cost and will apply to terminals that are rented.

Contingency A matter of judgment by the financial analyst to cover for unknown items, cost overruns, poor judgment, and any other factor that will be an expense to the company not otherwise identified.

DISCOMALL REVENUES

DiscoMall, Inc. expects to earn revenue from three major sources: rental of DiscoMall terminals, commissions paid to the DiscoMall system by suppliers of merchandise sold through the system, and fees charged to suppliers for space in the systems catalog. The total of revenues from these sources will be called gross system revenue. DiscoMall revenue will be gross system revenue less commissions paid to retail and industrial installers.

The DiscoMall terminals will remain the property of DiscoMall, Inc. Retail and industrial installers will pay a monthly fee for use of the terminal and monthy updates of the video disks. Maintenance and repair of the terminals will be the responsibility of DiscoMall, Inc.

Suppliers retain title to the merchandise until an order is placed through a terminal. In this respect, DiscoMall and the installers are the suppliers' agents. Suppliers will pay a percentage of their sales as a commission. The tentative plan is to make these commissions 14 percent of industrial and 18 percent of retail sales. These commissions will be shared by DiscoMall and the installers. These charges do not include credit and billing costs nor do they include merchandise delivery charges, which remain the responsibility of the supplier.

Most industrial installers will view their share of commissions as a discount to the sales price. Suppliers may also be willing to list lower prices in the DiscoMall industrial catalog due to lower warehousing costs and, perhaps, distribution and scale economies.

To determine gross revenue, it is necessary to predict the average amount of sales that will be generated per terminal. Some experimental work has been done on this topic with the following conclusions:

1 Sales per terminal is dependent on average order size and number of orders generated. It is expected that the average retail order will be in the range from $18 to $24. The nature of

industrial buying, however, led DiscoMall to conclude that average order size would be less of a problem in the industrial segment, where orders in the range of $50 to $100 were expected.

2 A second factor in determining sales per terminal is the percent utilization of the terminal. Retail terminals have a utilization rate of about six orders per hour and retail establishments are open approximately 250 hours per month. However, the terminal will not be operating at capacity for that time period. Terminal utilization of 25 to 75 percent was anticipated, with a median of approximately 40 percent. Industrial utilization was expected to be four orders per hour and industrial establishments are open approximately 40 hours a week for 50 weeks a year. The industrial segment is expected to be a more stable environment and terminal utilization rates are expected to be somewhat higher than for the retail segment.

Marketing Research

DiscoMall had already conducted preliminary market research to determine the demand for the DiscoMall system. Although there still remained uncertainty around the estimates developed from the survey, DiscoMall felt that the information they received was a start toward estimating the potential market.

Terminal Sales

From the first discussion of DiscoMall, Harrison, McKinley, and Jackson felt that involving a large national retail chain in the venture was key to its success. This would serve not only to lock in a substantial portion of the terminals needed to get the system started but would also be a selling tool in both convincing other retailers to join the system and convincing suppliers to buy space on the catalog.

In the process of finding the right partner in the venture, the three principals had contacted several national merchandisers. Compton's, a discount department chain with 1600 stores, was singled out as the ideal candidate. Compton's lacked its own catalog and had been interested in entering the mail-order business for some time. Negotiations had proceeded to the point now of talking about financial arrangements, although no firm Compton commitment to DiscoMall had been made. Compton's had informally guessed the number of terminals they might be willing to install in terms of the monthly rental fee (see Table 1 below). Through the preliminary

TABLE 1 TERMINAL INSTALLATIONS

Rental fee, $/month	Compton's installations, # of units	Other retail, # of units per year
$300	900	550
450	600	350
600	450	250
750	325	175
900	225	140

TABLE 2 SURVEY OF INDUSTRIAL PLACEMENT OF TERMINALS

Rental fee, $/month	Number placed in 500 companies	Number per company
300	33	.066
450	22	.044
600	15	.030
750	11	.022
900	9	.018

discussions, DiscoMall also had information on how many terminals could be placed each year in other retail outlets they had contacted (Table 1), assuming Compton's was a participant in the venture.

David Jackson had carried out some preliminary investigation of the potential in the industrial market. He had surveyed and received responses from 500 randomly sampled (and representative as to size and other characteristics) companies out of a potential market of about 5000 companies. In the survey the DiscoMall concept was briefly described and the respondents asked whether they would purchase a terminal and how many they would purchase at various prices. The results for the 500 companies gave DiscoMall an estimate for yearly industrial terminal sales and are shown in Table 2.

Catalog Sales

One of the primary goals of the marketing research was to determine supplier interest in buying space on the DiscoMall catalog. DiscoMall contacted approximately 1700 suppliers, out of an estimated potential market of 20,000 and received 862 responses to a survey asking the supplier a series of questions concerning their interest in the DiscoMall concept.

The first question tried to determine the sensitivity of supplier interest in DiscoMall relative to the price charged per book, assuming there were 3000 terminals in operation (books and 30-second spots were grouped together and considered a single item). The suppliers were asked first, if they were interested in the DiscoMall concept. Those who were interested were then given a list of prices per book and asked if they were willing to pay that much per book. Of the 862 responses, 548 said they were not interested at any price. Of those that were interested, their responses to the price they were willing to pay per book produced the results in Table 3.

Those suppliers that said they were interested in the DiscoMall concept were then asked how many books they were willing to buy. The survey produced the results of Table 4.

TABLE 3 PRICE SENSITIVITY AND CATALOG BOOK SALES

Price of book	Number of interested suppliers willing to pay price	% of interested suppliers*
$300	314	100.0
400	255	81.2
500	210	66.9
600	152	48.4
700	85	27.1
800	34	10.8
1000	0	0.0

*Number of interested suppliers = 314. Total number of suppliers in sample = 862.

TABLE 4 BOOKS BOUGHT PER SUPPLIER†

Books needed	# of suppliers	% of suppliers*
1	195	62.1
2	74	23.6
3	26	8.3
4	14	4.4
5	5	1.6
	314	100.0

*Sample size = 314 suppliers.
†Question asked only to those interested in DiscoMall concept.

TABLE 5

Terminals in operation	Suppliers interested in buying books	% of suppliers interested*
3000+	314	100.0
2500	256	81.5
1500	195	62.1
1000	127	40.4
500	74	23.6

*Sample size = 314 suppliers.

Finally, it was apparent that it could be several years before the system had 3000 terminals in operation. The suppliers who had shown interest in the system were asked if the number of terminals in operation would affect their decision to buy space on the catalog. They were given different levels of terminals in operation and asked if they would still be interested in the DiscoMall catalog. The survey produced the results presented in Table 5.

PRICING

The pricing of services was planned only tentatively at this time. The pricing should ensure a reasonable return for suppliers, installers, and DiscoMall, Inc.

DiscoMall had established four items or services to be priced:

1 Books (regular or motion sequences)
2 Commissions on sales for retail products
3 Commissions on sales for industrial products
4 Terminal rental

Some tentative objectives to be achieved in pricing had been outlined:

1 The pricing for books will cover the cost of catalog operations.

2 The terminal rental will cover manufacturing and sales cost of the terminals. Compton's would pay the same rental as other retailers and industrial installers.

3 Commissions will be sufficient to provide revenue for the terminal owner that will be profitable, and provide DiscoMall enough revenue to cover all additional expense and profit.

4 Revenue will be sufficient to fund growth through internal cash sources by the fifth year of operation. By that time DiscoMall hopes to have a 250-mps IBM 3033 system or equivalent, operating at 60 to 80 percent of its capacity. A fall-back position is to have by that time a 60-mps IBM 4341 system.

Messrs. Jackson, McKinley, and Harrison had developed the following list of proposed prices and commission split. None of the prices, however, had been evaluated in the context of the information received from the marketing research and all were open to change.

Books (catalog)	$500
Terminal rental	$600/month
Retail commission	
Terminal installer	12%
DiscoMall	6%
Industrial commission	
Terminal installer	10%
DiscoMall	4%

FINANCIAL ARRANGEMENTS

All of the financial arrangements were yet to be negotiated among the various parties interested in the new business. Although the principals of DiscoMall had not settled on the offer they would make to the other parties, their current plan was as follows:

1 Obtain equity financing from VentCap in the amount of 50 percent of the initial investment required.

2 Borrow 30 percent of required initial investment from FMG to be paid back at the end of 10 years (or sooner) with 20 percent interest. Manufactured terminals and other property would be used as collateral. As part of the loan agreement, FMG would be required to commit a line of credit to finance future working capital requirements (that is, to the extent that cumulative cash flow was negative) at the prime rate (currently 20 percent), with a cap of $20 million.

3 Obtain the other 20 percent of initial investment from Compton's Department Stores as an equity position along with a commitment to install a number of terminals consistent with Table 1. Compton's would act strictly as a retailer, not as a supplier.

Ownership of the firm would be as follows under this preliminary plan:

DiscoMall principals (6 percent to Jackson and 5 percent to two others)	16%
Venture Capital Associates	60%
Compton's Department Stores	24%

The DiscoMall principals had checked on the resources of the firms and knew that the plan fit into their capabilities. They knew that VentCap would invest $100 million this year in new, risky businesses, although they would be reticent to put a very large fraction of the $100 million in one place. FMG, a bank with assets of $74 billion, was looking for new opportunities with innovative firms. Compton's was known to have $6 million in the bank to be used for expansion purposes (they might even push for an equity position in DiscoMall larger than 20 percent).

One question that nagged at David Jackson was whether they would be leaving money on the table with their planned offer; perhaps they should ask for larger ownership shares. It had also occurred to him that maybe by starting only in the industrial segment they could get by without Compton's. VentCap could, perhaps, come up with more equity money. Or the bank, by setting up an independent firm, could potentially take an equity position. While there were lots of possibilities, he was going to worry just about the plan above for the time being.

David Jackson found it difficult to predict what the other parties would want to do because he had not completed his own financial analysis of the business. For one thing, he was not sure how much initial investment was needed. That depended on buy or lease decisions for the central facility and the computer, which had not yet been made. He was aware that the first order of business was to make pro forma projections of revenue, costs, and capital requirements for DiscoMall, Inc. for the next 10 years. Compton's, VentCap, and the bank would want to see projections, and some form of financial analysis, for the financial return from DiscoMall. Then he could address the lease or buy decisions and pricing decisions and plan his offer to the other parties. It would also be wise to plan a strategy for what to do next in developing the DiscoMall concept if the parties were interested in it. Should he do more market or cost research? Should he develop a pilot study with mocked-up terminals? How should he plan the introduction of the business?

Presentation to the Financial Committees

David Jackson would decide what information he and his colleagues needed to have on hand for his meeting with the financial committees of VentCap, Compton's, and FMG. He knew he needed to prepare for questions on the assumptions of his financial analysis and his treatment of the numerous uncertainties. They would surely ask about the chances for various possible

financial outcomes. There were also some questions about whether the suppliers and retail operators would make enough money with DiscoMall to justify their participation in the system. Mr. Jackson knew that it was time to get to work on developing this information if he was to have it ready for the presentation scheduled for October 1, 1981, now a week away.

NONPROFIT CASES

WESTVIEW ENVIRONMENTAL PLANNING (A)

Many large cities and towns were facing a problem similar to that of Westview. John Mills, the City Manager of Westview, had to find new facilities for the disposal of solid wastes. In Westview, as in other cities, the public was generating wastes at a steadily increasing rate which was rapidly outgrowing disposal capabilities. In New York City, for example, the Sanitation Department presently disposed of about 24,000 tons of refuse per day; it was estimated that this figure would increase by 50 percent in the next 15 years. At the current rate, present disposal facilities would be depleted by 1975. In a recent national survey for the Public Health Service, it was found that approximately 50 percent of the cities with over 25,000 in population did not know where they would be able to dispose of solid wastes in 6 years.

There were currently two primary means of waste disposal: landfill and incineration. Westview was using landfill as its means of disposal. Direct landfill methods buried raw refuse under a layer of soil, usually in marshes, tidelands, or other lowlands of marginal economic value. The refuse was compacted and allowed to settle for several years, after which it was used for recreation, residential housing, or commercial/industrial building. Proponents of landfill pointed out that it was the cheapest method to use based on operating costs. In addition, it created new usable lands. On the other hand, landfilling might cause ground water pollution unless special precautions were taken (a moderate expense). As pointed out by conservation groups, this method often destroyed wildlife sanctuaries and other natural assets which were important to maintaining a balanced ecological system.

Because of the limited number of sites available, many cities had been forced to use incineration as a means of reducing the volume of solid wastes. Incineration reduced the volume of refuse by 80 percent and consequently significantly extended the lifetime of available disposal sites. Disadvantages of incineraton included its contribution to air pollution and its cost. In addition, operating costs were about three times that of the landfill operation and capital costs were high.

There were also other means of processing refuse which involved combinations of combustion and compaction processes. These processing units were at the prototype stage and had been subjected to extensive testing. Manufacturers claimed they were ready to deliver full-scale

This case was written by Professor Charles A. Holloway for purposes of classroom discussion. It draws on material in H.M. Ting, "Aggregation of Attributes for Multiattributed Utility Assessment," Technical Report No. 66, Operations Research Center, Massachusetts Institute of Technology (August, 1971) but does not represent any actual city. © 1974 by the Board of Trustees of the Leland Stanford Junior University. Used with permission.

TABLE 1 GROWTH IN REFUSE FOR WESTVIEW

Year	Refuse, tons/day
1974	10,000
1975	10,500
1976	11,000
1977	11,700
1978	12,500
1979	12,500
1980	13,000
1981	13,500
1982	14,000
1983	14,500
1984 and beyond	15,000

models, but as yet none had been placed in operation. The economics of these plants were somewhat uncertain due to the lack of relevant operating experience and the limited availability of government grants to offset some of the capital costs.

Westview currently collected 10,000 tons of refuse daily. The expected growth in refuse, based on both population growth and the increasing trend toward disposable products, suggested that total waste requirements would increase significantly (see Table 1).

A leveling off in waste generation was expected to occur in 1984 as a result of exhaustion of available residential capacity and a recognition that many goods in short supply would be recycled due to our limited resources. In fact, many thought this phenomenon would set in earlier.

With these waste disposal requirements, current landfill sites would be at their design capacity by 1979. Mr. Mills and his assistant had identified a number of alternatives over the last few months. The choice seemed to have narrowed down to the following:

1 Continue to use the current landfill site by mounding the land above its design level. This would extend the usable life to 1989 or 1994.

2 Use landfill but open up new lands bordering the lake, which was now basically a swamp. Engineering surveys have shown that this swamp has the capacity for 10 to 15 years worth of refuse at 15,000 tons/day.

3 Build a large incinerator with a capacity of 15,000 tons/day and landfill the residue at the existing site.

4 Purchase a combined plant to separate, compact, and burn the refuse, creating some power and virtually eliminating the disposal problem.

In discussing these alternatives, Mr. Mills and his assistants identified a variety of concerns. Of course, cost was a major issue for the city; but beyond that, the alternatives would impact other areas of the city. Environmental issues were becoming increasingly more important to the citizens of Westview. The local chapter of the Sierra Club was active in demonstrating to the citizens and the school children the importance of maintaining open space, watersheds, and other natural lands (including swamps) to prevent severe disruptions in the ecological life cycle. In addition, issues of air and water pollution had been raised by several groups as well as independent citizens.

Although not diametrically opposed to the Sierra Club stand, certain groups tended to discount the environmental concerns as a fad. They felt there was plenty of open space and land to support continued growth in both industry and population. Just last year, the city council adopted a master plan which specified growth targets of 10,000 people and 8000 jobs by 1980. Since the council adoption came after an election in which the two winners supported the

growth platform, it seemed clear to Mr. Mills that not only the city council, but also the citizens were in favor of the growth goals.

Each of the alternatives seemed to relate to these concerns in different ways. Alternative 1 appeared to be lowest in cost. However, mounding landfill areas would make the land unsuitable for residential or industrial construction; its only use would be limited recreation (such as archery, pistol and rifle ranges, or possibly a golf driving range). This relatively restricted land use capability would raise significant objections in the council as well as the citizenry, but Mills thought he could win approval if he went all out for the alternative.

Reclaiming the swamp was also a low-cost program but it might possibly cause ground water pollution. Naturally it would also be opposed by the conservationists and those who felt the swamp was a necessary part of the ecology of the region.

Alternative 3 would require a $30 million capital expenditure. Because it essentially burned waste, it would also contribute to air pollution in the area. This would be very unpopular with those who lived in the foothills where the pollution was expected to accumulate.

The city's share of the capital costs for the fourth alternative would be $40 million. This alternative had the advantage of extending the capability of the current landfill site indefinitely. Air pollution would be less than that produced by alternative 3 and the processed waste material used for final landfill would not contribute to ground water pollution.

PRODUCTION, OPERATIONS MANAGEMENT CASE

GREEN VALLEY FOODS

Green Valley Foods processes and markets canned and frozen vegetables. The profitability of the company depends to a great degree on their ability to harvest and package the year's worth of crops efficiently in a 6-week period. The company was undertaking a project in late 1983 to improve the planting schedule and management of the harvest for the canned corn crop. A pilot study was being conducted in the Illinois division of the company.

Each year the amount of corn needed is budgeted at the end of the calendar year. Then contracts are procured for the number of acres needed to produce the budgeted crop. This decision had already been made for the 1984 growing season; contracts had been arranged for 4147 acres in Illinois.

There were two limitations to meet in accomplishing the budget yield. First, there was a limited growing season. Secondly, there was a fixed capacity at the cannery of 2520 pounds per 48-hour period. Ideally, the crops would mature in a uniform, continuous way to use the cannery most efficiently. If, because of equipment constraints, the corn could not be harvested at peak maturity within a 48-hour period, it would turn out as substandard quality and sell at a reduced price. If too much corn became ripe at once, then some acres might be "passed," at a cost of $800 per acre.

The planting schedule affects everything that happens in the growing season. The time when acres are planted determines how fast they mature and their yield. It was also felt by some experts that the location of the planting might affect yield and the time to maturity.

Unfortunately, it is impossible to do any exact planning. Weather, of course, varies from year to year providing variability in yields and the time between planting and maturity. Also, the time of planting cannot be guaranteed. If the ground is too wet or dry, planting must be postponed.

A useful method for crop planning being explored by Green Valley is the heat unit method. The method is based on the fact that there is some minimum temperature at which growth occurs and a maximum temperature above which growth stops. Within the range between minimum and maximum temperatures, crop development progresses roughly linearly with the number of heat units, where for a particular day,

$$\text{Heat units} = \text{average temperature} - \text{threshold temperature}$$

Crop maturity would be determined by the cumulative number of heat units it has received since planting. It had been estimated that 1680 heat units (using Fahrenheit temperature) were needed for corn to mature.

Green Valley now wanted a model to help them with their planning of the corn crop planting and harvest.

RISK MANAGEMENT CASES

HORSE SENSE

Charlie Paddock was a racetrack enthusiast. He bet on the horses for thrills, not money, since his Acme Glue Company pension and American Fertilizer Corp. dividends provided financial security. Mr. Paddock explained his betting policy as "First of all, I know that the odds at a racetrack are set by the amount of money being bet on a particular horse by the general public. The odds, therefore, represent the general public's opinion on the probability that each horse will win the race. I also know that the odds on the tote board are adjusted so that the racetrack can keep 10 percent of all bets as its commission. Using this information I convert the tote board payoffs into the implied win probability distribution for all bettors. I compare this to my own probability assessment before deciding if I should place a bet. I bet 5 big ones ($5000) or nothing. My rule has been to bet only if I can get 10 percent return on average; if a filly posts 1 to 1 odds,[1] I will not bet on her unless I think she has at least a 55 percent chance of winning. But that rule doesn't tell me which is the best bet; I'm not sure it's the best rule I could use, either."

As post time neared for the eighth race at Sasparilla Springs, Charlie consulted his worksheet (Exhibit 1) and wondered which horse, if any, warranted a wager. Conversations with Charlie have revealed that constant risk aversion with a risk-aversion coefficient of .00004 approximates his attitude toward risk. Use the appropriate utility function to determine certainty equivalents for bets that he should be willing to place in the eighth at Sasparilla Springs. Can you narrow down alternatives before you begin to run the numbers? What do you think of Charlie's rule of 10 percent average return?

EXHIBIT 1 CHARLIE'S WORKSHEET

Horse #	Name	Listed odds	P(W) public*	P(W) Charlie†
1	Tea Biscuit	17 to 1	.05	.15
2	Save Your Money	4 to 1	.18	.30
3	Waste O' Time	3 to 2	.35	.35
4	Fool's Folly	3 to 1	.22	.10
5	First in Ninth	7 to 2	.20	.10

*Probability assessment of horse winning for the general public.
†Charlie's probability assessment of horse winning.

[1] $2 returned for each $1 bet, assuming the horse wins.

HEDGING

John C. Hicks is a farmer in Middletown, Nebraska. John, like all of the farmers around him, grows corn and ships his harvest to Kansas City for sale at the prevailing market price. The farm has been in John's family for three generations; from his inheritance and prudent management John has built his net worth to $200,000. Since the family has kept detailed records of the amount of corn harvested each year and the revenue generated by the sale of the crop, John felt he could make accurate probability forecasts for the amount of corn he could expect to harvest and the price that it would bring at market. John found that the farm was most likely to produce 100,000 bushels of corn in a normal year. A bad year could push production as low as 50,000 bushels, and in a really good year the farm might produce as much as 150,000 bushels of corn. John also found that it was most likely for corn to be selling for around $3.10 a bushel, but over the past 5 years it had fallen as low as $2.40 a bushel or risen as high as $4.00 a bushel. Both the harvest and price frequency distributions approximated a triangular shape. The records also indicated that the farm incurred about $230,000 a year in fixed costs and there was a variable cost of about $.50 for each bushel of corn produced. The tax schedule for farmers meant that John did not pay any taxes unless he earned over $25,000 in a year. On $50,000 profit he would have to pay about 12 percent in taxes and this increased gradually to a maximum of 40 percent taxes on profit of $300,000 or more.

Like a lot of his neighbors, John was being pressed by increased costs and the failure of revenues to keep up with this increase. John was worried that a really bad year could wipe him out and he might lose the farm. A neighbor had informed John about the opportunity to hedge his crop. He could sell forward either 50,000 or 100,000 bushels prior to harvest for a guaranteed price of $3.10 a bushel. In doing this, John would have to guarantee the delivery of a fixed amount of corn on a given date before he knew how much corn his own farm would produce. If John's own crop fell below the amount of corn he had sold forward he would have to go into the open market, buy enough corn to make up the difference at the market price, and sell this corn to the investor who had bought his crop at the specified price. Any corn that John's farm produced above the amount specified in the contract would be sold on the open market at the prevailing market price.

John's goal was to reduce the risk of losing money (or the farm) due to a poor harvest or a drop in the price he received for his corn. He thought hedging might be a way of doing this, but he was not sure.

EXHIBIT 1 IFPS MODELS FOR HEDGING EXERCISE

1 Harvest and price dependent

```
MODEL HEDGE2  VERSION OF  05/25/82  15:36
5 COLUMNS 1-3
10 BUSHELS = TRIRAND(50,100,150)
15 PRICE = 6.25 - 0.01 * XPOWERY(BUSHELS * 1000, .5)
20 BUSHELS FORWARD = 0, 50, 100
25 RATE FORWARD = 3.10
30 *
35 SALES = (BUSHELS - BUSHELS FORWARD) * PRICE + BUSHELS FORWARD * L25
40 *
45 FIXED COSTS = 230
50 VARIABLE COSTS = .5 * BUSHELS
55 PBT = SALES - FIXED COSTS - VARIABLE COSTS
60 TAXES = INTERPOLATION ON(PBT,-230,0,25,0,50,6,300,120)
65 PAT = PBT - TAXES
```

2 Harvest and price independent

```
MODEL HEDGE1  VERSION OF  05/25/82  15:28
5 COLUMNS 1-3
10 BUSHELS = TRIRAND(50,100,150)
15 PRICE = TRIRAND(2,40,3,10,4.00)
20 BUSHELS FORWARD = 0, 50, 100
25 RATE FORWARD = 3.10
30 *
35 SALES = (BUSHELS - BUSHELS FORWARD) * PRICE + BUSHELS FORWARD * L25
40 *
45 FIXED COSTS = 230
50 VARIABLE COSTS = .5 * BUSHELS
55 PBT = SALES - FIXED COSTS - VARIABLE COSTS
60 TAXES = INTERPOLATION ON(PBT,-230,0,25,0,50,6,300,120)
65 PAT = PBT - TAXES
```

ASSIGNMENT

Using the IFPS Models in Exhibit 1, which computes profit after tax, determine whether John should hedge 0, 50,000, or 100,000 bushels for each of the following two cases, using a logarithmic utility function. Explain your result. Just to check your intuition, check what answer you would have with logarithmic utility if John's net worth were much lower (say $60,000) and therefore he were more risk averse. Does this answer conform to your intuition?

1 Assume that past history has shown that the price of corn varies directly with the amount of corn produced in any given year. Specifically that the price of corn per bushel = $6.25 - .01(\text{Avg. Crop})^{.5}$. Further assume that the crop John produces on his farm is representative of the average crop for all farmers. When John has a good year, everyone has a good year; when John has a bad year, everyone has a bad year.

2 Now assume that the market price is independent of the amount of corn produced by John on his farm. Use the distributions for price and bushels harvested that John had established from his records. Would John now be better off if he hedges his crop? If the answer to this question is different from the answer to the first question explain the reasons behind the change.

SMALL BUSINESS CASES

LAND AND ESTATE BROK.

STEVENS & COMPANY

Bob Patterson thought, "What a great idea!," complimenting himself on the novel way to make a real estate sale. He had been working for some time with Gordon D. Sandler, who was interested in buying the Blandfield estate, but so far he had met with little success. Mr. Sandler, President of Sandler Electronics, liked Blandfield as a place to live, but he was concerned about the value of the estate as an investment. Blandfield, with a price of $4.5 million, was a large, risky, long-term investment. Mr. Sandler had made investment decisions almost as large and complex as this for his business, but he normally had available a thorough analysis of the investment opportunities. Bob Patterson decided that that was what he would give him.

Bob's idea was to develop a computer model that would detail the financial characteristics of the Blandfield investment. He could use the model for any party interested in the property, but an interactive model would be especially useful for Sandler because Gordon could try what-if sensitivity analysis and risk analysis to his heart's content. This would be a great experiment, Bob thought. In another month, Bob would be getting his own IBM personal computer, and if a computer model would help him sell large properties, then he could use financial investment models in his business on a routine basis.

MR. SANDLER

Over the past 30 years, Gordon Sandler had built his father's electrical components manufacturing firm into a growing company valued at $5 million. He was now president and, at 55, was ready to shed some of his heavy operating responsibility. His plan was to become chairman of the board, hiring a replacement to take over as president of the company, which would allow him to reduce his time commitment to the firm. It appeared that, for at least the next 15 years, his salary, which was his only current source of income since his company did not pay out dividends, would provide enough money for him to maintain his current lifestyle. This would make it possible for him to pursue his dream of living in semiretirement on an estate like Blandfield.

Sandler Electronics produced real-time control devices for specialized applications. Up to now, they had been used almost exclusively in control devices for certain weapons on ships and land vehicles. A new industrial division, however, had been developing a market for the devices to automatically control certain large machines in heavy-duty manufacturing applications.

Mr. Sandler would be leaving the new president in the fortunate position of having just

This case was prepared by Michael McEnearney, Research Assistant, and Samuel E. Bodily, Associate Professor, for a supervised business study by Charles Barnard, as a basis for class discussion rather than to illustrate effective or ineffective handling of an administrative decision. Although the situation is real, the specific investor is ficticious and should not be construed to represent any specific client of Stevens & Company. Copyright 1982 by the Colgate Darden Graduate Business School Sponsors of the University of Virginia.

signed a 10-year contract with the Department of Defense. The contract included the delivery of components in quantities equal to that of 1981 at the same price adjusted for inflation (using the Consumer Price Index). In the industrial division, which in 1981 accounted for only 10 percent of total sales, growth was expected to be similar to that of the electronics industry as a whole. Forecasts indicated that the most likely growth in the industry over the next decade would be 10 percent per year, with a 90 percent chance that growth would be in the 5 to 23 percent range.

The plan that Mr. Sandler was contemplating was to sell 49 percent ownership of his company for $2.45 million and use the proceeds to buy Blandfield and develop it to its potential as an agricultural and forestry operation. Mr. Sandler had received an offer for that amount from an investor who was willing to allow the new president to buy a small part of the 49 percent. Giving the new president an opportunity to own a piece of the company would be an advantage in attracting and keeping a strong executive.

Mr. Sandler realized that to insure his position as chairman and to keep control of the company in his family, he must retain 51 percent ownership. One of his concerns with the plan for Blandfield was that it might cause him to reach so deeply into his pocket in order to buy and operate the property in the early years that he would need to sell more of the stock in his company. His own calculations seemed to indicate that the $2.45 million would be enough money to keep things going until he could begin to harvest timber from the property, but he wanted to understand the investment requirements over the life of the project much better before he would be convinced to buy Blandfield.

BLANDFIELD PLANTATION

Blandfield Plantation was located in northeastern Virginia on the south bank of the Rappahannock River just two hours south of Washington, D.C., and one hour north of Richmond, Virginia. The plantation consisted of the manor house and 3500 acres of land. The majority of the land had been forested for future sale of the timber while the remainder of the land was cropland and marshland.

Manor House

The manor house, as shown in Exhibit 1, was a beautiful mid-Georgian style mansion built around 1771. The house was clearly one of the finest 18th-century Georgian houses built in America. Consequently, the house was included on both the Virginia and National Register of Historic Places.

The Blandfield Mansion was still in the hands of the original family and had received careful attention to repair and maintenance during its 210 years. The result was that the house was in excellent condition and would not require any repairs or renovation prior to the Sandler's moving into the estate.

Marshland

Blandfield included approximately 800 acres of marshland and represented one of the largest privately owned hunting and wildlife preserves in the Middle Atlantic area. Over 30 species of water fowl, including Canada geese, mallards, and even an occasional bald eagle had been sighted in the area.

Blandfield also included over 3 miles of riverfront property along the Rappahannock River. Rock fish, gray trout, and catfish were some of the most popular for anglers, however, inland herring, shad, large-mouth bass, and blue-gill perch could also be found in the river. Mr. Patterson had contacted several hunting and fishing clubs in the area and found that the marshland could be rented for about $10,000 a year.

ON THE VIRGINIA REGISTER OF HISTORIC PLACES □ ON THE NATIONAL REGISTER OF HISTORIC PLACES

BLANDFIELD • CIRCA 1771 • ESSEX COUNTY, VIRGINIA

BLANDFIELD, circa 1771 is the second of two houses built upon the 3,500 acre plantation of the same name and is still in the original family ownership. Major Robert Beverley, the first to arrive in Virginia, left the quiet community of Beverley in Yorkshire, England and quickly rose to prominence in his adopted country. Major Robert, grandfather of the Beverley who conceived and built the first house on BLANDFIELD, settled near Jamestown, Virginia. He was elected Clerk of the House of Burgesses in 1670 and Justice of his county of residence in 1673. Strong in his belief that the British Crown should be served, he took the British governor's side against Colonial insurgents until the unusually harsh punishment and degradation perpetrated by the authorities proved too much for the Colonials' humanist standard bearer. He refused any more cooperation and was eventually imprisoned without trial on a ship in the Rappahannock River not far from where his grandson would built BLANDFIELD. Major Robert died on the 15th of March 1687 and is believed to have been buried at Jamestown.

EXHIBIT 1 Blandfield manor.

Timberland

The largest single resource on Blandfield was timberland. As Exhibit 2 indicates, these wooded areas covered approximately 2000 of Blandfield's 3500 acres. Using natural boundaries whenever possible, the timberland had been divided into 16 distinct stands. Each stand was planted at a given time so that the age of all the timber was easily established. (See Exhibit 4.)

All of Blandfield's stands had been planted in loblolly pines. This pine was chosen due to its characteristics of rapid growth and good form which resulted in high-quality logs when mature. However, loblolly pine grows slowly during its early years and requires a process called *hardwood control* to prevent suppression of the pine growth due to the inevitable growth of hardwoods in the stands.

Based on recommendations of the Virginia Division of Forestry, the stands would be harvested in a two-stage process. During the twentieth year of growth local loggers would be given the rights to remove the hardwoods and low-quality pines (hardwood control) so that optimal growth of the high-quality pines could be achieved during years 21 through 35. These loggers would remove approximately 10 cords of wood for every acre of timber and pay $8 to $9 per cord of wood removed. In the thirty-fifth year, considered the optimal year to harvest loblolly pine, loggers would be sold the rights to clear-cut (remove all the remaining trees in the stand) for $45 to $50 per cord of wood removed. The loggers were responsible for replanting the stand with loblolly pine after the clear-cutting operation was finished.

Mr. Sandler had never been involved in timber farming. He was assured, however, that efficient timber farming required little more than following the advice of the Virginia Division

EXHIBIT 2 TIMBER SUMMARY

Stand	Acres	Age	Volume in cords*
1	50	5	30
2	250	20	8670
3	225	17	6100
4	40	26	1910
5	60	28	3090
6	525	13	8610
7	40	1	0
8	90	13	1480
9	140	23	5820
10	100	25	4580
11	110	23	4570
12	80	23	3320
13	20	11	220
14	40	1	0
15	40	26	1910
16	200		wooded buffer (not sold)
Total	2010		50,310

*Bob Patterson had in his files the results of a statistical study done at the Southeastern Forest Experiment Station, Asheville, North Carolina. This could be used to predict the yield from loblolly pine using the age of the stand and a site index (which was 80 for Blandfield). A regression on historical data gave an R^2 of 90 percent with a standard error of 1 percent of the estimated volume at the mean values for age and site and 3 percent at the extreme values of age and site. The standard deviation of individual plot volumes ranged from -12 to $+14$ percent of estimated volumes throughout the range of age and site. A predicted volume could be obtained by first calculating the logarithm of volume in standard cords per acre (LOGVOL) from the equation below, then converting to actual volume of merchantable pulpwood by taking 10 to the power of LOGVOL (that is 10^{LOGVOL}). The equation, which assumes there has been hardwood control in the twentieth year, is:

$$\mathrm{LOGVOL\ (base\ 10)} = 1.2059 - 12.07050(1/\mathrm{age}) + .01172\ (\mathrm{site\ index})$$

of Forestry and the willingness to tie up a large amount of money in land while waiting for the trees to grow. It would be 7 years before Mr. Sandler could clear-cut the first stand and this would be stand number 5, which was one of the smallest. It would be a total of 10 to 12 years before Mr. Sandler could expect any sizeable income from the timber on Blandfield.

Farmland

Approximately 500 acres of Blandfield were highly productive sandy loam soils, a product of thousands of years of sedimentation along the Rappahannock River Basin. The land was currently used for cash grain crops and beef cattle production. Mr. Patterson was convinced, however, that the farmland was not being used as efficiently as it could be.

Mr. Sandler told Bob Patterson that timber farming was ideal because it would require a minimal amount of management. Mr. Sandler was concerned, however, that crop farming would be too time consuming and was not very profitable. Bob told Mr. Sandler that several other landowners in the area had taken the same position. They realized that farming was a good use of their land, yet they lacked the time or knowledge to manage it profitably. The result was that arrangements had been made with local farmers, whereby the farmers would do all of the work and incur all of the expenses of farming the land and would pay the landowners from 30 to 40 percent of their operating income for the use of the land. Furthermore, the landowner could indicate which crops and livestock would be raised on the farm.

Bob also told Mr. Sandler that a linear programming (LP) model, developed by the Department of Agronomy and Wildlife Management at the Virginia Polytechnic Institute, was available to help him choose the most profitable crops and livestock to raise on the farmland.

EXHIBIT 3 LINEAR PROGRAM OUTPUT

Activity name	Units	Price/cost per unit	Total Price/cost
Fescue	16.16	$ 354.00	$ 5722.32
Hay	33.03	129.00	4261.42
Hogs	190.44	922.48	175,676.87
Soybeans #1	487.00	188.00	91556.00
Mixed Hay	20.00	60.00	1200.00
Soybeans #2	199.00	150.50	29949.50
Fescue pasture/hay	27.16	−70.00	−1901.22
Pasture	112.84	−15.00	−1692.60
Corn	38552.76	−3.00	−115661.29
Jan–March labor	860.38	−3.75	−3226.41
April–May labor	957.84	−3.75	−3591.89
June–Aug labor	3000.00	−3.75	−11250.00
Sept–Oct labor	1756.34	−3.75	−6586.27
Nov–Dec labor	2000.00	−3.75	−7500.00
Grain storage	38553.76	−0.19	−7325.22
Summer squash	3.88	3400.00	13,175.89
		Total	$162,807.10

Notes: Information on the cost and revenue structure for a variety of different farm products were fed into the computer and the linear program solved for the optimal mix. The output above is an indication of what should be grown on the 500 acres of farmland at Blandfield. The units represent acres for all crops (soybeans, hay, pasture, etc.) with the total exceeding 500 because of multiple plantings. The units of hogs represent the number of animals to be raised and the units of labor are those in addition to the farmer who would manage the farm. Negative unit costs are for expense items (e.g., corn), which are needed to operate the farm but which are not necessarily produced on the farm.

The linear program was a resource allocation model which used the expected costs and revenues for various crops to pick the best mix for any given farm. Exhibit 3 shows the output of the LP model. The expenses and revenues used in the program were averages based on a number of working farms both in Virginia and the southeastern United States.

Selling Price

In their earlier conversations, Mr. Patterson had quoted Mr. Sandler a price of $4,500,000 for the Blandfield Plantation. After several preliminary discussions between Mr. Sandler and the owner, each felt that a value of $4.25 million would buy the property, and Patterson divided it as follows:

Land	$3,500,000	1000/acre × 3500 acres
Timber	402,480	$8/cord × 50,310 cords
House	347,520	
	$4,250,000	

Mr. and Mrs. Sandler could afford this selling price, but they were not sure that it constituted the right value for the property. Mr. Sandler had arranged for a 20-year mortgage at 13 percent with a 40 percent down payment. Considering the cost of the property and the prospect of its continuing drain on Mr. Sandler's resources for the first 5 to 10 years, he was concerned that this be a sound financial decision as well as a choice of a comfortable home.

Bob Patterson was convinced that Blandfield was a good investment for the Sandlers and that, if he could convince them of this, they would buy Blandfield. Bob arranged (through a

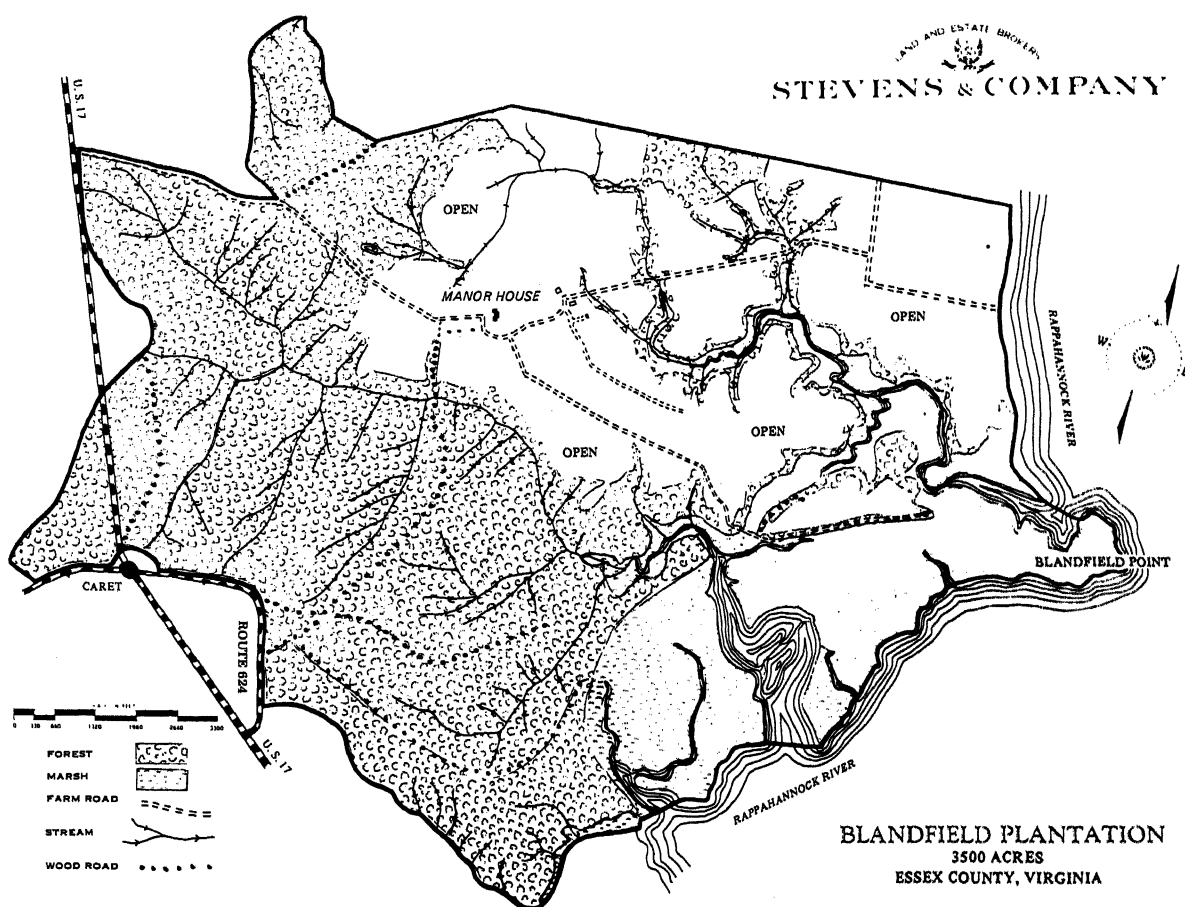



EXHIBIT 4 Map of Blandfield Plantation.

professor he had met taking an executive course in decision analysis) for an MBA student, Steve Condlin, to develop a financial model of the investment. In his second year of the MBA program, Steve did not have a lot of time, but he decided that, by using a new financial modeling language, he could make some headway with this kind of problem without taking a lot of time. He planned to use the work for a required project, anyway.

First Meeting Bob and Steve met with Gordon Sandler over lunch to discuss his concerns about the purchase of Blandfield. Mr. Sandler explained that he had two major objectives for Blandfield, one personal and one business. On the personal side, the estate would serve as Gordon's home. He and his wife had been attracted to it from the start. For many years they had had a dream of retiring to a farm, but Blandfield was much more than they had ever imagined. It provided amenities that did not exist on other farm properties, due to Gordon's love of hunting and fishing. But their interest was not strictly for themselves; Gordon saw this as an opportunity to make Blandfield the Sandler family home for generations to come.

From the business perspective, Gordon's primary objective was for Blandfield to be a wise financial investment, one where he could expect a return comparable to other investment opportunities available to him. A secondary business interest was to use the estate for business meetings and entertaining. Blandfield would serve well as a place to entertain clients and employees of the company. Since he was president-elect of an industry association, he thought

he could also make use of the estate for professional retreats for other members of the association. Finally, since in recent years he had become involved in various political causes, he thought he could use Blandfield to bring officials and congressmen down from Washington for informative retreats and entertainment.

One reason that made Blandfield financially attractive to Mr. Sandler was that he saw it as an opportunity to diversify his investments. To him, Blandfield offered a different type of investment from stock in his company, one that was affected differently by uncertain economic forces. But again, Gordon was not strongly convinced that overall it was less risky than his stock, particularly when he worried that Blandfield might force him to liquidate more of his stock and relinquish control of his company. Gordon left them at the end of lunch with the questions: Is it a good financial investment for me to make? What are the risks associated with buying Blandfield and how big are they compared to just keeping 100 percent interest in my company?

Bob explained the difficulty he was having in developing a way of evaluating a project with these risks and the lengthy time horizon and yet come up with a model that could be explained without giving a course in financial analysis. Steve, eager to use his MBA skills to tackle the impossible and have it done by tomorrow, borrowed the Blandfield file from Bob and promised to call Bob after he had thought about it some more.

Second Meeting Two weeks later, Steve came to talk to Bob Patterson about interim results.

Condlin: Bob, it seems that I've raised more questions than I've answered. Let me explain to you some of the problems I am having. My biggest question is what time horizon to use. I started out planning on a time horizon of 5 to 10 years, a period of time I thought appropriate for Mr. Sandler's personal use of the property. One of the problems with that, however, is that harvesting of timber really doesn't start for 10 years and the value we place on the timber inventory at the horizon is going to have a significant impact on the return from the investment. That was one of the advantages of using a 35-year horizon since this would incorporate a full growing cycle for the timber and the salvage value would not have as great an impact. Another problem I am having is that I don't know how to forecast timber, agriculture, and land prices and these estimates are crucial to what I am going to do.

Patterson: I know what you mean. But that is one of the reasons I wanted you to help. I can give you data on typical land appreciation and changes in farm prices and revenues (Exhibit 4) going back any number of years. And I can direct you to data on growth in prices for wood. However, I am looking for you to tell me the best way to go about building the model itself.

Condlin: Well then, maybe you can help me with this. Clearly buying the estate has uncertainties in it, such as the influence of inflation on timber and agricultural prices and the amount of timber that is produced each year. At the same time there is some uncertainty in the value of a company like Sandler Electronics. The company is affected by inflation too, just like Blandfield, but there is also risk specific to the operation of the company. Sales to the industrial segment, for example, are uncertain. I'm not sure how to model all of these uncertainties.

Patterson: I've got some information here from the Virginia Division of Forestry on the growth of timber. If I remember right it deals with the expected growth of loblolly pine over the life of the tree (see note on Exhibit 2). Isn't there some data or forecasts available that will help you to predict some of the other uncertainties like inflation or stock prices?

Condlin: Well, yes, I guess so. But it is not simply a matter of putting numbers on things. There are some real modeling issues that have to be settled before I write the model. For instance, what variables to make explicit in the model. There are a lot of different items which go into forecasting the operating income for the 500 acres of farmland. I am not sure if I should model only the operating income or whether the analysis would be significantly improved by modeling all of the different components of farm revenue and expenses. Another problem is

EXHIBIT 5
INDEXES OF PRICES RECEIVED AND PRICES PAID BY FARMERS, 1940–1980 (1967 = 100)

Year or month	Prices received by farmers			Prices paid by farmers						Addendum: Average farm real estate value per acre§
					Production items					
	All farm products	Crops	Livestock and products	All commodities, services, interest, taxes, and wage rates*	Total†	Tractors and self-propelled machinery	Fertilizer	Fuels and energy	Wage rates‡	
1940	40	40	40	36	43				15	19
1941	49	48	50	39	45				18	19
1942	64	64	62	44	52				23	21
1943	77	83	72	50	57				31	23
1944	79	88	71	53	60				38	26
1945	83	90	77	56	61				42	29
1946	94	102	88	61	67				46	32
1947	110	117	105	70	78				49	36
1948	115	113	115	76	87				52	39
1949	100	100	99	73	83				51	41
1950	103	103	102	75	86				50	40
1951	121	118	122	82	95				55	46
1952	115	119	111	84	95				59	51
1953	102	107	97	81	89				61	52
1954	98	108	90	81	89				60	51
1955	93	103	85	81	87				61	53
1956	92	104	82	81	87				53	55
1957	94	100	89	84	90				66	58
1958	100	99	99	86	92				68	61
1959	96	98	93	87	93				72	66
1960	95	99	92	88	92				74	68
1961	96	101	91	88	93				76	69
1962	98	103	93	90	94				78	73
1963	97	107	89	91	95				80	77
1964	95	106	86	92	94				82	82
1965	98	103	94	94	96	92	103	98	86	86
1966	106	106	106	99	100	96	102	98	93	93
1967	100	100	100	100	100	100	100	100	100	100
1968	102	100	104	103	100	104	94	101	108	107
1969	107	97	117	108	104	111	87	102	119	113

1970	110	100	118	112	108	116	88	104	128	117
1971	113	108	118	118	113	122	91	107	134	122
1972	125	114	136	125	121	128	94	108	142	132
1973	179	175	183	144	146	137	102	116	155	150
1974	192	224	165	164	166	161	167	159	178	187
1975	185	201	172	180	182	195	217	177	192	213
1976	186	197	177	192	193	217	185	187	210	242
1977	183	192	175	202	200	238	181	202	226	283
1978	210	203	217	219	217	259	180	212	242	308
1979	241	223	257	250	248	289	196	276	265	351
1980	245	239	251	280	275	323	243	380	286	404
1979:										
Jan	233	209	253	235	231	272	179	226	257	
Feb	242	216	265	239	236	272	179	229	257	351
Mar	246	214	274	244	244	280	187	235	257	
Apr	245	213	274	247	247	280	187	246	269	
May	246	220	270	249	248	280	194	256	269	
June	244	234	255	249	248	293	194	270	269	
July	244	238	249	252	251	293	194	285	266	
Aug	238	236	242	251	249	293	194	298	266	
Sept	240	226	254	255	254	302	194	308	266	
Oct	236	224	247	257	256	302	211	314	269	
Nov	238	226	251	258	256	302	211	318	269	379
Dec	239	222	255	260	258	302	222	324	269	
1980										
Jan	236	220	252	269	263	302	222	345	284	
Feb	238	220	255	271	266	302	222	365	284	404
Mar	234	220	247	274	270	317	244	378	284	
Apr	224	217	232	274	268	317	244	384	284	
May	227	223	232	275	268	317	248	385	284	
June	232	226	237	278	270	325	248	387	284	
July	247	242	252	280	273	325	248	388	288	
Aug	256	250	262	283	278	325	248	385	288	
Sept	261	259	263	286	282	337	248	382	289	
Oct	260	259	263	288	284	337	246	383	288	
Nov	264	270	260	290	287	337	246	385	288	
Dec	261	266	258	291	287	337	247	390	288	

*Includes items used for family living, not shown separately.
†Includes other items not shown separately.
‡Seasonally adjusted; annual data are averages of seasonally adjusted data.
§Average for 48 states. Annual data are for March 1 of each year through 1975 and for February 1 beginning 1976. Monthly data are for first of month.
Source: *Economic Report of the President, January 1981.*

expressing the relationship between the variables. Which variables are dependent on which and what is the nature of that relationship? I know inflation influences the value of property and helps to increase the prices that are charged for farm products and timber, but I'm not exactly sure how that relationship works. These are some of the issues we have to resolve before we get to the stage of putting down numbers for variables.

Patterson: Right.

Condlin: And then there is the problem that Sandler should really look at his portfolio, not just the separate pieces in his package of investments. His options are 51 percent of his stock along with Blandfield or keeping 100 percent of his stock. There are some things that affect the risk of both the stock and the estate. If we model the risk explicitly in each piece before combining them, we will be double counting the risk that is common in the two investments.

Patterson: It would seem advisable to analyze the total investment in one model, taking account of the uncertainties explicitly that affect both. But the effects on the two will not necessarily be identical.

Condlin: Now, assuming that I get the risks modeled right, I still should somehow incorporate Sandler's preferences. He told us already he would use a 7 percent discount rate for after-tax proceeds where there is no risk. But what about this situation where there is risk? How do we account for his tolerance for risk?

Patterson: Well, I remember from the decision analysis course some methods for treating attitude toward risk. But I would suggest that whatever you do, you make it understandable to the client. It's easy to get in over one's head.

Condlin: The last problem I'm having is in trying to incorporate some of Mr. Sandler's other objectives into the model. He mentioned that the property has both personal and business uses which really aren't included in the financial analysis we are doing on the estate. A lot of his reasons for buying the property seem subjective and I'm not sure how I would go about measuring these objectives or how I would incorporate subjective measures along with objective measures into the analysis.

Patterson: I think the best we can do right now is to concentrate on the financial aspects of the estate which we can model and then let Mr. Sandler deal with the other issues himself.

Condlin: Well, I guess I've just got to struggle a little more with the puzzle. Let me get some of those sources for land appreciation and growth of wood prices and I'll work some more on this.

Steve Condlin left the office of Stevens and Company with a promise that he would have some kind of model to show Bob Patterson in 10 days. It occurred to him on the way home that he would really rather spend the 10 days relaxing at Blandfield and thinking about his own retirement years. But he would have plenty of time for that later.

Ten days later Bob Patterson was readying his office for the visit of Steve Condlin. Steve had phoned to say that he would give Bob a 30-minute presentation as a dry run before giving it to Gordon Sandler later. While Steve had prepared his model on a mainframe computer, he fully expected to be able to adapt it to Bob's personal computer when it arrived. "This should be interesting," Bob thought to himself.

WILLIAM TAYLOR AND ASSOCIATES (A)*

"The one thing I can say for certain is that I cannot measure the benefits I derive from running my own business in dollars alone." The speaker was William Taylor, owner of Taylor and Associates. He continued, "Many in business used to say that the impact on the bottom line was all that mattered. But today, even big businesses—or at least some of them—seem to be learning what most small businessmen have known all along—that you frequently are unable to measure total benefit derived from an enterprise on the single scale of profits."

Taylor and Associates was currently a single proprietorship. After considering the possibility that objectives other than profit were important to his overall satisfaction, Mr. Taylor wondered whether the current organizational form was the most appropriate. Mr. Taylor's son, Brian, an MBA student with a master's degree in engineering, had often discussed with him the role of conflicting objectives in the evaluation of his company's performance. Mr. Taylor hoped that, with the aid of his son, he could choose the best structure for his company using a more formal analysis, one which accounted for multiple objectives.

"I agreed to go through with this multiattribute process for several reasons. First, any formal process which would help me commit to spending some time on my strategic problems seemed to me to be positive. Secondly, I am an engineer as well as a businessman. For this reason, the formally quantified approach to my problem of which legal form to adopt for my organization has some appeal. I must admit, though, that the business side of me is rather leary of using any magic formula to obtain 'the' answer to such a complex problem. But since my goals and objectives in making this decision were not well formulated, I felt that this process of structuring objectives and measuring alternatives against them would help me to better understand them."

COMPANY HISTORY

Taylor and Associates was a 21-year-old engineering consulting firm with its home office in Lynchburg, Virginia. Though it had operated under other forms of ownership, in 1979 it was a single proprietorship operated by William Taylor. During its existence, the firm had employed design professionals from several disciplines. In recent years, however, the firm had worked primarily in civil engineering, deriving most of its income from work in sanitary engineering design. Its primary market area had included the states of Virginia, Tennessee, and North Carolina.

*Names and locations are disguised at the request of the owner of the firm.

This case was prepared by Michael McEnearney, Research Assistant, and Samuel E. Bodily, Associate Professor, as a basis for class discussion rather than to illustrate effective or ineffective handling of an administrative decision. Copyright 1981 by The Colgate Darden Graduate Business School Sponsors of the University of Virginia.

Operations

The company was a small design firm with a gross income in 1978 of $810,000 and pretax profits of $53,000. The firm had been mildly innovative both in design and in the area of "process" or production. The company's major innovation was its early shift to the use of the computer, having had its own machine installed in 1967. Automated drafting and graphic arts had been used to a much greater extent and started much earlier than even many of the larger engineering firms. Finally, the firm obtained an IBM System 6 word processor in 1978. This processor did for specifications and text in general some of the same things the graphic arts machinery had done for engineering drawing.

Marketing

Taylor and Associates had been heavily involved in the sanitary engineering market segment which, in the 1960s and early 1970s, boomed as a result of the concerns for the environment. It had sought to provide the high level of service which small towns in its market area had needed in dealing with the Environmental Protection Agency and Farmer's Home Administration. Most new clients in this area had come to the firm as a result of its reputation with previous clients.

There had always been a background level of work in other areas of civil engineering—from subdivision work to structural design. However, the degree to which the firm had become primarily involved in sanitary work during the late 1970s is clear from the makeup of the firm's design section. In 1979, all design engineers were civil engineers, and all advanced degrees held were in sanitary engineering. Mr. Taylor had a dedication to the "marketing concept" which said that the best way to serve the needs of the client was through a multidisciplinary approach, but no specific market segments had been targeted. Due to the company's current industry segment and barriers to entry into the other disciplines, such as architecture, the multidisciplinary approach had never caught on.

While little formal strategic market analysis had been done, the efforts of the two men involved in project development had produced some results. In 1979, the two areas in which the most marketing and selling effort was being focused were energy-related work (coal and gas) and the private industrial sector. It was hoped that growth in these new areas would help to counter the effects on the firm of the reduction of demand within the sanitary segment.

Another way in which the company had sought to open markets was based on geographic expansion. The opening of the office in Asheville, North Carolina, had been intended to provide access to North Carolina markets never successfully cracked by the Lynchburg office. As of 1979, staffing in Asheville would not support engineering, but the surveying practice appeared to have established itself there.

Finance

Unlike any other functional area, the area of finance and accounting had been solely under the purview of the owner-operator himself. Mr. Taylor's computerized accounting and control system seemed to give him the type of control that was critical for a design firm. In spite of his otherwise conservative financial stance, Mr. Taylor had chosen to use his line of credit as a means of reducing his mortgages and other long-term finances on a regular basis. This had led to relatively high short-term debt (in comparison with the industry). Though this policy may have added some expense, it apparently had created little danger of loss of short-term financing because of the firm's record with the bank.

New Ventures In 1971 Taylor and Associates was one of the three professional firms involved in Freedom Inc., which began putting together a package which was to become the Freedom Ski development in North Carolina. After most of the planning and early development

work was finished, the company withdrew from the venture in 1976. However, Mr. Taylor thought the enterprise was an effective way to expand the business and wished to pursue other "new ventures" if possible. In 1979 the company had several opportunities in this area.

In 1979 Taylor and Associates purchased over 700 acres of prime land in and around the city of Asheville, North Carolina. The ultimate plan for the property included an executive park and executive housing areas. The likely first phase, however, would be King's Lodge, a high-quality conference center, overlooking the lake site.

Another opportunity available to the company would be to enter the systems analysis and computer consulting business. The firm's in-house IBM computer and word processor would provide a starting point for a move into this expanding field.

Finally the company could seek other land development projects similar to Freedom Inc. (which the firm participated in but did not control). Such projects would make use of the consulting firm's project management and engineering skills.

ORGANIZATIONAL STRUCTURE

The firm was a single proprietorship organized along functional lines (see Exhibit 1), with one thin layer of reasonably strong middle management. Mr. Taylor's hope was to eventually delegate profit and loss responsibility for projects to project managers although, in 1979, this had not yet occurred. While Mr. Taylor recognized he couldn't separate organizational consideration from business strategy, his main purpose now was a study of the organizational arrangement.

Four general conditions existed which had led Mr. Taylor to consider a possible change:

1 The last 2 or 3 years had brought some major changes in the environment. Demand in the sanitary engineering segment was declining while the demand in other areas, such as energy

EXHIBIT 1 Present organization of the firm. (Source: discussion with firm's owner-operator.)

*These individuals are licensed Professional Engineers. The design section includes six graduate engineers, four of whom are licensed Professional Engineers, and three of whom hold advanced technical degrees. Any licensed engineer could serve as a Project Manager, and in some cases, a licensed surveyor might also. A number of people, including the head of the Asheville Office, are registered Land Surveyors.

and computer consulting, was growing rapidly. Mr. Taylor questioned whether the firm was well structured to deal with these changes.

2 The owner-operator wished to be involved in new venture activities. Real estate development and management projects appeared to be particularly attractive. Some of the new ventures, however, might not present a good organizational fit with the present structure. Even though they were conceptually related to the business, they would require functional and managerial skills not present in the existing firm.

3 The goals of the middle management of the existing firm and those of the owner seemed inconsistent with one another. Activities Mr. Taylor would take that were beneficial to himself or to the long-term strength of the company might have a negative effect on the managers' profit-sharing plan.

4 The question of finances was an area of continuing concern. The form of legal organization adopted by the firm would affect its ability to raise capital. Expansion of new ventures would require large amounts of capital and it would be easier for a corporation to raise the funds than for a sole proprietorship. In addition Mr. Taylor could limit his personal financial liability with a corporate organization, whereas with a sole proprietorship he was personally liable for all business debts.

ORGANIZATION STUDY

With Brian's help, Mr. Taylor spent a lot of time drawing up concrete statements of his objectives. Though he covered a lot of ground, focus was slow in coming. The problem centered on the fact that, as in most small businesses organized as single proprietorships, there was no clear distinction between the owner's personal and business objectives. As Mr. Taylor's attempts at structuring his problem continued, it became increasingly clear that he felt the personal objectives of his employees and his objectives for himself and for the business were in conflict with one another—and that he valued all three sets of objectives. Although the structure of the whole problem remained unfocused, Mr. Taylor decided to center hs efforts on these three areas.

Maintaining the Business

Mr. Taylor's primary objective with respect to the business was ensuring its stability and future growth even if, for some unknown reason, he should have to leave the firm. His primary means of providing for the business's continued existence had been to carry sufficient life insurance to cover all debts and taxes. Though the majority of this insurance was carried because of the business the substantial insurance premiums Mr. Taylor paid had to come out of after tax dollars because of the firm's current structure. Reorganizing the business might not only help ensure its future but could also provide an added tax benefit: tax-free insurance premiums (deducted as a business expense) for Mr. Taylor.

Some of the legal aspects of operating a professional organization also concerned Mr. Taylor. The law held a professional responsible for his actions and those of his employees under his supervision. Anyone assuming ownership in a professional firm might assume not only typical business financial risk but professional liability as well. In addition Virginia state law restricted ownership to 25% of a professional firm by persons not registered in that profession. Finally, the nonprofessional owner had to be an employee of the company.

Other areas of the business that concerned Mr. Taylor included strengthening the functional areas, finance, marketing, and particularly new ventures, and strengthening the management and control areas.

Employee Welfare

In addition to insuring the income and job satisfaction of his employees, Mr. Taylor was particularly concerned with the strength of his management and their job satisfaction. For

EXHIBIT 2 ALTERNATE ORGANIZATIONAL ARRANGEMENTS

1. *Status quo*—Mr. Taylor would retain 100 percent ownership and, therefore, full financial responsibility for the company. Mr. Taylor would still maintain full control over the operations and policy decisions. Growth potential for the company and advancement possibilities for the employees would be limited since Mr. Taylor would be forced to restrict the size of the company to keep it manageable.
2. *Single partnership*—Mr. Taylor would enter into a partnership with one other individual, retaining at least 51 percent interest in the company. A single partnership would provide increased job security for the employees (the company no longer relies solely on Mr. Taylor for continuity), increased financial leverage (the company can borrow against both partners, not just Mr. Taylor), and reduced financial liability for Mr. Taylor. Major disadvantages include reduced control over the company's operation for Mr. Taylor, reduced personal income for Mr. Taylor (profits must now be shared with the partner), and reduced personal freedom for Mr. Taylor since the partnership agreement would tie him to the business and limit his ability to change his own working conditions without prior approval of the partner.
3. *Multiple partnerships*—Mr. Taylor considered the possibility of breaking the business up into separate operating entities (i.e., surveying and engineering and various new ventures) and forming a separate partnership for each one. Mr. Taylor would retain at least a 51 percent interest in each new enterprise. The advantages are the same as with a single partnership only to a higher degree with the added advantages of increased professionalism and growth potential due to specialization within each partnership. The major disadvantages include a reduction in Mr. Taylor's ability to control the different businesses and a reduction in the synergistic benefits of operating a single business (i.e., unified marketing and planning).
4. *Professional corporation plus proprietorship*—A professional corporation would be formed to run the engineering and surveying portions of the business, Mr. Taylor would retain at least a 51 percent interest, and the remainder of the business would remain a single proprietorship (i.e., new venture, land development, and nonengineering consulting). The major advantages include increased stability from the employees' point of view, an increase in the professional stature of the business, and an increase in personal freedom for Mr. Taylor since he would be free to concentrate on the parts of the business which interest him the most (the segments of the business kept in the sole proprietorship). The disadvantages include a reduction in synergy (as with multiple partnerships), a possible* reduction in tax benefits to Mr. Taylor (who may pay corporate tax in addition to personal tax), and a reduction in the "entrepreneurial" nature of the business.
5. *Professional corporation plus one or more general corporations*—Establish a professional corporation for engineering and surveying and set up one or more general corporations for the nonprofessional (new ventures, consulting, and land development) aspects of the business. This would combine most of the advantages of the multiple partnerships: growth, professionalism†, and employee satisfaction due to specialization, along with the added protection against personal liability for the financial performance of the company. The major disadvantages would be reduced personal tax benefits to Mr. Taylor (any company income not from subchapter S corporations would be subject to corporate taxes) and reduced control since all of the companies would be publicly owned.
6. *Proprietorship with responsibility centers*—This form of organization is basically the same as the status quo with the exception that employees at the upper and middle levels of management would become more responsible for the successful operation of the company. Mr. Taylor would retain all of the personal benefits and financial liabilities of the sole proprietorship he is currently operating but would increase the job satisfaction of his employees.
7. *Single corporation with employee stock option plan*—A single corporation would be formed with Mr. Taylor retaining at least 51 percent interest in the company. The remainder of the stock would be owned by employees under a stock option plan. This plan incorporates the benefits of incorporating (reduced personal financial, but not professional, liability) with some of the control of a single proprietorship. Mr. Taylor would remain in control of the operations of the business and the direction the company would take. Major disadvantages include a possible* reduction in the personal tax benefits to Mr. Taylor, a reduction in Mr. Taylor's ownership of the company, a limitation on the company's growth potential due to lack of specialization, and a loss of professionalism† that the formation of multiple partnerships or a professional corporation would bring about.

Source: These alternatives were prepared by the owner-operator during the early part of this study.

*The corporation may be a subchapter S corporation, in which case the taxes to Mr. Taylor would not change.

†By Virginia law, a professional corporation is entitled to practice a single professional specialty and no other form of business. Therefore the professional corporation, like a partnership, is perceived by clients as a source of more specialized and expert professionals than a general corporation.

EXHIBIT 3 ORGANIZATIONAL OBJECTIVES

1. Ample income and retirement security for all employees.
2. A sense of position, or of being part of a moderately sized organization, for employees (e.g., president, executive vice president, etc.). This objective is to see that all employees understand their positions within the organization.
3. Some form of cost-center arrangement whereby profit sharing can be related to performance and employees will not feel profits from their efforts are unfairly siphoned off to other activities.
4. Permanence of organization not dependent on my life, but this need be only to the extent that the employees can see that there is a permanence—not necessarily legal permanence of the ownership entity.
5. Opportunity for employee growth and advancement—as each individual chooses—i.e., along purely technical engineering lines or in areas of business and/or management.
6. A sense of providing input and at least some measure of control by senior employees—related to pride of ownership as opposed to mere employee status.
7. Avoidance of irrevocably "giving away the store," at least in the near term.
8. Maintenance of a professional engineering entity as it is generally understood by organizations like Consulting Engineers Council and our professional liability insurance carrier.
9. Limitation of my personal financial liability.
10. Adequate financing for all resultant organizations and activities.
11. Ability to "sell" each phase of the organization as being large and having adequate support facilities (for use in marketing efforts).
12. Maximum tax-shelter benfits for all—myself and employees.
13. Tax-free life insurance premiums.
14. Retention of ultimate policy decision control over all units.
15. Removal of any real or imagined restraints on pursuing "blue-sky" projects.
16. Assurance that ultimate benefits of blue-sky projects accrue primarily to those having capital (or effort) at risk on these projects.
17. Compatibility with King's Lodge development.
18. Compatibility with development of a strong systems analysis and computer consulting capability.
19. Compatibility with pursuit of large land development projects (e.g., Freedom Inc.) with our role being managers as well as engineers and surveyors. This relates primarily to North Carolina.
20. Minimize government interference, particularly IRS, to maximum extent possible.
21. Provide for growth in all phases.
22. Compatibility with development of design management.
23. Conservation and efficient use of resources (single accounting system, centralized management, etc.).

Source: This list of objectives was prepared by the owner-operator during the early part of this study.

several years, the firm had operated a profit-sharing plan which placed a large portion of anything over $50,000 in net income achieved by the firm into investments for the employees' retirement. Geared mainly toward management and senior employees, the amount received by each person was based on relative salary level. Mr. Taylor knew that in order to maintain a strong management group he had to insure not only their income (i.e., profit sharing) but increase their sense of control over the organization.

Owner's Welfare

Mr. Taylor had to admit that although he was concerned with the business and the welfare of his employees, his major concern was his own security and satisfaction. Mr. Taylor wanted to insure not only his current income but wanted some guarantee of security in the future. He was also concerned that any change in the company's organization would not "give away the

store." He expected to remain in control of the company, whatever form it took, and wanted to retain ultimate policy control over all functions. Mr. Taylor did see, however, advantages to limiting his actually "running" the day-to-day operation of the business. Not only could he have more time to spend in areas that interested him (i.e., King's Lodge) but he was anxious to limit his personal financial liability which, under a single proprietorship, was very high. As already mentioned (i.e., life insurance) a change in the organization's structure may also provide personal tax benefits to Mr. Taylor.

After having considered the three major areas in more detail, Mr. Taylor sat back and looked at all his options. The first thing he did was draw up a list of all the organizational arrangements he thought were feasible for his company. Exhibit 2 gives a listing of these alternatives and a brief description of each.

Mr. Taylor still was not satisfied that all of his objectives had been clearly stated. He decided to spend some time at a vacation home where he could isolate himself and spend more time on the problem. When he returned he had a list of 23 objectives (Exhibit 3) which he felt covered all of his goals with respect to any new organizational structure. Mr. Taylor felt there was still room for improvement. Some of the objectives covered more than one of the three major areas of concern and needed to be broken out further. In addition, Mr. Taylor felt that 23 objectives were too many to use. Although he didn't see that any of the objectives could be eliminated, he liked the idea of grouping the objectives into related areas and then dealing with this smaller number of grouped objectives. Then it would be necessary to identify specific attributes and describe how to measure or estimate each one.

The problem of picking a new organization was not a simple one. Mr. Taylor realized, however, that before he could ever consider which organizational structure to pick, he had to place his objectives into a more formalized structure to help him analyze the problem better.

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APPENDIX: SAMPLE SOLUTIONS TO SELECTED EXERCISES

CHAPTER 2

2.1.

a. There is a separate binary variable for each of the bonds. Thus $x_i = \{0 \text{ or } 1\}$ signifies that bond i is {refused or accepted}. It might erroneously be assumed that one discrete variable with values (1, 2, . . . , n) would suffice for choosing among n bonds. However, this does not allow for various combinations of bonds. The number of possible choices from the set of bonds is much greater than n (it is 2^n).

b. Here, one discrete variable with possible values (1, 2, . . . , n) will indicate which applicant is selected.

c. Whether to release the new product is associated with a binary decision variable. The price to charge is a continuous decision variable.

2.2. Consider the problem of selecting a microcomputer. *One* possible solution consists of the following objectives and associated attributes:

Objectives	Attributes
1. Low cost	Cost of purchase
2. Reliable maintenance	2a. On-site maintenance (*)
	2b. Manufacturer's record and ability to stay in business (*)
3. Compatability with other systems	3a. Number of units sold
	3b. Similarity to other systems (*)
	3c. Ability to link to other systems (*)
4. Ease of use	Simplicity of manuals (*)
5. Data protection	Password protection on data (yes/no)
6. Training	Hours of training provided by vendor
7. Vendor support	Committment and knowledge of vendor (*)
8. Software availability	8a. Number of useful programs available now
	8b. Number of useful programs available future
9. Ease of expansion	flexibility of computer (*)

Those marked by an asterisk are estimated by direct preference assessment. That is, they will be subjectively assigned numbers between 0 and 100 with 100 meaning it is the best available and 0 meaning the worst available.

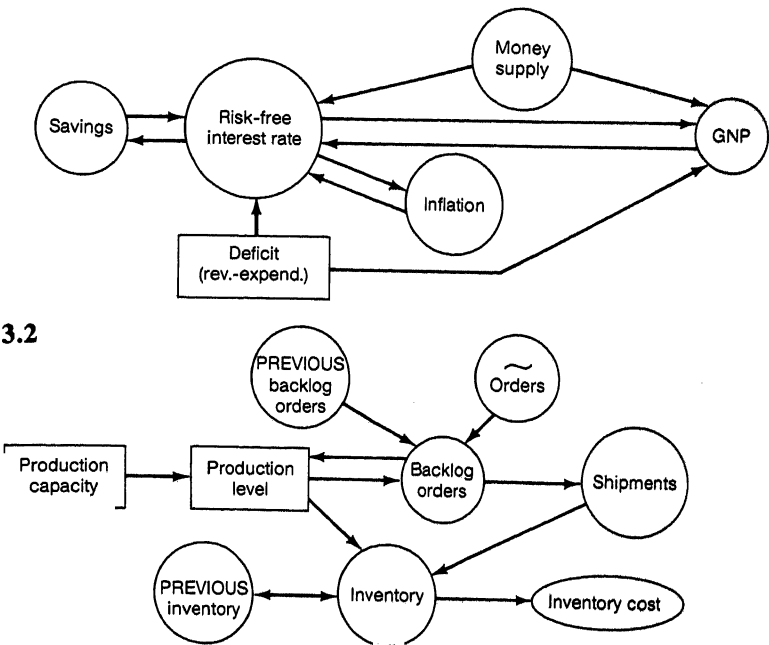
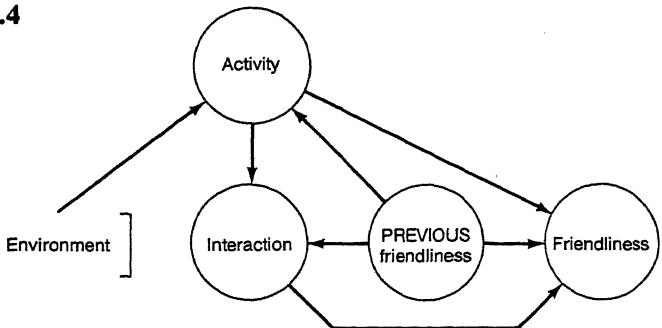
Note that for some of the 10 objectives there are two attributes. These attributes are associated with separate (unstated) subobjectives.

2.4. The identification of variables below would be consistent with a model whose prime use would be to select the number of beds to make available in the hospital.

Decision variable	Intermediate variables	Attribute
Number of Beds	Morbidity Rate (E,R) Length of Hospital Stay (E,R) Occupancy Rate	Number of Patients Rehabilitated

The E and the R signify exogenous and random variables. Morbidity in the population served by the hospital is clearly exogenous to the hospital. It is a matter of judgment to decide to treat it as a random (uncertain) variable. Length of Stay is here labeled exogenous, although it might be argued that the hospital decides how long patients stay, in which case it would be appropriate to call it a decision variable. Occupancy Rate is determined by the other variables. Of course, the attribute (or performance measure) included in the list of variables is Number of Patients Rehabilitated.

CHAPTER 3

3.1. A possible way to relate influences (one provided by an economist specializing in macroeconomics) is given below:

3.4

This diagram implies that activity is determined by demands set by the environment, a decision variable for the MBA situation. The level of activity then influences the level of interaction within the group and both the level of activity and the level of interaction influence the level of friendliness among members of the group. Time dynamics can be brought in through PREVIOUS friendliness, which influences all variables (except environment). Of course many other influence diagrams could be drawn to reflect other perceptions of the problem and other decision situations.

CHAPTER 4

4.1.

a. The mathematical statement could be written

Amount of Usable Product = Initial Amount ($e^{-\text{Spoilage Rate (Time)}}$)

or using the conventions of IFPS as

Amount of Usable Product = Initial Amount * NATEXP(− Spoilage Rate * Time)

b. There is a model called the gravitational model that is often used in transportation engineering that is written

$$\text{Demand for Travel} = \frac{\text{(Population of City 1) (Population of City 2) (Constant)}}{\text{(Distance Apart)}^2}$$

In IFPS, this would appear as

DEMAND FOR TRAVEL = (POPULATION OF CITY 1 * POPULATION OF CITY 2 *'
CONSTANT)/(DISTANCE APART * DISTANCE APART)

where the apostrophe (') signifies continuation of the line. It may be that Distance Apart is taken to a different power, but it still would make a lot of sense for Demand for Travel to be proportional to the product of the populations of the cities.

c. In IFPS terms, it would be easiest to write two expressions:

NUMBER OF BASIC OXYGEN FURNACES = ROUNDUP(CAPACITY NEEDED/B)
SIZE OF PLANT TO BUILD = STEP(NUMBER OF BASIC OXYGEN FURNACES, ...)

The ROUNDUP function accounts for the fact that you would have an integer number of furnaces. The constant B would express the capacity (say in tons per day) of a furnace. The STEP function recognizes that the size of the plant will be taken from a discrete number of possible sizes, rather than a continuous range of possible sizes. Of course those specific size alternatives would need to be placed into the STEP function in order to complete the model.

d. A curve such as that pictured below may be appropriate here. One way to write the function in IFPS is

```
AMOUNT OF PRODUCTION = IF AMOUNT OF EQUIPMENT .GE.'
                       MANPOWER CAPACITY THEN C'
                       ELSE A + B * XPOWERY(AMOUNT OF EQUIPMENT,F)
```

where F is an appropriate fraction (say F = .5).

Alternatively the curve could be sketched and entered using a function such as INTERPOLATION ON in IFPS.

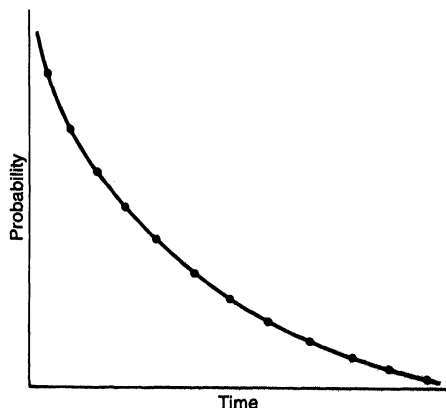
4.2. Breakeven Sales = (Fixed Costs)/(Unit Price–Unit Variable Cost)

4.6. What follows is only one of many possible answers.

a. CONSUMER PRICE INDEX = NORRAND(MEAN, STANDARD DEVIATION)

b. Share of Market = TRIRAND(1/3n,1/n,3/n), assume n > 3

c. Day of first availability = UNIRAND(0, 365)

d. Use the GENRAND option and have the curve declining as below (as a negative exponential):

CHAPTER 6

6.1. The weight and rate analysis would give the following results:

WEIGHTED SCORES FOR EACH ALTERNATIVE

	HAL	MSO	HNS
1. $ cost of system	.7	1.0	.7
2. Manufacturer's commitment	1.0	.1	.6
3. # of units sold	.9	.09	.45
4. On-site maintenance	.875	.875	0.0
5. Ease in understanding manuals	.375	.3	.3
6. Password protected	.075	.75	.075
7. Link to national data	.7	.56	.56
8. Training	.42	.7	.21
9. Vendor commitment	.35	.49	.07
10. Ease of use	.3	.42	.42
11. Software available now	.27	.45	.45
12. Software in future	.4	.5	.12
13. Ease of expansion	.3	0.0	.04
Totals	6.665	5.935	3.905

Thus based on these scores, HAL wins out.

CHAPTER 8

8.3. Because of difference independence, the value function in the two attributes can be written

$$v(c,\$) = w_c v_c(c) + w_\$ v_\$(\$) + w\ w_c v_c(c)\ w_\$ v_\$(\$)$$

where c represents Convenience and $ stands for Cost. Then by scaling the one-dimensional value functions between 0 and 1 over the range for the attributes, the weights are the overall values assigned to the corner points. This gives the weights,

$$w_c = .4$$
$$w_\$ = .3$$

Using these weights the interaction weight can be solved using the expression

$$.3 + .4 + w\ (.3)\ (.4) = 1.0$$

to find w = 2.5. Because the interaction weight is positive, there is complementarity between the attributes.

CHAPTER 9

9.1.

a. Alternative D will lose to any of the other alternatives in a majority vote, therefore it can be compared at any time. B, however, is the only other

contender that would beat C in a paired comparison. Since 2 prefers C, he or she would want to remove B from contention and not compare it to C. There are several ways to do this. One way is to choose to have alternatives A and B compared first to yield A. Then when A is compared to C, C will win.

b. Individuals 1 and 3 may collaborate to vote together on B, even though this is a second-best alternative for 1. Then B will be chosen, which is a better result for both individual 1 and individual 3 than C.

c. Individual 3 benefits most, and therefore may be willing to offer some (legal) side payment to individual 1 to provide some incentive to collaborate.

d. Individual 1 would choose to have a runoff between B and C. Then the winner, which would be B, would be compared to A. Thus A would be chosen. As in a above, D is always a loser and could be compared at any time.

INDEX

MODERN DECISION MAKING

A GUIDE TO MODELING WITH DECISION SUPPORT SYSTEMS